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Brief paper

Stochastic self-triggered model predictive control for linear systems with probabilistic constraints[☆]

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ABSTRACT

A stochastic self-triggered model predictive control (SSMPC) algorithm is proposed for linear systems subject to exogenous disturbances and probabilistic constraints. The main idea behind the self-triggered framework is that at each sampling instant, an optimization problem is solved to determine both the next sampling instant and the control inputs to be applied between the two sampling instants. Although the self-triggered implementation achieves communication reduction, the control commands are necessarily applied in open-loop between sampling instants. To guarantee probabilistic constraint satisfaction, necessary and sufficient conditions are derived on the nominal systems by using the information on the distribution of the disturbances explicitly. Moreover, based on a tailored terminal set, a multi-step open-loop MPC optimization problem with infinite prediction horizon is transformed into a tractable quadratic programming problem with guaranteed recursive feasibility. The closed-loop system is shown to be stable. Numerical examples illustrate the efficacy of the proposed scheme in terms of performance, constraint satisfaction, and reduction of both control updates and communications with a conventional time-triggered scheme.

1. Introduction

Networked control systems are usually subject to constraints and uncertainties. The constraints include not only the traditional system constraints, such as state constraints, but also communication constraints, such as a limited bandwidth in wireless communication networks. For such systems, an integrative model predictive control (MPC) and event-based control approach is a natural idea which could ensure the system constraint satisfaction and trade off the performance of control systems and the usage of communication resources. Thus, the research of event-based MPC is of great interest.

Two specific types of event-based control are event-triggered and self-triggered control. Different from event-triggered control which requires the continuous monitoring of system states, self-triggered control determines the next update time in advance based on the information at the current sampling instant. Also, self-triggered control allows the shut-down of the sensors between two updates, resulting in a lower sampling frequency to prolong the lifespan of sensors powered by batteries. Please refer to Heemels, Johansson, and Tabuada (2012) and Hetel et al. (2017) for an overview of event-based control.

This paper considers a self-triggered implementation of stochastic MPC (SMPC) for linear systems with stochastic disturbances. One main feature of SMPC is the presence of probabilistic constraints, which require the constraints to be satisfied with given probability thresholds. Such constraints can mitigate the conservativeness introduced by hard constraints of robust MPC (RMPC). SMPC has found applications in diverse fields, e.g., building climate control (Long, Liu, Xie, & Johansson, 2014) or chemical processes (Qin & Badgwell, 2003). To the best of our knowledge, stochastic self-triggered MPC (SSMPC) has not been explored up to now. One remarkable challenge is how to characterize the 'propagation' of uncertainties during two sampling instants and formulate a computationally tractable optimization problem for determining sampling instants and control design.

Some developments of self-triggered MPC are available. Many of these results are proposed for systems without uncertainties (Barradas Berglind, Gommans, & Heemels, 2012; Hashimoto, Adachi, & Dimarogonas, 2017; Henriksson, Quevedo, Sandberg,

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& Johansson, 2012). For systems with uncertainties, most results account for the synthesis of self-triggered control and RMPC which aims to guarantee robust constraint satisfaction. The interested reader can refer to Aydiner, Brunner, and Heemels (2015) and Brunner, Heemels, and Allgöwer (2014, 2016). By maximizing the inter-sampling time subject to constraints on the cost function, a robust self-triggered MPC (RSMPC) algorithm is presented for constrained linear systems with bounded additive disturbances in Brunner et al. (2014), which employs the robust Tube MPC method in Mayne, Seron, and Raković (2005) to guarantee constraint satisfaction. In Brunner et al. (2014), all constraint parameters are determined by fixing the maximal inter-sampling time, which has the drawback of leading to a conservative region of attraction. To alleviate the conservatism, a RSMPC algorithm based on a more advanced Tube method (Raković, Kouvaritakis, Findeisen, & Cannon, 2012) is proposed in Aydiner et al. (2015), where the cost function is defined depending on the length of the inter-sampling time such that the constraint parameters are not affected by the maximal sampling interval. By combining with the self-triggering mechanism in Aydiner et al. (2015), a recent RSMPC method is presented in Brunner et al. (2016) with the focus of extending the Tube method in Chisci, Rossiter, and Zappa (2001) to evaluate the effect of the uncertainty on the prediction of the self-triggered setup.

Inspired by Aydiner et al. (2015) and Brunner et al. (2016), we design a self-triggered strategy for SMPC. Notice that inherent differences between SMPC and RMPC make our SSMPC algorithm largely different from the ones presented in Aydiner et al. (2015) and Brunner et al. (2016). Following the ideas of Tube MPC (Kouvaritakis, Cannon, Raković, & Cheng, 2010), we construct stochastic tubes as tight as possible by explicitly using the distributions of the disturbances. Since a crucial assumption of feedback at every time step in Kouvaritakis et al. (2010) is not satisfied in the self-triggered setting (which allows open-loop operations between sampling instants), some appropriate and non-trivial modifications are needed: (i) by considering the multi-step open-loop operation between control updates, three predicted controllers are defined for different phases of the prediction horizon, making it more complex than (Kouvaritakis et al., 2010) to evaluate the effect of the uncertainty on predictions and construct equivalent deterministic constraints; (ii) the inter-sampling time as an optimizing variable is included in the cost function and a tuning parameter is introduced to provide a trade-off between performance and communication; (iii) an improved terminal set, which is adapted to different inter-sampling times, is designed to make the constraints recursively feasible.

The present paper is the first work on SSMPC, which extends the existing literatures on MPC considerably. The main contributions are summarized in the following. (i) Our joint design of the self-triggering mechanism and the SMPC controller effectively reduces the amount of communication, while guaranteeing control performance with specific level of trade-off. (ii) The MPC optimization problem is transformed into a tractable quadratic programming problem by using information on the disturbance distribution. (iii) For the self-triggering mechanism, the probability of constraint violation can be tight to the specified limit. (iv) Both recursive feasibility and closed-loop stability are guaranteed. To illustrate the effectiveness of the algorithm, numerical experiments are carried out to compare the proposed SSMPC with a periodically-triggered SMPC (PSMPC), RSMPC, and unconstrained MPC (LQR).

The remainder of this paper is structured as follows. Problem formulation is set up in Section 2. In Section 3, a multi-step open-loop MPC optimization problem is formulated incorporating probabilistic constraints and specific terminal sets. In Section 4, a SSMPC algorithm is developed and main results are established. Section 5 presents numerical simulations and Section 6 concludes.

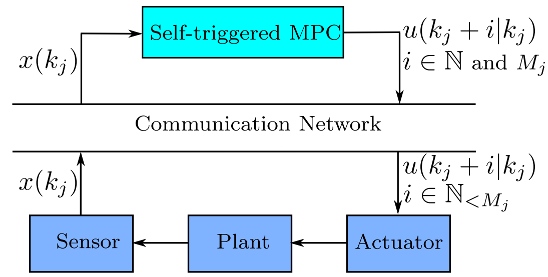

Fig. 1. The self-triggered MPC framework.

Notation 1.1. *Let $\mathbb{N} \triangleq \{0, 1, \ldots\}$. For some $q, s \in \mathbb{N}$ and $q < s$, let $\mathbb{N}_{\geq q}$, $\mathbb{N}_{>q}$, $\mathbb{N}_{\leq q}$, $\mathbb{N}_{<q}$, and $\mathbb{N}_{[q,s]}$ denote the sets $\{r \in \mathbb{N} \mid r \geq q\}$, $\{r \in \mathbb{N} \mid r > q\}$, $\{r \in \mathbb{N} \mid r \leq q\}$, $\{r \in \mathbb{N} \mid r < q\}$, and $\{r \in \mathbb{N} \mid q \leq r \leq s\}$, respectively. Let I and $\mathbf{0}$ denote an identity matrix and a zero matrix or zero vector of appropriate dimension. When $\leq, \geq, <, >$, and $|\cdot|$ are applied to vectors, they are interpreted element-wise. For $W \in \mathbb{R}^{n\times n}$, $W \succ 0$ means that W is symmetric and positive definite. For $x \in \mathbb{R}^n$ and $W \succ 0$, $\|x\|_W^2 \triangleq x^T W x$. For $x_i \in \mathbb{R}^n$, $i \in \mathbb{N}$, define $\sum_{i=a}^{b} x_i = \mathbf{0}$ if $a > b$. Pr denotes the probability, $\mathbb{E}$ the expectation, $\mathbb{E}_k$ the conditional expectation of a random variable given the state at time k, and $(k+i|k)$ a prediction of a variable i steps ahead from time k.*

2. Problem formulation

The self-triggered MPC framework of this paper is shown in Fig. 1, in which the notations are introduced below. Consider a linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k) + w(k),\ \ k \in \mathbb{N}, \tag{1}$$

where $x(k) \in \mathbb{R}^{N_x}$ is the state, $u(k) \in \mathbb{R}^{N_u}$ the control input, $w(k) \in \mathbb{R}^{N_w}$ the stochastic disturbance, and (A, B) a stabilizable pair. Notice that $N_x = N_w$. Assume that $w(k)$, $k \in \mathbb{N}$, are independent and identically distributed (i.i.d.) and the elements of $w(k)$ have zero mean. The distribution F_i of the ith element of $w(k)$ is assumed to be known and continuous with a bounded support $[-\sigma_i, \sigma_i]$, $\sigma_i > 0$, and correspondingly we have $w(k) \in \mathcal{W} \triangleq \{w \mid |w| \leq \sigma\}$, $\sigma = [\sigma_1\, \sigma_2 \ldots \sigma_{N_w}]^T$. Moreover, system (1) is subject to n_c probabilistic constraints $Pr\{g_\ell^T x(k) \leq h_\ell\} \geq p_\ell$, $\ell \in \mathbb{N}_{[1,n_c]}$, $k \in \mathbb{N}$, where $g_\ell \in \mathbb{R}^{N_x}$, $h_\ell \in \mathbb{R}$, and $p_\ell \in [0, 1]$. In the sequel, we will focus on one probabilistic constraint

$$Pr\{g^T x(k) \leq h\} \geq p,\ \ k \in \mathbb{N}, \tag{2}$$

as the other constraints can be treated in a similar way.

In a periodically-triggered MPC scheme, the predictive control input at time k can be designed as

$$u(k+i|k) = Kx(k+i|k) + c(k+i|k),\ \ i \in \mathbb{N}, \tag{3}$$

where $K \in \mathbb{R}^{N_u\times N_x}$ is chosen offline such that the matrix $\Phi \triangleq A + BK$ is Schur stable and for a prediction horizon $N \in \mathbb{N}_{\geq 1}$, $c(k+i|k)$ for $i \in \mathbb{N}_{\leq N-1}$ are optimization variables and $c(k+i|k) = \mathbf{0}$ for $i \in \mathbb{N}_{\geq N}$. At each time instant k, $u(k) = Kx(k) + c(k|k)$ is applied to the system.

To reduce the amount of communication, in the self-triggered scheme, the states $x(k)$ are only measured and transmitted to the controller at sampling instants $k_j \in \mathbb{N}$, $j \in \mathbb{N}$, which evolve as $k_{j+1} = k_j + M_j$ with $k_0 = 0$. The inter-sampling time $M_j \in \mathbb{N}_{[1,N-1]}$ is determined by a self-triggering mechanism based on the state at sampling instant k_j. Since the values of $x(k_j + i|k_j)$, $i \in \mathbb{N}_{[1,M_j-1]}$, cannot be determined at time k_j in the presence of