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Brief paper

Rejection of sinusoidal disturbances for known LTI systems in the presence of output delay☆

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ABSTRACT

This paper focuses on estimation and cancellation of unknown sinusoidal disturbances in a known LTI system with the presence of a known output delay. Parametrizing the disturbance and representing the delay as a transport PDE, the problem is converted to an adaptive control problem for ODE–PDE cascade. An existing state observer is used to estimate the ODE system states. The exponential stability of the equilibrium of the closed-loop and error system is proved. The perfect estimation of the disturbance and state is shown. Moreover, the convergence of the state to zero as $t \to \infty$ is achieved in the closed loop system. The effectiveness of the controller is demonstrated in a numerical simulation.

1. Introduction

Cancellation of sinusoidal disturbances has been among difficult challenges faced by control engineers, with numerous applications such as active suspension systems (Basturk, 2016), active noise control (Bodson, Jensen, & Douglas, 2001) and marine vessels (Basturk & Krstic, 2013). A common method to address this problem is to model the disturbance as the output of a linear dynamic system which is called an exosystem. Including the exosystem in the feedback loop, disturbance effect can be compensated in the plant response. This method is known as internal model principle (Francis & Wonham, 1975).

The problem of disturbance rejection for linear systems by output feedback is studied in Kim and Shim (2015) and Marino and Tomei (2013). Authors assume that the linear system is known but the disturbance is the output of an uncertain exosystem. However, the Refs. Kim and Shim (2015) and Marino and Tomei (2013) consider no delay in output channels while designing the controllers.

Since time delay is a common phenomenon observed in most real-world applications, the studies have focused on developing control methods in which delays arise. Adaptive control design techniques for systems with unknown ODE parameters and input delay are given in Evesque, Annaswamy, Niculescu, and Dowling (2003) and Niculescu and Annaswamy (2003). The problem of adaptive stabilization is solved for the systems with unknown parameters and distributed input delay in Bekiaris-Liberis, Jankovic, and Krstic (2013). The idea of representing time delay as dynamic of PDE is introduced in Xu, Yung, and Li (2006). Inspiring by Xu et al. (2006), an adaptive observer for PDEs is developed in Krstic and Smyshlyaev (2008) with a backstepping like design technique to compensate a delay.

The cancellation of sinusoidal disturbance for known and unknown LTI systems with input delay is studied in Basturk and Krstic (2015), Pyrkin and Bobtsov (2016) and Pyrkin et al. (2015), respectively. Moreover, the case where the delay appears in the state is considered in Basturk (2017). The output regulation problem is addressed in Kerschbaum and Deutscher (2016), Li, Tang, Zhang, and Zou (2012), Tang and Li (2008), Yu and Wang (2015) and Zhang, Tang, and Zhang (2010), for output-delayed known linear systems. An observer design for output delayed systems with model parameter uncertainty is given in Ahmed-Ali, Giri, Krstic, and Lamnabhi-Lagarrigue (2016). Moreover, for known linear systems with simultaneous state, input and output delay, disturbance cancellation algorithms are proposed in Lu and Huang (2014) and Yoon and Lin (2016). However, the studies (Kerschbaum & Deutscher, 2016; Li et al., 2012; Lu & Huang, 2014; Tang & Li, 2008; Yoon & Lin, 2016; Yu & Wang, 2015; Zhang et al., 2010) assume that exosystem is known and can be used in the controller. To the best of our knowledge, no attempt has been made to reject the disturbance, which is the output of unknown exosystem, in LTI systems with output delay.

The problem that we consider in this paper is the combination of disturbance cancellation by output feedback and delay in the

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measurement. Contrary to Kerschbaum and Deutscher (2016), Li et al. (2012), Lu and Huang (2014), Tang and Li (2008), Yu and Wang (2015), Yoon and Lin (2016) and Zhang et al. (2010), the unknown disturbance in our system is generated by an uncertain exosystem. Our main contribution is to solve this type of a problem by combining two methods. We first use the technique given in Nikiforov (2004) to express the disturbance in a parametrized form and then, employ an adaptive observer proposed in Ahmed-Ali et al. (2016). In addition to this, by employing the perfect estimation of the disturbance and the state, we design an adaptive controller that rejects the disturbance and makes the equilibrium of the closed-loop system exponentially stable.

The paper is organized as follows. In Section 2, the problem definition is stated. The disturbance representation and disturbance parametrization are given in Sections 3 and 4, respectively. In Section 5, the controller design and stability theorem are presented. In Section 6, the proof of stability theorem is given. Finally, an example simulation is illustrated in Section 7.

Notation. Throughout the paper, we use the following notations; B_i is a column vector whose ith element is 1 and the rest is 0, state/parameter estimation and estimation errors are denoted with the symbols "$\hat{}$" and "$\tilde{}$", respectively. As an example, estimation error of X state is $\tilde{\boldsymbol{X}} = \hat{\boldsymbol{X}} - \boldsymbol{X}$ where $\hat{X}$ is the estimation of X. We use subscript i for ith scalar element in general, however I_i and 0_i denote $i \times i$ identity matrix and $i \times 1$ column zero vector, respectively. The Euclidean norm is denoted by $|\cdot|$. We use ∂_t and ∂_x to denote time and spatial derivatives of a function respectively.

2. Problem statement

We consider the single-input single-output LTI system

$$\dot{X}(t) = AX(t) + B\big(U(t) + \nu(t)\big), \tag{1}$$

$$Y(t) = CX(t - D), \tag{2}$$

where $D \in \mathbb{R}$ is the known delay, $X = [X_1, \ldots, X_n]^T \in \mathbb{R}^n$, $U(t) \in \mathbb{R}$ is the input and

$$A = \begin{bmatrix} -a_{n-1} & \\ \vdots & I_{n-1} \\ \vdots & \\ -a_0 & 0_{n-1}^T \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \tag{3}$$

with $0_{n-1} = [0, \ldots, 0]^T \in \mathbb{R}^{n-1}$. The particular form of A can always be achieved if (C, A) is observable. The unknown sinusoidal disturbance $\nu(t) \in \mathbb{R}$ is given by

$$\nu(t) = d + \sum_{i=1}^{q} g_i \sin(w_i t + \phi_i) \tag{4}$$

where $d, g_i, w_i, \phi_i \in \mathbb{R}$ are unknown.

The following PDE can represent the output Eq. (2) as

$$\partial_t y(x, t) = \partial_x y(x, t), \quad x \in [0, D] \tag{5}$$

$$y(D, t) = CX(t), \tag{6}$$

$$Y(t) = y(0, t). \tag{7}$$

The solution of the transport PDE is given by $y(x, t) = CX(t+x-D)$.

The sinusoidal disturbance $\nu(t)$ can be represented as the output of a linear exosystem,

$$\dot{W}(t) = SW(t), \tag{8}$$

$$\nu(t) = h_\nu^T W(t), \tag{9}$$

where the state $W(t) \in \mathbb{R}^{2q+1}$. The matrix S comprises the unknown frequency of the sinusoidal disturbance $\nu(t)$. Constant bias term d, amplitude g_i and phase ϕ_i are determined by initial condition of (8), are thus unknown. Without loss of generality, one can choose output vector h_ν^T such that (h_ν^T, S) becomes observable pair.

The disturbance $\nu(t)$ is not measured. The output $Y(t)$ is the only available measurement. Regarding the plant (1)–(2) and the exosystem (8)–(9), we make the following assumptions:

Assumption 1. The frequencies of the disturbance are distinct, $\omega_i \neq \omega_j$ for $i \neq j$, and the number of the distinct frequencies q is known.

Assumption 2. The bias $d \neq 0$ and amplitude $g_i \neq 0$ for all $i \in \{1, \ldots, q\}$.

Our ultimate goal is to design an observer achieving accurate online estimation of state $X(t)$ as well as the disturbance $\nu(t)$. Using the observer states, we design a controller stabilizing the equilibrium of the closed loop system. Moreover, we aim the state $X(t)$ to converge to zero as $t \to \infty$ in the presence of the output delay and unmeasured sinusoidal disturbance.

3. Disturbance representation

Our main interest here is a preparation for disturbance observer design which is presented in the next section. Firstly, we employ a filter introduced in Krstic and Smyshlyaev (2008) for systems under no disturbance effect. However, because of the unknown disturbance in our system, we show that the error between the system states and the filter states is driven by unknown sinusoidal terms. Main motivation of obtaining this error is to use it in disturbance representation and then, disturbance parametrization.

Inspiring (Krstic & Smyshlyaev, 2008), we propose the following filter

$$\dot{\hat{X}}_d(t) = A\hat{X}_d(t) + BU(t) + e^{AD}L(Y(t) - \hat{y}_d(0, t)), \tag{10}$$

$$\partial_t \hat{y}_d(x, t) = \partial_x \hat{y}_d(x, t) + Ce^{Ax}L\big(Y(t) - \hat{y}_d(0, t)\big), \tag{11}$$

$$\hat{y}_d(D, t) = C\hat{X}_d(t), \tag{12}$$

where L is chosen such that $A - LC$ is Hurwitz. Since the pair (A, C) is observable, there exists an L such that this condition is satisfied. The error is given as follows,

$$\tilde{X}_d(t) = \hat{X}_d(t) - X(t), \tag{13}$$

$$\dot{\tilde{X}}_d(t) = A\tilde{X}_d - e^{AD}L\tilde{y}_d(0, t) - B\nu(t), \tag{14}$$

$$\partial_t \tilde{y}_d(x, t) = \partial_x \tilde{y}_d(x, t) - Ce^{Ax}L\tilde{y}_d(0, t), \tag{15}$$

$$\partial_t \tilde{y}_d(D, t) = C\tilde{X}_d(t). \tag{16}$$

The following transformation

$$\tilde{w}(x, t) = \tilde{y}_d(x, t) - Ce^{A(x-D)}\tilde{X}_d(t) \tag{17}$$

transforms (13), (14) into the form of

$$\dot{\tilde{X}}_d(t) = A_{aug}\tilde{X}_d(t) - e^{AD}L\tilde{w}(0, t) - B\nu(t), \tag{18}$$

$$\partial_t \tilde{w}(x, t) = \partial_x \tilde{w}(x, t) + Ce^{A(x-D)}B\nu(t), \tag{19}$$

$$\tilde{w}(D, t) = 0, \tag{20}$$

where

$$A_{aug} = A - e^{AD}LCe^{-AD}. \tag{21}$$

By using similarity transformation e^{AD} and noting that $A - LC$ is Hurwitz, it can be proved that A_{aug} is Hurwitz.

If there is no disturbance in the system as it is shown in Krstic and Smyshlyaev (2008), the error $\tilde{X}_d(t)$ converges to 0 as $t \to \infty$. However, $\tilde{X}_d(t)$ is driven by $\nu(t)$ and $\tilde{w}(0, t)$ as seen in (18). In Lemma 1, we show that $\tilde{w}(0, t)$ can be expressed as a sum of sinusoidal signals whose frequencies are same as $\nu(t)$.