Automatica 92 (2018) 49–55

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Low order stabilizing controllers for a class of distributed parameter systems[☆]

Hideki Sano

Department of Applied Mathematics, Graduate School of System Informatics, Kobe University, 1-1 Rokkodai, Nada, Kobe 657-8501, Japan

ARTICLE INFO

Article history:
Received 25 April 2017
Received in revised form 11 November 2017
Accepted 18 December 2017

Keywords:
Distributed parameter system
Finite-dimensional controller
Residual mode filter
Stability radius
Semigroup

ABSTRACT

This paper is concerned with reduction of the order of finite-dimensional stabilizing controllers for a class of distributed parameter systems. Since the middle of the 1980s, the design method of finite-dimensional stabilizing controllers of Sakawa type has been generalized for a wider class of parabolic distributed parameter systems with boundary control and/or boundary observation. The controller of Sakawa type consists of two kinds of observers: one is an observer of Luenberger type and the other is an estimator for residual modes. Especially, the latter is called residual mode filter (RMF), and it plays an essential role in the design of finite-dimensional stabilizing controllers when the order of RMF is "sufficiently large". The purpose of this paper is to propose the design method containing low order RMF. An approach based on stability radius is employed.

1. Introduction

In the control theory of distributed parameter systems, the system described by the following evolution equation with output equation has been used for a long time.

$$\dot{z}(t) = -Az(t) + Bu(t), \quad t > 0, \quad z(0) = z_0, \tag{1}$$

$$y(t) = Cz(t), \quad t > 0, \tag{2}$$

where $-A$ is the infinitesimal generator of a C_0-semigroup on a real Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. $B : \mathbf{R}^m \to H$ and $C : H \to \mathbf{R}^p$ are bounded linear operators. $z(t) \in H$ is the state variable, $u(t) \in \mathbf{R}^m$ the input variable, and $y(t) \in \mathbf{R}^p$ the output variable. For systems (1)–(2), the stabilization problem by finite-dimensional controllers have been investigated by many researchers (see e.g. Balas, 1988; Curtain, 1984; Curtain, 2003; Curtain & Zwart, 1995; El Jai & Pritchard, 1988; Fuentes & Balas, 1999; Ito, 1990; Nambu, 1985; Sakawa, 1983; Sano & Kunimatsu, 1994; Schumacher, 1983 and the references therein). Generally, when one constructs a finite-dimensional model for an infinite-dimensional system and applies a finite-dimensional controller to the original infinite-dimensional system, spillover phenomenon may occur due to the influence of unmodeled modes. Sakawa first introduced two kinds of finite-dimensional observers for linear diffusion systems to reduce the influence of unmodeled modes for the closed-loop system with the finite-dimensional controller (Sakawa, 1983). Then, Balas called one of them the residual mode filter (RMF), and clarified that the RMF plays an essential role for the construction of finite-dimensional stabilizing controllers (Balas, 1988).[1] Furthermore, Sano and Kunimatsu showed that the method could be extended to infinite-dimensional systems with A^γ-bounded output operators (Sano & Kunimatsu, 1994). In those papers, by choosing the order of the RMF "sufficiently large", the closed-loop stability was assured. Independently of Sakawa's work (Sakawa, 1983), Curtain gave a design method for finite-dimensional stabilizing controllers for linear parabolic systems with unbounded control and observation (Curtain, 1984), in which Schumacher's design method (Schumacher, 1983) for the case with bounded control and observation was extended to the unbounded case. Since there was no upper bound on the order of controller in both works (Curtain, 1984; Schumacher, 1983), they used the perturbation result of Weinstein–Aronszajn determinant (Kato, 1966) to make the controller design feasible. After that, Fuentes and Balas applied the perturbation theory of operators to obtain the lowest order of RMF (Fuentes & Balas, 1999). Also, in Curtain (2003), the method of LQG-balancing was developed for model reduction of a class of infinite-dimensional systems, and the method was successfully applied to construct robust controllers.

☆ This research is supported by KAKENHI (Grant-in-Aid for Scientific Research (C), No. 15K04999), Japan Society for the Promotion of Science. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Denis Dochain under the direction of Editor Miroslav Krstic.

E-mail address: sano@crystal.kobe-u.ac.jp.

[1] For nonlinear distributed parameter systems, Balas also introduced nonlinear RMFs to construct finite-dimensional stabilizing controllers (Balas, 1991).

https://doi.org/10.1016/j.automatica.2018.02.013
0005-1098/

In this paper, we consider the problem of reducing the order of RMFs in finite-dimensional controllers of Sakawa type, under the assumption that the eigenvalues and eigenfunctions of the state operator are completely known. As a technical tool, we use stability radius theory (Chicone & Latushkin, 1999; Pritchard & Townley, 1989), and the approach is different from that of Fuentes and Balas (1999). First of all, we survey the Sakawa's design method (Sakawa, 1983) and then give the modified version using the stability radius. But, to calculate stability radius, we need the value of H_∞-norm of a transfer function whose realization is described by infinite-dimensional operators in a Hilbert space. From the computational point of view, we need to prepare a family of approximate finite-dimensional operators and then to calculate the H_∞-norm of their transfer functions. However, it is not assured that they converge to the value of H_∞-norm of the original transfer function. The purpose of this paper is to justify the convergence and to propose an algorithm to reduce the order of RMFs. In addition, the case where the bounded output operator is replaced by an A^γ-bounded output operator is discussed. Finally, we give a numerical example to demonstrate the validity of the theory.

2. Sakawa's design method and its modification

2.1. System description

To explain the existing result (Balas, 1988; Sakawa, 1983) briefly for system (1)–(2), we consider the case where the operator A is defined by

$$Af = \sum_{i=1}^{\infty} \lambda_i \langle f, \phi_i \rangle \phi_i, \quad f \in D(A),$$

$$D(A) = \left\{ f \in H \,;\, \sum_{i=1}^{\infty} \lambda_i^2 \langle f, \phi_i \rangle^2 < +\infty \right\}, \tag{3}$$

where $\{\lambda_i, i \geq 1\}$ is a sequence of real numbers such that $\lambda_1 < \lambda_2 < \cdots < \lambda_i < \cdots$, $\lim_{i\to\infty}\lambda_i = \infty$, and $\{\phi_i, i \geq 1\}$ is a complete orthogonal system in H. From the definition, it is clear that the operator A is self-adjoint on H. By using Hille–Yosida's theorem (Engel & Nagel, 2000; Yosida, 1980), we see that $-A$ generates the C_0-semigroup e^{-tA} whose expression is given by $e^{-tA}f = \sum_{i=1}^{\infty} e^{-\lambda_i t} \langle f, \phi_i \rangle \phi_i$, $t \geq 0, f \in H$.

2.2. Partitioned system

In order to derive a finite-dimensional model for system (1)–(2), we use the orthogonal projection P_k defined by $P_k f = \sum_{i=1}^{k} \langle f, \phi_i \rangle \phi_i$. Here, using the operators P_l and P_n $(n > l)$, we decompose system (1)–(2) as follows: First, we decompose the state variable $z(t)$ as $z(t) = z_1(t) + z_2(t) + z_3(t)$, where $z_1(t) := P_l z(t)$, $z_2(t) := (P_n - P_l)z(t)$, $z_3(t) := (I - P_n)z(t)$. Then, the state space H has the expression

$$H = \overbrace{P_l H}^{\dim = l} \oplus \overbrace{(P_n - P_l)H}^{\dim = n-l} \oplus \overbrace{(I - P_n)H}^{\dim = \infty}.$$

Accordingly, system (1)–(2) is expressed as follows (e.g. Balas, 1988):

$$\begin{cases} \dot{z}_1(t) = -A_1 z_1(t) + B_1 u(t), & z_1(0) = P_l z_0, \\ \dot{z}_2(t) = -A_2 z_2(t) + B_2 u(t), & z_2(0) = (P_n - P_l) z_0, \\ \dot{z}_3(t) = -A_3 z_3(t) + B_3 u(t), & z_3(0) = (I - P_n) z_0, \\ y(t) = C_1 z_1(t) + C_2 z_2(t) + C_3 z_3(t), \end{cases} \tag{4}$$

where $A_1 := P_l A P_l$, $B_1 := P_l B$, $C_1 := C P_l$, $A_2 := (P_n - P_l)A(P_n - P_l)$, $B_2 := (P_n - P_l)B$, $C_2 := C(P_n - P_l)$, $A_3 := (I - P_n)A(I - P_n)$, $B_3 := (I - P_n)B$, $C_3 := C(I - P_n)$. In the above, the operator A_3 is unbounded, whereas all the other operators are bounded.[2]

Hereafter, we identify the finite-dimensional Hilbert space $P_l H$ with the Euclidean space $\mathbf{R}^l$ with respect to the basis $\{\phi_1, \phi_2, \ldots, \phi_l\}$. In this way, each element in $P_l H$ is identified with an l-dimensional vector, and the operators A_1, B_1, and C_1 are identified with matrices with appropriate size. Similarly, each element in $(P_n - P_l)H$ is identified with an $(n - l)$-dimensional vector, and the operators A_2, B_2, and C_2 are identified with matrices with appropriate size.

2.3. Finite-dimensional controllers with RMFs

For the decomposed system (4), we consider the finite-dimensional system

$$\begin{cases} \dot{z}_1(t) = -A_1 z_1(t) + B_1 u(t), \\ \eta(t) = C_1 z_1(t), \end{cases} \tag{5}$$

as a finite-dimensional model of system (1)–(2). For the model, we set the following assumption.

Assumption 1. (i) The integer $l (\geq 1)$ is chosen such that the eigenvalues of the matrix $-A_1$, $\sigma(-A_1)$ contains all unstable eigenvalues of the operator $-A$. (ii) The pair $(-A_1, B_1)$ is controllable and the pair $(C_1, -A_1)$ is observable (see e.g. Zhou, Doyle, & Glover, 1997 for the definitions and the related theorems).

Remark 1. The second assumption (ii) can be relaxed as (ii′) The pair $(-A_1, B_1)$ is stabilizable and the pair $(C_1, -A_1)$ is detectable.

Under (ii) of Assumption 1 (or (ii′) of Remark 1), we can choose a matrix F_1 such that $-A_1 - B_1 F_1$ is Hurwitz, and we can choose a matrix G_1 such that $-A_1 - G_1 C_1$ is Hurwitz (e.g. Zhou et al., 1997). Here, we consider the observer-based controller

$$\begin{cases} \dot{w}_1(t) = (-A_1 - G_1 C_1) w_1(t) + G_1 y(t) + B_1 u(t), \\ w_1(0) = w_{10}, \\ u(t) = -F_1 w_1(t). \end{cases} \tag{6}$$

The control law (6) works as a stabilizing controller for the finite-dimensional model (5), however, it is not assured for the original system (1)–(2). For that reason, we use an RMF (7) together with the control law (6). Then, the whole controller is described as follows (see Fig. 1):

$$\begin{cases} \dot{w}_2(t) = -A_2 w_2(t) + B_2 u(t), \quad w_2(0) = w_{20}, \\ \hat{y}_2(t) = C_2 w_2(t), \end{cases} \tag{7}$$

$$\begin{cases} \dot{w}_1(t) = (-A_1 - G_1 C_1) w_1(t) + G_1 (y(t) - \hat{y}_2(t)) \\ \qquad\qquad + B_1 u(t), \quad w_1(0) = w_{10}, \\ u(t) = -F_1 w_1(t). \end{cases} \tag{8}$$

Then, the following result is well-known.

Theorem 2 (*Balas, 1988; Sakawa, 1983*)**.** *Suppose that Assumption 1 is satisfied and let another integer n be chosen such that $n > l$. Then, the control law consisting of (7)–(8) becomes a finite-dimensional stabilizing controller for system (1)–(2), if the integer n is chosen sufficiently large.*

Remark 3. In Sano and Kunimatsu (1994), Theorem 2 was extended to the system whose output operator was A^γ-bounded.

[2] The projections have been widely used. For example, Byrnes et al. solved the output regulation problem for a class of infinite-dimensional systems (Byrnes, Gilliam, & Shubov, 2003). Christofides and Daoutidis applied approximate inertial manifolds to the stabilization problem of semilinear distributed parameter systems (Christofides & Daoutidis, 1997).