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Brief paper

Adaptive robust tracking control for uncertain nonlinear fractional-order multi-agent systems with directed topologies☆

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ABSTRACT

This paper addresses the robust consensus tracking problem for a class of uncertain nonlinear fractional-order multi-agent systems (FOMASs) under general directed topologies. More specifically, FOMASs in the presence of heterogeneous unknown nonlinearities and external disturbances are considered in this paper, which include the second-order MASs as its special cases. First, we design two distributed saturated observers to overcome the deficiency of the traditional tracking control strategies. Second, when there exists a dynamics leader with unknown and bounded state trajectory, a discontinuous observer-based distributed controller with σ-modification adaptive schemes is presented to guarantee the tracking error converges to zero asymptotically. Next, a continuous observer-based distributed controller is further proposed, under which the consensus tracking error is uniformly ultimately bounded (UUB) and can be reduced as small as desired. A neural network (NN), whose weights are tuned online, is used in the designed controllers to approximate the unknown nonlinearities. Motivated by the σ-modification adaptive method, all the proposed adaptation algorithms require only local information and allow for robust even in the presence of heterogeneous unknown disturbances and fractional-order dynamics. Finally, the simulation results validate the efficacy of our proposed method.

1. Introduction

The study of coordination of MASs has attracted more and more attention over the past decades due to its broad applications in such areas as formation flight of unmanned aerial vehicles, swarming in insects, flocking in birds, and synchronization and phase transitions in physical and chemical systems. A typical cooperative control problem of MASs is the consensus problem, where each agent using only local information such that all the agents reach an agreement, i.e., achieve the same position, velocity, rendezvous and attitude. Generally speaking, existing consensus problem of MASs can be categorized into leaderless consensus problem (Olfati-Saber & Murray, 2004) and consensus tracking problem when there exists a leader for other agents (called followers) to track (Ren & Beard, 2005).

Up to date, most of the existing results on consensus problem of MASs assume an integer-order dynamics, such as first-order dynamics (Chen, Chen, Xiang, Liu, & Yuan, 2009; Ren & Beard, 2005), second-order dynamics (Meng, Lin, & Ren, 2013; Yu, Zheng, Lü, & Chen, 2013), and higher-order dynamics (Ferik, Qureshi, & Lewis, 2014; Zhang & Frank, 2012). It is founded that many practical coordinated behaviors of agents in complex environment often demonstrate non-integer-order (fractional-order) dynamics, such as vehicles moving on the top of macromolecule fluids and porous media (Sabatier, Agrawal, & Machado, 2007), high-speed aircraft traveling in dust storm, rain, or snow environment (Cao, Li, Ren, & Chen, 2010) and so on. It is worth noting that the integer-order dynamics can be regarded as a special case of the fractional-order dynamics. As we known, there is few report in the previous studies on the distributed coordination of MASs with fractional-order dynamics. The distributed coordination control problem of linear FOMASs with directed graphs has been investigated in Cao et al. (2010). It is indicated in Cao et al. (2010) that the convergence speed of the fractional-order consensus algorithms can be increased by adopting a varying-order fractional-order strategy. Further, Gong (2016) has studied the fractional-order leaderless and leader-following consensus of nonlinear FOMASs with directed topologies. Recently in Gong (2017), the consensus tracking problem has been investigated for Lipschitz-type nonlinear FOMASs subject to heterogeneous control gains and an unknown

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leader. In order to track the leader with second-order dynamics, a distributed fractional-order observer with dynamics order less than two has been designed in Yu, Li, Wen, Yu, and Cao (2017).

As is well known, we are often confronted with the case when there exist heterogeneous uncertainties as well as the external disturbances in practical MASs. For general MASs with unknown nonlinearities and disturbances, the NN-based adaptive approach is originally proposed in Hou, Cheng, and Tan (2009) to solve the consensus problem of first-order MASs under a fixed undirected graph. Shortly afterwards, such approach has been used to address the consensus tracking problem for uncertain nonlinear MASs with first-order integrator dynamics (Das & Lewis, 2010), with second-order integrator dynamics (Das & Lewis, 2011), and with high-order integrator dynamics (Ferik et al., 2014; Zhang & Frank, 2012). Note that one limitation in Das and Lewis (2010, 2011), Ferik et al. (2014), Hou et al. (2009) and Zhang and Frank (2012) is that the design of the consensus protocols depends on some global or unknown knowledge, such as the Laplacian matrix's eigenvalue information and the unknown nonlinear dynamics. One of the effective approaches to overcome these shortcomings is to adopt distributed adaptive gain updating laws. The distributed adaptive gains updating laws for both the cases of leaderless and leader-follower consensus of second-order Lipschitz-type nonlinear MASs with undirected topologies have been designed in Yu et al. (2013). In Yoo (2013), the containment control problem of uncertain nonlinear strict-feedback systems has been considered by applying distributed adaptive gain control algorithm. Some fully distributed consensus algorithms have been given in Mei, Ren, and Chen (2016) to address the consensus problem for a group of heterogeneous Lipschitz-type second-order MASs under general directed topologies. However, there is no paper to design fully distributed algorithms for solving the consensus problem of FOMASs with general nonlinear dynamics and general directed network topologies.

The above mentioned works and observation inspire us to consider the tracking problem for FOMASs subject to heterogeneous unknown nonlinearities and external disturbances over a general directed graph by designing fully distributed algorithms. Such problem is nontrivial and rather challenging due to the following reasons. At first, due to the existence of heterogeneous unknown nonlinearities as well as the unknown external disturbances in the nonlinear systems, it is desirable first to identify the heterogeneous unknown nonlinearities and then try best to compensate them in the designed controller. On the other hand, since the loss of symmetry in the directed network and the well-known Leibniz rule for fractional derivatives is invalid, how to construct a suitable Lyapunov function for analyzing the stability of nonlinear FOMASs is very challenging. Besides, it should be noted that the problem about designing fully distributed algorithms is much more challenging.

In this paper, by applying the fractional Lyapunov direct method, a discontinuous and continuous observed-based fully distributed algorithms are provided to address the robust consensus tracking problem for uncertain nonlinear FOMASs, respectively. The contributions of this paper are mainly threefold. (i) This paper extends the robust consensus tracking problem to the case where each agent has heterogeneous unknown nonlinear fractional-order dynamics and the fixed topology contains a directed spanning tree, which takes the second-order MASs in Das and Lewis (2011), Meng et al. (2013) and Yu et al. (2013) as its special cases. (ii) Unlike the existing adaptive algorithms in Das and Lewis (2010, 2011), Gong (2016), Hou et al. (2009), Meng et al. (2013), Yoo (2013), Yu et al. (2013) and Zhang and Frank (2012), the proposed discontinuous observed-based adaptive algorithm with σ-modification schemes here not only guarantees the tracking error converges to zero asymptotically, but also allows for robust even in the presence of heterogeneous unknown nonlinear fractional-order dynamics. (iii) Both of the proposed discontinuous and continuous robust adaptive algorithms can be implemented in a fully distributed fashion without requiring any global and unknown information. Note that in Yu et al. (2013) the design of fully distributed algorithms for second-order MASs with general directed topologies is clarified as an unsolved challenging problem.

Notations: Throughout the paper, let $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{Z}^+$ denote, respectively, the sets of all real numbers, nonnegative real numbers, and positive integers. Let $\mathbb{R}^{m\times n}$ denote the set of $m\times n$ real matrices, and $\mathbb{R}^m$ be the m-dimensional Euclidean space. Let $I_n \in \mathbb{R}^{n\times n}$ ($O_n \in \mathbb{R}^{n\times n}$) be the $n \times n$ identity (zero) matrix, and $\mathbf{1}_n \in \mathbb{R}^n$ ($\mathbf{0}_n \in \mathbb{R}^n$) denote the $n \times 1$ column vector of all ones (zeros). Denote by $\mathrm{diag}(d_1,\ldots,d_n) \in \mathbb{R}^{n\times n}$ a diagonal matrix with diagonal entries d_1 to d_n, and $\mathcal{I}_N = \{1,\ldots,N\}$. For a vector $x = [x_1,\ldots,x_n]^T \in \mathbb{R}^n$, denote $\|x\|_\infty = \max_{i=1,\ldots,n}|x_i|$, $\|x\| = \sqrt{x^Tx}$, and $\|x\|_1 = \sum_{i=1}^n |x_i|$. Let $\otimes$ be the Kronecker product, $\mathrm{tr}(\cdot)$ be the trace of a matrix, and $\lambda_{\min}(\cdot)$ ($\lambda_{\max}(\cdot)$) be the minimum (maximal) nonzero eigenvalue of a real symmetric square matrix.

2. Preliminaries and problem statement

2.1. Caputo fractional operators and Mittag-Leffler function

Some basic definitions and properties of fractional operators and Mittag-Leffler function are presented below in this section.

Definition 1 (*Riemann–Liouville Integral* Podlubny, 1999**).** The Riemann–Liouville fractional integral of function $f \in C^n(\mathbb{R}^+,\mathbb{R})$ is defined as follows:

$$I^q f(t) = I^q[f(\cdot)](t) = \int_0^t \frac{(t-\tau)^{q-1}}{\Gamma(q)} f(\tau)d\tau,$$

where $q \in (n-1, n]$, $n \in \mathbb{Z}^+$, $t \in \mathbb{R}^+$, and $\Gamma(\cdot)$ is the well-known Gamma function, defined as $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$.

Definition 2 (*Caputo Derivative* Podlubny, 1999). The Caputo fractional derivative of function $f \in C^n(\mathbb{R}^+,\mathbb{R})$ is defined as follows:

$$D^q f(t) = I^{n-q} f^{(n)}(t) = \int_0^t \frac{(t-\tau)^{n-q-1}}{\Gamma(n-q)} f^{(n)}(\tau)d\tau,$$

where $q \in (n-1, n]$, $n \in \mathbb{Z}^+$, and $t \in \mathbb{R}^+$.

In the Caputo settings, it is given by the following properties (Podlubny, 1999).

Property 1. *For any two continuous functions $h(t), g(t) \in C^n(\mathbb{R}^+,\mathbb{R})$, then $D^q\,(ah(t) + bg(t)) = aD^q h(t) + bD^q g(t)$ and $D^q c = 0$ hold, where a, b, and c are any three constants.*

Property 2. *Let $f(t) \in C^n(\mathbb{R}^+,\mathbb{R})$, then for all $t \in \mathbb{R}^+$,*

$$I^q D^q f(t) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{k!} t^k,$$

where $q \in (n-1, n]$, $n \in \mathbb{Z}^+$. In particular, if $q \in (0, 1]$, then $I^q D^q f(t) = f(t) - f(0)$.

Definition 3 (*Mittag-Leffler Function* Podlubny, 1999**).** The Mittag-Leffler function with two positive parameter a and b is defined as follows:

$$E_{a,b}(z) = \sum_{k=1}^{\infty} \frac{z^k}{\Gamma(ka+b)},$$

where z is a complex number. Let $E_{a,1}(z) = E_a(z)$ as $b = 1$, further, $E_{1,1}(z) = e^z$.