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Brief paper

Generalized convergence conditions of the parameter adaptation algorithm in discrete-time recursive identification and adaptive control☆

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ABSTRACT

In this paper, we extend convergence conditions for the parameter adaptation algorithm, used in discrete-time recursive identification schemes, or in adaptive control. Whereas the classical stability analysis of this algorithm consists in checking the strictly real positiveness of an associated transfer function, we demonstrate that convergence can be obtained even when this condition is not fulfilled, under some assumptions on the algorithm forgetting factors. These results regarding both deterministic and stochastic contexts are obtained by analyzing convergence with a prescribed degree of stability.

1. Introduction

The parameter adaptation algorithm (PAA) described in Landau, Lozano, Saad, and Karimi (2011) is a cornerstone in adaptive control. It provides an on-line parameter estimation of a discrete-time system and is extensively used in recursive identification schemes. The issue of PAA convergence analysis has been addressed for a long time (Landau, 1965) by considering that this algorithm can always be represented as an equivalent closed-loop including a linear time-varying (LTV) feedback system in interaction with a discrete feedforward linear time-invariant (LTI) system. Hyperstability theory imposes the strictly real positiveness condition of the transfer function linked to the feedforward LTI system and, as this condition is only sufficient, in some cases when it is not fulfilled, the PAA convergence can be nevertheless observed. The purpose of this paper is to provide less restrictive convergence conditions for the PAA. We show that under classical assumptions on the algorithm forgetting factors, such as those considered in, e.g., Lozano (1983), the algorithm stability can be proved even if the LTI system strictly real positiveness is not satisfied. These results provide new analysis tools able to cope, in particular, with an LTI system transfer function having poles on the unit circle. The hereafter developments are based upon controlled LTI systems with a prescribed degree of stability that have been studied in Anderson and Moore (1971) and Bourlès (1987) in the continuous-time case, and in Bourlès, Joannic, and Mercier (1990) in the discrete-time-case. Likewise Kalman filters with a prescribed degree of stability have been developed in Anderson and Moore (1979). Since the PAA is a variant of the Kalman filter, we combine these approaches in the sequel with developments achieved in a deterministic context (Landau & Silveira, 1979) and in a stochastic context (Landau, 1982) to obtain these generalized convergence conditions.

2. Deterministic context

In the beginning of this section, we refer systematically to Landau et al. (2011) (pp.102–103) for the description of the PAA, and in particular we reuse the same notation. The PAA aims at making as close to zero as possible a prediction (or adaptation) error between the output of the system to be identified and an adjustable model output. Let us denote by:

$\nu(t+1)$: The a-posteriori adaptation error (scalar),
$\phi(t)$: The observation vector (size $(n_\phi, 1)$),
θ: The parameters vector to be estimated (size $(n_\phi, 1)$),
$\widehat{\theta}(t)$: The current estimated parameters vector (size $(n_\phi, 1)$).

We consider systems for which the a-posteriori error is given by:

$$\nu(t+1) = H(q^{-1})(\theta - \widehat{\theta}(t+1))^T \phi(t). \quad (1)$$

In this expression $H(q^{-1})$ is the operator associated with the (bi-causal) transfer function $H(z^{-1})$, which is a ratio of two monic

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polynomials. The PAA equations are:

$$\widehat{\theta}(t+1)=\widehat{\theta}(t)+F(t)\phi(t)\varepsilon(t+1) \quad (2)$$

$$F^{-1}(t+1)=\lambda_1F^{-1}(t)+\lambda_2\phi(t)\phi^T(t) \quad (3)$$

$F(t)$ is the adaptation gain (positive definite matrix), $0<\lambda_1\leq 1$, $0\leq\lambda_2<2$ are the forgetting factors. A sufficient convergence condition for the PAA, is that the transfer function:

$$H(z^{-1})-\frac{\lambda_2}{2}$$

be strictly positive real. From Eqs. (1), (2), (3) an equivalent closed-loop can be drawn. For this purpose let us denote by: $\tilde{\theta}(t)=\widehat{\theta}(t)-\theta$: the parameters vector error (size $(n_\phi,1)$),

$u(t+1)=-\phi^T(t)\tilde{\theta}(t+1)$: the input of $H(q^{-1})$ (scalar),
$y(t+1)=\nu(t+1)$: the output of $H(q^{-1})$ (scalar),
$\tilde{u}(t+1)=\nu(t+1)$: the associated LTV system input,
$\tilde{y}(t+1)=\phi^T(t)\tilde{\theta}(t+1)$: the LTV system output (scalar),
$\tilde{x}(t)=\tilde{\theta}(t)$: the LTV system state vector (size $(n_\phi,1)$),
$x(t)$: the state vector of the operator $H(q^{-1})$ (size $(n,1)$).

The LTV system state space equations are:

$$\tilde{x}(t+1)=\tilde{A}(t)\tilde{x}(t)+\tilde{B}(t)\tilde{u}(t+1) \quad (4a)$$

$$\tilde{y}(t+1)=\tilde{C}(t)\tilde{x}(t)+\tilde{D}(t)\tilde{u}(t+1). \quad (4b)$$

With:
$\tilde{A}(t)=I_{n_\phi}$,
$\tilde{B}(t)=F(t)\phi(t)$, (size $(n_\phi,1)$),
$\tilde{C}(t)=\phi^T(t)$, (size $(1,n_\phi)$),
$\tilde{D}(t)=\phi^T(t)F(t)\phi(t)$, (scalar).

In what follows, a controllable and observable linear system is considered as an operator associating the output to the input, with zero initial conditions. The LTI system state space equations are written:

$$x(t+1)=Ax(t)+Bu(t) \quad (5a)$$

$$y(t)=Cx(t)+Du(t). \quad (5b)$$

With A of size (n,n),
B of size $(n,1)$,
C of size $(1,n)$
D a scalar.

The equivalent closed-loop is represented in Fig. 1. For any signal $s=\{s(t)\}$ (determinist or stochastic), denote by s_ρ the signal $\{\rho^t s(t)\}$, $\rho\geq 1$. Considering the signal $y(t)=H(q^{-1})u(t)$, the relation between $y_\rho(t)$ and $u_\rho(t)$ is $y_\rho(t)=H_\rho(q^{-1})u_\rho(t)$, with $H_\rho(q^{-1})=H(\rho q^{-1})$, (Bourlès et al., 1990). An equivalent closed-loop can be derived from the loop represented in Fig. 1, in which the feedback LTV system input and output are now $\tilde{u}_\rho(t+1)$, $\tilde{y}_\rho(t+1)$, and the feedforward LTI system input and output correspond to $u_\rho(t)$ and $y_\rho(t)$. The state space equations of the so-called ρ-LTV system are given by:

$$\tilde{x}_\rho(t+1)=\tilde{A}_\rho(t)\tilde{x}_\rho(t)+\tilde{B}_\rho(t)\tilde{u}_\rho(t+1) \quad (6a)$$

$$\tilde{y}_\rho(t+1)=\tilde{C}_\rho(t)\tilde{x}_\rho(t)+\tilde{D}_\rho(t)\tilde{u}_\rho(t+1). \quad (6b)$$

With:
$\tilde{A}_\rho=\rho I_{n_\phi}$,
$\tilde{B}_\rho=F(t)\phi(t)$ (size $(n_\phi,1)$),
$\tilde{C}_\rho(t)=\rho\phi^T(t)$ (size $(1,n_\phi)$),
$\tilde{D}_\rho=\phi^T(t)F(t)\phi(t)$ (scalar).

Fig. 2 describes the equivalent PAA closed-loop, with ρ-signals.

Imposing that $\tilde{y}_\rho(t)$ and $\tilde{u}_\rho(t)$ converge towards 0 is equivalent to impose a degree of stability ρ to the classical closed-loop in Fig. 1.

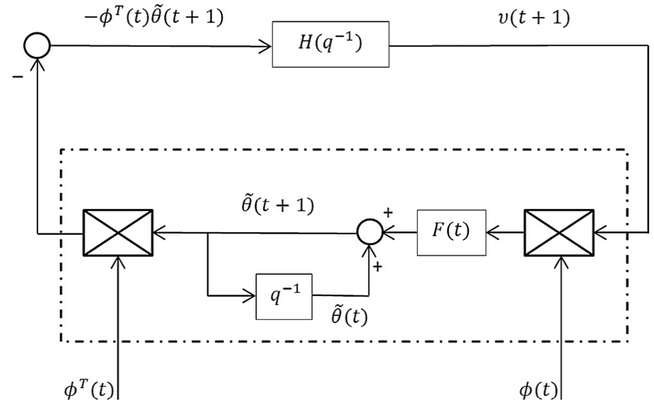

Fig. 1. Classical PAA closed-loop.

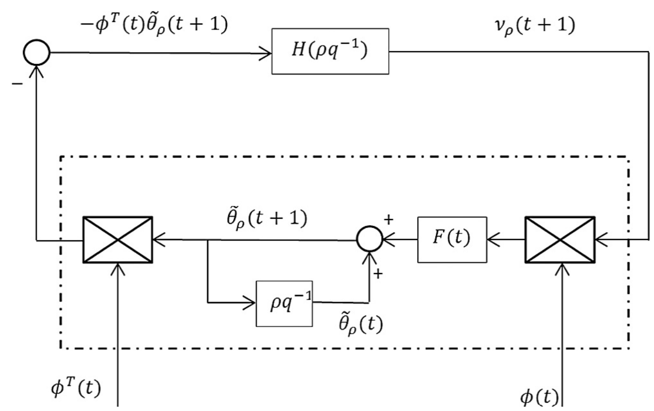

Fig. 2. Equivalent PAA closed-loop with ρ-signals.

Theorem 1. *Consider the PAA algorithm given by (1), (2), and (3). Assume that there exists $\rho\geq 1$ such that the following conditions hold:*

(1) $\lambda_1\leq 2-\rho^2$, $0\leq\lambda_2<2$
(2) *The transfer function $H(\rho z^{-1})-\frac{\lambda_2}{2}$ is strictly positive real.*

Then one has:

$$\lim_{t\to\infty}\nu_\rho(t+1)=0 \quad (7)$$

$$\lim_{t\to\infty}\left[\theta-\widehat{\theta}(t+1)\right]^T\phi(t)\rho^t=0 \quad (8)$$

$$\lim_{t\to\infty}\left[\rho\left(\theta-\widehat{\theta}(t+1)\right)-\left(\theta-\widehat{\theta}(t)\right)\right]^TF^{-1}(t)\cdots$$
$$\cdots\left[\rho\left(\theta-\widehat{\theta}(t+1)\right)-\left(\theta-\widehat{\theta}(t)\right)\right]\rho^{2t}=0 \quad (9)$$

$$\left[\widehat{\theta}(t)-\theta\right]^TF^{-1}(t)\left[\widehat{\theta}(t)-\theta\right]\rho^{2t}<\text{const}<\infty. \quad (10)$$

Proof. Since the case $\rho=\lambda_1=1$ has already been treated in (Landau et al., 2011, Thm. 3.1) we assume $\rho>1$. The transfer function $H(\rho z^{-1}-\frac{\lambda_2}{2}$ is strictly real positive if and only if (by definition) $H(\rho z^{-1})$ belongs to the class $L(\Lambda)$, as defined in Landau et al. (2011), p. 556. The feedback loop using ρ-signals is stable and $\tilde{y}_\rho(t)$, $\tilde{u}_\rho(t)$ converge towards 0 if the LTV system (6), belongs to the class $N(\Gamma)$, as defined in Landau et al. (2011), p. 558, with $\Gamma=\lambda_2$. According to Lemma C.7 of the same reference, the system (6) belongs to the class $N(\Gamma)$ if there exist three sequences of non-negative definite symmetric matrices $\{P(t)\}$, $\{R(t)\}$, $\{Q(t)\}$ and a