Automatica 92 (2018) 173–180

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Finite-time command filtered backstepping control for a class of nonlinear systems ☆

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ARTICLE INFO

Article history:
Received 25 September 2016
Received in revised form 11 December 2017
Accepted 2 March 2018

Keywords:
Nonlinear systems
Command filtered backstepping
Finite-time convergence

ABSTRACT

This paper considers the problem of finite-time tracking control for a class of nonlinear systems. A novel finite-time command filtered backstepping approach is proposed by using the new virtual control signals and the modified error compensation signals. The new design technique not only has the advantages of the conventional command-filtered backstepping control, but also guarantees the finite-time convergent property. Two examples are included to show the effectiveness of the obtained theoretical results.

1. Introduction

Backstepping control is one of the most commonly used methods to deal with nonlinear systems (Bribiesca-Argomedo & Krstic, 2015; Kanchanaharuthai & Mujjalinvimut, 2018; Khalil, 2002; Krstic, Kanellakopoulos, & Kokotovic, 1995), in which the states are used as virtual control signals in control law design, and the virtual signals and their derivatives are required in each step of the design process. However, the backstepping design procedure has the problem of "explosion of complexity" (Bribiesca-Argomedo & Krstic, 2015; Swaroop, Hedrick, Yip, & Gerdes, 2000; Zhang & Ge, 2008) caused by repeated differentiations of the virtual control signals, especially for systems with high-order dynamics. To address the above issue, the dynamic surface control (DSC) approaches were first proposed, in which the first-order filters were introduced in the backstepping design, and the adaptive fuzzy technique was further combined with DSC to eliminate the influence of uncertain nonlinearities (Sun, Li, & Ren, 2015; Swaroop et al., 2000; Tong, Li, Feng, & Li, 2011; Tong, Sui, & Li, 2015; Yu, Shi, Dong, Chen, & Lin, 2015; Zhang & Ge, 2008). But how to compensate the errors caused by the first-order filters was not considered in Sun et al. (2015), Swaroop et al. (2000), Tong et al. (2011), Tong et al. (2015), Yu, Shi, Dong, Chen et al. (2015) and Zhang and Ge (2008), which influence the control quality (Yu, Shi, Dong, & Yu, 2015). The command-filtered backstepping is another modified backstepping control method, in which the command filters are introduced to approximate the derivative of the virtual (Dong, Farrell, Polycarpou, Djapic, & Sharma, 2012; Farrell, Polycarpou, Sharma, & Dong, 2009), and the errors caused by the command filters can be reduced with the combination of compensation signals. It should be pointed out that the conventional backstepping control laws in Khalil (2002), Krstic et al. (1995) and the modified backstepping control laws in Dong et al. (2012), Farrell et al. (2009), Sun et al. (2015), Swaroop et al. (2000), Tong et al. (2011), Tong et al. (2015), Yu, Shi, Dong, Chen et al. (2015); Yu, Shi, Dong, and Yu (2015) and Zhang and Ge (2008) are all asymptotically stable control laws, which means that the closed-loop convergence is achieved as time goes to infinity.

Compared with the asymptotic control approach, the finite-time control technique has many advantages such as faster response, higher tracking precision and better disturbance-rejection ability (Bhat & Bernstein, 2000; Du, Li, & Qian, 2011; Yu, Yu, Shirinzadeh, & Man, 2005). Therefore, many finite-time control methods have been developed for various nonlinear systems during the past few years (Gao, Wu, & Zhang, 2015; Hou, Zhang, Deng, & Duan, 2016; Huang, Lin, & Yang, 2005; Lu & Xia, 2013; Qian & Lin, 2002; Shen & Huang, 2012; Zhang, Feng, & Sun, 2012; Zhao & Jia, 2015). For example, the works in Gao et al. (2015), Hou et al. (2016), Huang et al. (2005), Qian and Lin (2002), Shen and Huang (2012) and Zhang et al. (2012) studied the finite-time stabilization problem of high-order nonlinear systems by a power integrator technique.

☆ This work was partially supported by the National Nature Science Foundation, China (61573204, 61603204, 61773131, and U1509217), the Australian Research Council (DP170102644), Taishan Scholar Special Project Fund (TSQN20161026) and 111 Project (B17048, B17017). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Shuzhi Sam Ge under the direction of Editor Miroslav Krstic.
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https://doi.org/10.1016/j.automatica.2018.03.033
0005-1098/

In many practical applications, the tracking problem is required to be solved, and the finite-time tracking is more desirable (Lu & Xia, 2013; Zhao & Jia, 2015). Note that the command-filtered backstepping is an effective strategy to deal with the tracking problems of high-order nonlinear systems, but the finite-time convergence cannot be achieved by employing the existing control frameworks. Then a natural question is: how to extend the command-filtered backstepping control such that the finite-time tracking issue can be resolved for high-order nonlinear systems? This is the motivation of our work in this paper.

In this paper, we will study the problem of finite-time tracking control for high-order nonlinear systems by a new finite-time command filtered backstepping approach. Compared with the conventional command-filtered backstepping control in Dong et al. (2012), Farrell et al. (2009), Yu, Shi, Dong, and Yu (2015), the main contributions are summarized as follows:

(1) At each step of the backstepping, the finite-time filter is introduced to replace the command filter with asymptotic convergence rate in Dong et al. (2012), Farrell et al. (2009) and Yu, Shi, Dong, and Yu (2015), which can guarantee that the output of filter approximates the derivative of the virtual control;

(2) Compared with Dong et al. (2012), Farrell et al. (2009), Yu, Shi, Dong, and Yu (2015), the new finite-time error compensation mechanism is first proposed, which can timely reduce the filtering errors than that in Dong et al. (2012), Farrell et al. (2009) and Yu, Shi, Dong, and Yu (2015);

(3) The finite-time virtual control signals can guarantee the closed-loop systems with faster response and higher tracking precision than the virtual control signals proposed in Dong et al. (2012), Farrell et al. (2009) and Yu, Shi, Dong, and Yu (2015) by choosing proper control parameters.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. The control law design is presented in Section 3. Two examples are given in Section 4 to verify the effectiveness and advantages of the proposed new design method, and the paper is concluded by Section 5.

2. Problem formulation and preliminaries

Consider the following class of nth order SISO nonlinear systems:

$$\begin{aligned} \dot{x}_1 &= f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 \\ \dot{x}_2 &= f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3 \\ &\vdots \\ \dot{x}_n &= f_n(x) + g_n(x)u \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbf{R}^n$ is the state vector with $x(0) = x_0$ and $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, y is the system output and u is the control signal. The functions $f_i(\cdot)$ and $g_i(\cdot)$ $(i = 1, 2, \ldots, n)$ are assumed to be known. Denote x_{1d} as the desired tracking signal and its first time derivative is assumed to be a smooth, bounded and known function. The control objective of this paper is to construct the control law u such that the output x_1 tracks the reference signal x_{1d} from any initial conditions in finite-time and all the signals and states of the closed-loop system are bounded in finite-time. The following assumption is imposed on system (1).

Assumption 1. There exists an open set $\Omega_d \subset R^n$ that includes the origin and the initial condition x_0. For system (1), for $i = 1, \ldots, n-1, p = 1, \ldots, (n-i)$: (1) $f_i^{(p)}(\cdot)$ and $g_i^{(p)}(\cdot)$ are bounded in closed set $\bar{\Omega}_d$; (2) $f_n(\cdot), g_n(\cdot)$ and their first-order partial derivatives are bounded in closed set $\bar{\Omega}_d$; and (3) there exist known positive constants η and ρ such that $\eta < |g_i| < \rho$.

By Assumption 1, both functions f_i and g_i are each Lipschitz on $\bar{\Omega}_d$. Let us recall the following results in order to develop our main results in sequel.

Lemma 1 (*Bhat & Bernstein, 2000*). *Suppose $V(x)$ is a C^1 smooth positive-definite function (defined on $U \subset \mathbf{R}^n$) and $\dot{V}(x) + \lambda V^{\alpha}(x)$ is a negative semi-definite function on $U \subset \mathbf{R}^n$ and $\alpha \in (0, 1)$, then there exists an area $U_0 \subset \mathbf{R}^n$ such that any $V(x)$ which starts from $U_0 \subset \mathbf{R}^n$ can reach $V(x) \equiv 0$ in finite time. Moreover, if T_r is the time needed to reach $V(x) \equiv 0$, then $T_r \le \frac{V^{1-\alpha}(x_0)}{\lambda(1-\alpha)}$ where $V(x_0)$ is the initial value of $V(x)$.*

Lemma 2 (*Yu et al., 2005*). *For any real numbers $\lambda_1 > 0, \lambda_2 > 0, 0 < \gamma < 1$, an extended Lyapunov condition of finite-time stability can be given $\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\gamma}(x) \le 0$ where the settling time can be estimated by $T_r \le t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 V^{1-\gamma}(t_0)+\lambda_2}{\lambda_2}$.*

Note that Lemmas 1–2 provide a general Lyapunov condition of finite-time stability, which cannot be always guaranteed under the designed control law, and the state will be driven into the bounded region in finite-time, which is defined as practical finite-time stability in Zhu, Xia, and Fu (2011). It should be pointed out that Zhu et al. (2011) presented the Lyapunov condition of practical finite-time stability under Lemma 1. We will further give the Lyapunov condition of practical finite-time stability under Lemma 2 in this paper. Based on Lemma 2, we then have the following result.

Corollary 1. *Consider the system $\dot{x} = f(x)$. If there exist continuous function $V(x)$, scalars $\lambda_1 > 0, \lambda_2 > 0, 0 < \gamma < 1, 0 < \eta < \infty$ such that $\dot{V}(x) \le -\lambda_1 V(x) - \lambda_2 V^{\gamma}(x) + \eta$, then the trajectory of system $\dot{x} = f(x)$ is practical finite-time stable, and the residual set of the solution of system $\dot{x} = f(x)$ is given by*

$$\{\lim_{t \to T_r} |V(x) \le \min\{\frac{\eta}{(1-\theta_0)\lambda_1}, (\frac{\eta}{(1-\theta_0)\lambda_2})^{\frac{1}{\gamma}}\}\}$$

where θ_0 satisfies $0 < \theta_0 < 1$. The setting time is bounded as

$$T_r \le \max\{t_0 + \frac{1}{\theta_0\lambda_1(1-\gamma)} \ln \frac{\theta_0\lambda_1 V^{1-\gamma}(t_0)+\lambda_2}{\lambda_2}, \quad t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 V^{1-\gamma}(t_0)+\theta_0\lambda_2}{\theta_0\lambda_2}\}.$$

Proof. Note that there exists a scalar $0 < \theta_0 < 1$ such that the inequality $\dot{V}(x) \le -\lambda_1 V(x) - \lambda_2 V^{\gamma}(x) + \eta$ can be expressed as

$$\dot{V}(x) \le -\theta_0\lambda_1 V(x) - (1-\theta_0)\lambda_1 V(x) - \lambda_2 V^{\gamma}(x) + \eta \tag{2}$$

or

$$\dot{V}(x) \le -\lambda_1 V(x) - \theta_0\lambda_2 V^{\gamma}(x) - (1-\theta_0)\lambda_2 V^{\gamma}(x) + \eta. \tag{3}$$

From (2), we have

$$\dot{V}(x) \le -\theta_0\lambda_1 V(x) - \lambda_2 V^{\gamma}(x)$$

if $V(x) > \frac{\eta}{(1-\theta_0)\lambda_1}$. Then, by Lemma 2 and Zhu et al. (2011), it follows that the decrease of $V(x)$ in finite time will drive x into the region

$$x \in \{V(x) \le \frac{\eta}{(1-\theta_0)\lambda_1}\}. \tag{4}$$

The time needed to arrive (4) is given as

$$T_r \le t_0 + \frac{1}{\theta_0\lambda_1(1-\gamma)} \ln \frac{\theta_0\lambda_1 V^{1-\gamma}(t_0)+\lambda_2}{\lambda_2}.$$

From (3), it can be seen that

$$\dot{V}(x) \le -\lambda_1 V(x) - \theta_0\lambda_2 V^{\gamma}(x)$$