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Control design with guaranteed cost for synchronization in networks of linear singularly perturbed systems ☆

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ABSTRACT

This work presents the design of a decentralized control strategy that allows singularly perturbed multi-agent systems to achieve synchronization with global performance guarantees. The study is mainly motivated by the presence of two features that characterize many physical systems. The first is the complexity in terms of interconnected subsystems and the second is that each subsystem involves processes evolving on different time-scales. In the context of interconnected systems, the decentralized control is interesting since it considerably reduces the communication load (and the associated energy) which can be very important when dealing with centralized policies. Therefore, the main difficulty that we have to overcome is that we have to avoid the use of centralized information related to the interconnection network structure. This problem is solved by rewriting the synchronization problem in terms of stabilization of a singularly perturbed uncertain linear system. The singularly perturbed dynamics of subsystems generates theoretical challenges related to the stabilizing controller design but also numerical issues related to the computation of the controller gains. We show that these problems can be solved by decoupling the slow and fast dynamics. Our theoretical developments are illustrated by some numerical examples.

1. Introduction

The main goal of this paper is to design a decentralized control strategy that allows singularly perturbed multi-agent systems to achieve synchronization with global performance guarantees. Decentralized coordination control of multi-agent systems attracted a lot of attention during the last decade. An important feature of this class of large scale systems is the fact that local information plays a key role. In the decentralized control design each system is able to implement and design its own control law without the help of a central entity that requires important amounts of communication and computation. Consequently, decentralized control aims at reducing the communication and computation costs. When these costs are neglected the centralized control strategies generally outperform the decentralized ones. However, energy aware strategies have to take into account the overall cost and should reduce the communication and computation loads (Hassan & Shamma, 2016). Therefore, in this paper we design decentralized controllers that provide a guaranteed cost.

Synchronization of singularly perturbed systems is mainly motivated by two features that characterize the nowadays systems. The first one is the complexity in terms of subsystems interconnected together in order to accomplish a global goal while the second is that physical subsystems often involve processes that evolve on different time-scales. Generally these features are tackled independently one from another. Indeed, the multi-agent formalism allows treating problems coming from a wide application domain such as engineering (Bullo, Cortés, & Martinez, 2009), biology (Pavlopoulos et al., 2011), sociology (Hegselmann & Krause, 2002; Morărescu & Girard, 2011). Consensus and synchronization were mainly studied for linear agents interacting through a directed or undirected graph with a fixed or dynamically changing topology (Jadbabaie, Lin, & Morse, 2003; Moreau, 2005). However, there are also studies on nonlinear agents such as oscillators dynamics (Morărescu, Michiels, & Jungers, 2016; Steur, Tyukin, & Nijmeijer, 2009), nonholonomic robots (Bullo et al., 2009) or general nonlinear systems (Buşoniu & Morărescu, 2014).

On the other hand, one can find many applications ranging from biological systems such as gene expression systems (Chen

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& Aihara, 2002), neurons behavior (Hodgkin & Huxley, 1952) to engineering problems (Malloci, Daafouz, & Iung, 2009) that involve processes evolving on different time-scales. General stability and stabilization of such linear and nonlinear systems, called singularly perturbed, can be found in Khalil (2001) and Kokotović, Khalil, and O'Reilly (1999).

In a preliminary work (Rejeb, Morărescu, & Daafouz, 2016), we have combined the two features presented above to study the synchronization of singularly perturbed systems. In that work we have designed decentralized controllers able to achieve asymptotically the synchronization goal. Here, we address the more challenging problem of synchronization with global performance guarantees. This problem was also considered separately in the context of multi-agent systems and singularly perturbed systems. For instance, a linear quadratic (LQ)-based optimal linear consensus protocol for multi-vehicle systems with single integrator dynamics was investigated in Cao and Ren (2010) in both continuous time and discrete time. The authors of Kim and Mesbahi (2006) considered an iterative algorithm that maximizes the second smallest eigenvalue of a state-dependent graph Laplacian yielding to optimization of the convergence speed toward consensus. A nonlinear distributed coordination law was presented in Cortés and Bullo (2005) to achieve optimal consensus under a switching directed communication graph.

On the other hand there exist studies that consider linear quadratic optimal control design for linear singularly perturbed systems (Garcia, Daafouz, & Bernussou, 2002; Kokotović et al., 1999). One of the most common approaches is the time-scale decomposition that leads to decoupled slow and fast subsystems and an appropriate combination of the corresponding results yields an optimal control design for the original system.

The main contribution of our paper is twofold. Firstly, we strengthen the main result in Rejeb et al. (2016) and correct the proof of Rejeb et al. (2016, Proposition 4) that contains a flaw in the reasoning since we had implicitly assumed a particular Lyapunov function for the systems under consideration. The corresponding result in this work is Proposition 3 which is instrumental for the design of a decentralized synchronizing controller presented in Theorem 4. Secondly, we go beyond those results by considering the more challenging problem of decentralized guaranteed cost control design. The supplementary difficulty that we face is related to the fact that we have to ensure bounds on a global cost in a decentralized manner *i.e.* by a local design. To the best of the authors' knowledge this is the first attempt of designing guaranteed cost controllers for singularly perturbed multi-agent systems. When we deal with optimal decentralized control, the Riccati equation, which, in the LQ case is the basis for the derivation of the optimal control law, involves the eigenvalues of the graph Laplacian describing the overall network. In order to get rid of this centralized information, instead of looking for an optimal controller, a guaranteed cost controller is designed to ensure a performance level of the closed-loop dynamics. Precisely we consider a multi-agent system under fixed undirected interaction graph. The dynamics of each agent is represented by linear singularly perturbed system. To solve the problem of decentralized guaranteed cost control design, we transform the synchronization problem in an uncertain system stabilization one. The uncertainty comes from the fact that the graph Laplacian eigenvalues are modeled as unknown but bounded uncertain parameters in order to avoid an explicit use of Laplacian's eigenvalues. This is motivated by the fact that the only available graph information consists in its connectivity.

The paper is organized as follows. Section 2 presents some basic definitions and notations in graph theory. In Section 3, we introduce a change of variables that allows us to reformulate the synchronization problem under consideration in terms of simultaneous synchronization of linear singularly perturbed systems. Section 4 provides conditions under which the decentralized synchronization is possible. Moreover, it presents a methodology to design a decentralized controller that achieves the synchronization goal. Section 5 is devoted to the design of the decentralized guaranteed cost control law. Conditions on the gain matrix such that the closed-loop singularly perturbed systems achieve asymptotic synchronization while an upper bound on the performance index is minimized, are expressed through linear matrix inequalities (LMIs). Simulation results are presented in Section 6. The paper ends with some concluding remarks.

Notation: The following standard notation is used throughout the paper. $\mathbb{R}$ is the set of real numbers, $\|x\|$ is the Euclidean norm of the vector x and $\otimes$ denotes the Kronecker product of two matrices. We also denote by $I_n \in \mathbb{R}^{n\times n}$ the identity matrix of size n and by $\mathbf{1}_n, \mathbf{0}_n \in \mathbb{R}^n$ the column vector whose components are all 1 and 0, respectively. By $\mathbf{0}_{n\times m} \in \mathbb{R}^{n\times m}$ we denote the matrix whose all components are 0. Given a matrix $A \in \mathbb{R}^{n\times n}$ and $A > \mathbf{0}$ ($A \geq \mathbf{0}$) means that A is positive (semi-) definite. The transpose of A is denoted by $A^\top$. We denote $diag(A_1, \ldots, A_n)$ the block diagonal matrix having the matrices A_1 to A_n on the diagonal and 0 everywhere else.

2. Preliminaries and problem formulation

We consider a network of n identical singularly perturbed linear systems. For any $i = 1, \ldots, n$, the ith system at time t is characterized by the state $(x_i(t), z_i(t)) \in \mathbb{R}^{n_x+n_z}$ and a small $\varepsilon > 0$ such that its dynamics is given by:

$$\begin{cases} \dot{x}_i(t) = A_{11}x_i(t) + A_{12}z_i(t) + B_1u_i(t) \\ \varepsilon\dot{z}_i(t) = A_{21}x_i(t) + A_{22}z_i(t) + B_2u_i(t), \end{cases} \tag{1}$$

where $u_i \in \mathbb{R}^m$ is the control input and

$$A_{11} \in \mathbb{R}^{n_x\times n_x},\ A_{12} \in \mathbb{R}^{n_x\times n_z},\ \ B_1 \in \mathbb{R}^{n_x\times m}$$
$$A_{21} \in \mathbb{R}^{n_z\times n_x},\ A_{22} \in \mathbb{R}^{n_z\times n_z},\ B_2 \in \mathbb{R}^{n_z\times m},$$

such that $rank(B_1) = rank(B_2) = m$.

Assumption 1. The matrix A_{22} is invertible.

The previous assumption is standard in singular perturbation theory (see (Kokotović et al., 1999)) but it is not verified for the case of simple integrators which are standard in multi-agent systems. Nevertheless, our analysis applies for a wide range of systems that have an internal dynamics. With the network of n systems we associate a graph $\mathcal{G}$ which is a couple $(\mathcal{V}, \mathcal{E})$. Here, $\mathcal{V} = \{1, \ldots, n\}$ represents the vertex set and $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$ is the edge set. In the sequel we suppose that the graph is undirected meaning that $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$. We also assume that $\mathcal{G}$ has no self-loop (i.e. $\forall i = 1, \ldots, n$ one has $(i, i) \notin \mathcal{E}$). A weighted adjacency matrix associated with $\mathcal{G}$ is $G = [g_{ij}] \in \mathbb{R}^{n\times n}$ such that $g_{ij} = g_{ji} > 0$ if $(i, j) \in \mathcal{E}$ and $g_{ij} = 0$ otherwise. The corresponding weighted Laplacian matrix is $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ defined by $\begin{cases} l_{ii} = \sum_{j=1}^{n} g_{ij},\ \forall i = 1, \ldots, n \\ l_{ij} = -g_{ij} \text{ if } i \neq j \end{cases}$. By definition L is symmetric and all of its rows sums are zero.

Definition 1. A **path of length** p in the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an union of edges $\bigcup_{k=1}^{p}(i_k, j_k)$ such that $i_{k+1} = j_k,\ \forall k \in \{1, \ldots, p-1\}$. The node j is **connected** with node i in $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if there exists at least a path in $\mathcal{G}$ from i to j (*i.e.* $i_1 = i$ and $j_p = j$). A **connected graph** is such that any of its two distinct elements are connected.

Throughout the rest of the paper the following hypothesis holds.

Assumption 2. The undirected graph $\mathcal{G}$ is connected and all the non-zero weights $g_{i,j} \neq 0$ of the associated weighted Laplacian matrix are within the interval $[g_m, g_M]$ with $g_M \geqslant g_m > 0$.