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Brief paper

Adaptive consensus of multi-agent systems under quantized measurements via the edge Laplacian☆

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ABSTRACT

This paper investigates consensus problems for a first-order nonlinear multi-agent system with unknown non-identical parameters when quantized measurements are available. We explore the utility of edge Laplacian for designing adaptive consensus protocol with quantized state information. A tree structure plays an important role in designing adaptive control with quantized information. Then, all agents achieve consensus mainly based on tree edges information. The proposed protocol has the advantage of using an equivalent simpler graph to avoid the redundant edge information in the cycle structure of the graph. Furthermore, when the multi-agent systems have unknown identical control directions, a Nussbaum-type item is used in the protocol for each agent to seek control direction adaptively and cooperatively. Finally, simulation examples are given to illustrate the effectiveness of the proposed method in this article.

1. Introduction

There has been a great deal of interest in the development of multi-agent systems (Fax & Murry, 2004; Li & Li, 2014; Ren & Atkins, 2007; Xiao, Wang, & Wang, 2006). The consensus problem is known as agreement on certain quantities of interest for groups of agents. Early efforts focus on the real-valued data exchanging among agents with high-precision, since the agents must exchange information over a digital communication channel, and quantization is one of the basic limitations induced by finite bandwidth channels (Liu, Ho, & Niu, 2010; Niu & Ho, 2014; Wang, Dong, Shen, & Gao, 2013; Yao, Wu, & Zheng, 2013). Therefore, the quantized information is available to the agents instead of the precise information. Recently, in the very active area of consensus, synchronization, and coordinated control for multi-agent systems, the use of quantized measurements and control has received compelling attention.

The existing works on coordination problems under quantization effect have various results for discrete-time systems (Dong, Wang, & Gao, 2012; Frasca, Carli, Fagnani, & Zampieri, 2009; Fu & Xie, 2010; Li, Fu, Xie, & Zhang, 2011; Li, Ho, & Lu, 2013). In Li et al. (2013), a precise mathematical treatment was provided for the continuous multi-agent network with quantization and time delay. The existence of a global solution to the resulting system was firstly proved in the Filippov sense and then the authors proved that the solution converges to a practical consensus set asymptotically. Authors in Fu and Xie (2010) studied robust control problems under the setting of quantized feedback and they presented a number of results for different controller-quantizer configurations. Quantization effect on continuous-time systems has attracted considerable attention more recently (Ceragioli, Persis, & Frasca, 2011; Dimarogonas & Johansson, 2010; Frasca, 2012; Fu & Wang, 2014; Jiang & Liu, 2013). The authors studied the adaptive coordinated tracking problem for first-order integrator systems with quantized information in Fu and Wang (2014). In Jiang and Liu (2013), an overview of recent developments was presented on the control of linear and nonlinear systems when the control input was subject to quantization or the quantized states or outputs were used as feedback measurements.

The main limitation of most of the existing works is that they focused the quantization effect on the coordination problems for linear multi-agent systems. Some interesting works (Fu & Wang, 2015; Persis & Jayawardhana, 2012) studied the quantized consensus for nonlinear multi-agent systems. The authors in Persis and Jayawardhana (2012) investigated a passivity approach for

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coordination problems with quantized measurements. Also, the authors in Fu and Wang (2015) applied a new design of adaptive control law to the coordination problems under quantization effect. In Persis and Jayawardhana (2012) and Fu and Wang (2015), the coordination problem has been solved if the multi-agent system was a strictly passive system. However, when the nonlinear multi-agent system is not a strictly passive system, the consensus problem cannot be solved by utilizing the protocols designed in Persis and Jayawardhana (2012) and Fu and Wang (2015).

In many applications, control directions might not always be known in priori. Recently, some interesting results involving multi-agent systems with unknown control direction have been investigated (Chen, Li, Ren, & Wen, 2014; Ding, 2015). However, there are few results on the cooperation control problem under quantization effect for multi-agent systems with unknown control direction. To tackle the difficulty caused by unknown control direction and quantized information, the Nussbaum-type function is necessary to be used in controller protocol based on quantized information. Therefore, this paper is one of the first attempts to analyse the quantized consensus problem for nonlinear multi-agent systems with unknown parameters and control gains.

In this paper, we consider consensus problems for nonlinear multi-agent systems with non-identical unknown parameters and control gain when quantized measurements are available. Edge-based adaptive protocols under quantization effect are designed based on the tree edges of the graph. Hence, all agents achieve consensus by employing the designed protocol. The contributions of this paper are summarized as follows: (i) This paper studies the consensus problem under quantization effect for nonlinear multi-agent systems, and the dynamic of each agent does not satisfy the passive condition. Therefore, the protocols designed in Fu and Wang (2014), Persis and Jayawardhana (2012) and Fu and Wang (2015) cannot solve this consensus problem for the multi-agent systems of this paper. (ii) The distinguished feature of this paper is that the control gain of each agent is unknown. When each agent has unknown identical control direction, a Nussbaum-type item is then used in the protocol for each agent to seek control direction adaptively and cooperatively. (iii) Besides, this work explores the advantages of using edge-based adaptive protocols of which it is not available in the conventional graph Laplacian approach in the literature. It shows that the edge Laplacian provides a new perspective and useful approach to study adaptive consensus problem for nonlinear multi-agent systems. In addition, it is interesting that we can use the information on a tree structure to solve the consensus problem proposed in this paper, and to avoid the effect of the redundant information in the cycle structure.

The remainder of the paper is organized as follows. In Section 2, some preliminaries are briefly outlined. Problem formulation is presented in Section 3. Edge-based adaptive protocols under quantized measurements are proposed in Section 4. In Section 5, the effectiveness of the distributed adaptive consensus algorithms is demonstrated by the simulations. Finally, conclusions are drawn in Section 6.

2. Preliminaries

2.1. Quantizer

A logarithmic quantizer is an odd map $q : R \to R$ with the definition that

$$q_l(x) = \begin{cases} \exp(q_u(\ln(x))) & x > 0, \\ -q_l(-x) & x < 0, \\ 0 & x = 0 \end{cases} \tag{1}$$

where the uniform quantizer q_u is defined as $q_u(x) = \delta_u \lfloor \frac{x}{\delta_u} + \frac{1}{2} \rfloor$, and $|q_u(x) - x| \leq \delta_u/2$, where δ_u is a positive number. The quantization error for a logarithmic quantizer satisfies $|q_l(x) - x| \leq \delta|x|$, where δ is determined by $\delta = \exp(\delta_u/2) - 1$. It still holds that $xq_l(x) \geq 0, \forall x \in R$, the equality sign holds if and only if $x = 0$.

2.2. Graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote an undirected graph. $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of the graph $\mathcal{G}$. A path of length r from i to j is a sequence of $r + 1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. When $i = j$, this path is a cycle. If there is a path between any two vertices of graph $\mathcal{G}$, then graph $\mathcal{G}$ is connected. A connected graph is a tree if it contains no cycles. Equivalently, a tree is a connected graph with N vertices and $N - 1$ edges. The Laplacian matrix of graph $\mathcal{G}$ is L with $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$.

For an undirected graph $\mathcal{G}$ with N nodes and M edges, denote one end of each edge the positive end and the other negative end, then the $N \times M$ incidence matrix $D = [d_{il}]$ is defined as

$$d_{il} = \begin{cases} +1 & \text{if agent } i \text{ is the positive end of edge } l, \\ -1 & \text{if agent } i \text{ is the negative end of edge } l, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

By definition of D, it holds that $D^{\mathrm{T}}\mathbf{1}_N = \mathbf{0}$. The edge Laplacian matrix is defined as $L_e = D^{\mathrm{T}}D$. The Laplacian matrix $L = DD^{\mathrm{T}}$ has the property that all of nonzero eigenvalues of L are real and positive.

3. Problem formulation

Consider the first-order linearly parameterized multi-agent system and the dynamics of ith agent is described by

$$\dot{x}_i(t) = \theta_i\xi_i(x_i) + b_iu_i(t) \tag{3}$$

where $x_i \in R$ and $u_i \in R$ are the state and the input of ith agent. $\xi_i(x_i)$ is a known continuous nonlinear function and θ_i is an unknown constant. b_i is an unknown control gain for agent i.

Our control objective is to design an adaptive control strategy using quantized relative state information of each agent such that all agents achieve consensus, i.e., $|x_i(t) - x_j(t)| \to 0$ as $t \to \infty$ while the overall system maintain bounded.

Remark 1. We are not dealing with estimation of uncertain coefficients as in the existing adaptive control, but dealing with quantized information among nodes. The quantizer has a specific form of model as indicated in (1), while uncertain model may only require the uncertainty to be bounded. Hence a specific distributed adaptive coordination controller will be designed according to the local information.

If agents i and j are connected by the lth edge, then we can obtain $z_l = \sum_{k=1}^{N} d_{kl}x_k$. By the definition of matrix D, the variable $z(t)$ can be represented as

$$z(t) = D^{\mathrm{T}}x(t) \tag{4}$$

where $z = [z_1, \ldots, z_M]^{\mathrm{T}}$ and $x = [x_1, \ldots, x_N]^{\mathrm{T}}$.

Therefore, if a graph is connected, $z = 0$ implies that $x_1 = x_2 = \cdots = x_N$. This is due to that $z = 0$, then $Lx = 0$.

From the graph theory, if a connected graph is not a tree, it must have cycles. Then, for easy illustration of the main results in next section, we divide connected graphs into two different kinds as follows: (1) tree communication graph; (2) connected graph with cycle.

Lemma 1 (*Dimarogonas & Johansson, 2010*)**.** *If $\mathcal{G}$ is a tree, then the edge Laplacian matrix $L_e(\mathcal{G}) = D^{\mathrm{T}}D$ is positive definite.*

Remark 2. The literature (Zelazo, Rahmani, & Mesbahi, 2007) showed that there are strong connections between the node agreement problem and its edge version. The consensus problem for linear multi-agent systems can be solved based on node, and