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Adaptive output containment control of heterogeneous multi-agent systems with unknown leaders[☆]

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ABSTRACT

This paper investigates the adaptive output containment control of general linear heterogeneous multi-agent systems with multiple unknown leaders whose dynamics are only known to the neighboring followers. The output containment control objective is to assure the convergence of each follower's output to the dynamic convex hull spanned by the leaders' outputs. Compared with the situation where the leaders' dynamics are known to each follower exactly, local adaptive observers are presented at each follower to estimate the leaders' dynamics. Then, an adaptive tuning law is proposed to solve the associated output regulator equations without knowing leaders' dynamics. Two distributed control protocols, using state-feedback and dynamic output-feedback are locally designed, using the estimated leaders' dynamics and adaptive solutions to the output regulator equations. Both proposed control protocols are verified using numerical simulations.

1. Introduction

Cooperative control of multi-agent systems (MAS) has been an active research area given its broad applications in biology, physics, and engineering (Chen et al., 2012; Fax & Murray, 2004; Lewis, Zhang, Hengster-Movric, & Das, 2014; Zuo, Davoudi, Song, & Lewis, 2016). One of the important problems is called the leader–follower tracking problem, which aims at making each agent's state/output converge to that of a single leader (Ding, 2013). For the communication network with multiple leaders, the tracking problem is referred to as the containment problem to control each agent to enter a convex hull spanned by multiple leaders. State containment of homogeneous MAS has been studied for agents with different intrinsic dynamics (Cao, Ren, & Egerstedt, 2012; Mei, Ren, & Ma, 2012; Meng, Ren, & You, 2010; Yoo, 2013), where all agents have identical dynamics. In practice, the agents are heterogeneous in that the system matrices and even state dimensions can be different. There is limited amount of studies investigating the containment control of heterogeneous MAS (Haghshenas, Badamchizadeh, & Baradarannia, 2015; Wang, Hong, & Ji, 2014). Haghshenas et al. (2015) studied the problem in cooperative output regulation framework using state-feedback design. While system matrices can be different, the state dimensions of all agents are restricted to be identical. Hence, state containment is considered. Wang et al. (2014) studied the output containment of single-input single-output heterogeneous MAS. The relative degrees of all agents are restricted to be the same.

The above literature assumes that the leaders' dynamics are exactly known to each follower in the communication network. The output regulation of heterogeneous MAS with a single unknown leader was investigated in Cai, Lewis, Hu, and Huang (2015). In this paper, we consider the adaptive output containment of general linear heterogeneous MAS, where the leaders' dynamics are only known to the neighboring followers. There are three main contributions. First, local adaptive observers are proposed to estimate the leaders' dynamics. An adaptive algorithm solves the associated output regulator equations. Then, two different control protocols using state-feedback and dynamic output-feedback are proposed. Both control protocols use the estimated leaders' dynamics and the adaptive solutions to the output regulator equations. Third, the heterogeneous MAS considered can have different system matrices and different state dimensions.

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2. Preliminaries and problem formulation

2.1. Preliminaries in graph theory and notations

Suppose that the interaction among the followers is represented by a weighted digraph $\mathscr{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a finite set of n nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and the associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$. Here, $\mathcal{A}$ is constant. a_{ij} is the weight of edge (v_j, v_i), and $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. Define the in-degree matrix as $\mathcal{D} = \mathrm{diag}(d_i) \in \mathbb{R}^{n\times n}$ with $d_i = \sum_{j=1}^{n} a_{ij}$ and the Laplacian matrix as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Consider a group of agents composed of n followers and m leaders. Denote the sets of followers and leaders by $\mathscr{F} \triangleq \{1, \ldots, n\}$ and $\mathscr{L} \triangleq \{n + 1, \ldots, n + m\}$, respectively. $\mathcal{G}_k = \mathrm{diag}(g_i^k) \in \mathbb{R}^{n\times n}$ is the diagonal matrix of pinning gains from the kth leader to each follower. $g_i^k > 0$ if there is a link between the kth leader and the ith follower, otherwise, $g_i^k = 0$. Digraph $\bar{\mathscr{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ shows the interaction among the followers and the leaders. $\sigma_{\min}(X)$, $\sigma_{\max}(X)$, and $\sigma(X)$ are the minimum and maximum singular values, and the spectrum of matrix X, respectively. $\mathrm{vec}(X)$ is a column vector formed by all the columns of matrix X. $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix. $\mathbf{1}_n \in \mathbb{R}^n$ is a vector with all entries of one. Kronecker product is denoted by $\otimes$. The operator $\mathrm{diag}\{\cdot\}$ builds a block diagonal matrix from its arguments. The distance from $x \in \mathbb{R}^n$ to the set $\mathcal{C} \in \mathbb{R}^n$ in the sense of Euclidean norm is denoted by dist $(x, \mathcal{C})$, that is, dist $(x, \mathcal{C}) = \inf_{y\in\mathcal{C}} \|x - y\|_2$.

2.2. Problem formulation

The dynamics of the ith follower is given by

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i, \end{cases} \quad \forall i \in \mathscr{F} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^p$ are the state, input, and output of the ith follower, respectively. The dynamics of the kth leader is given by

$$\begin{cases} \dot{\zeta}_k = S\zeta_k, \\ y_k = R\zeta_k, \end{cases} \quad \forall k \in \mathscr{L} \tag{2}$$

where $S \in \mathbb{R}^{q\times q}$ and $R \in \mathbb{R}^{p\times q}$. $\zeta_k \in \mathbb{R}^q$ and $y_k \in \mathbb{R}^p$ are the state and output of the kth leader, respectively. The following assumptions are made in this paper.

Assumption 1. There exists a directed path from each leader $k \in \mathscr{F}$ to each follower $i \in \mathscr{L}$ in the digraph $\bar{\mathscr{G}}$.

Assumption 2. The real parts of the eigenvalues of S are non-negative.

Assumption 3. (A_i, B_i) is stabilizable and (A_i, C_i) is detectable, $\forall i \in \mathscr{F}$.

Assumption 4.

$$\mathrm{rank}\begin{bmatrix} A_i - \lambda I_{n_i} & B_i \\ C_i & 0 \end{bmatrix} = n_i + p, \forall \lambda \in \sigma(S), \forall i \in \mathscr{F}. \tag{3}$$

Definition 1 (Rockafellar, 2015). A set $\mathcal{C} \subseteq \mathbb{R}^n$ is convex if $(1 - \lambda)x + \lambda y \in \mathcal{C}$, for any $x, y \in \mathcal{C}$ and any $\lambda \in [0, 1]$. Let $Y_{\mathscr{L}} = \{y_{n+1}, y_{n+2}, \ldots, y_{n+m}\}$ be the set of the leaders' outputs. The convex hull $\mathrm{Co}(Y_{\mathscr{L}})$, spanned by the leaders' outputs, is the minimal convex set containing all points in $Y_{\mathscr{L}}$. That is, $\mathrm{Co}(Y_{\mathscr{L}}) = \left\{ \sum_{k=n+1}^{n+m} \gamma_k y_k \middle| y_k \in Y_{\mathscr{L}}, \gamma_k \in \mathbb{R}, \gamma_k \geqslant 0, \sum_{k=n+1}^{n+m} \gamma_k = 1 \right\}$.

Problem 1. The output containment control problem is to design control protocols u_i in (1) to assure the followers' outputs y_i converge to the convex hull spanned by the leaders' outputs y_k. That is

$$\lim_{t\to\infty} \mathrm{dist}(y_i(t), \mathrm{Co}(Y_{\mathscr{L}}(t))) = 0, \ \forall i \in \mathscr{F}. \tag{4}$$

Define the local relative output information as

$$e_{y_i} \equiv \sum_{j=1}^{n} a_{ij}(y_j - y_i) + \sum_{k=n+1}^{n+m} g_i^k (y_k - y_i). \tag{5}$$

The global form of (5) can be written as

$$e_y = -\sum_{k=n+1}^{n+m} (\Phi_k \otimes I_p)(y - \bar{y}_k), \tag{6}$$

where $e_y = [e_{y_1}^T, \ldots, e_{y_n}^T]^T$, $y = [y_1^T, \ldots, y_n^T]^T$, $\bar{y}_k = \mathbf{1}_n \otimes y_k$, and $\Phi_k = \frac{1}{m}\mathcal{L} + \mathcal{G}_k$. Define the following global output containment error vector

$$e = y - (\sum_{r=n+1}^{n+m} (\Phi_r \otimes I_p))^{-1} \sum_{k=n+1}^{n+m} (\Phi_k \otimes I_p)\bar{y}_k \tag{7}$$

where $e = [e_1^T, \ldots, e_n^T]^T$ and $e_y = -\sum_{k=n+1}^{n+m}(\Phi_k \otimes I_p)e$. The following technical results are needed to solve Problem 1.

Lemma 1 (Haghshenas et al., 2015). *Given Assumption 1, one has the following: (i) Matrices Φ_k and $\sum_{k=n+1}^{n+m}\Phi_k$ are positive-definite and non-singular M-matrices; (ii) $(\Phi_k)^{-1}$ and $(\sum_{k=n+1}^{n+m}\Phi_k)^{-1}$ exist and are non-negative.*

Lemma 2 (Zuo, Song, Lewis, & Davoudi, 2017). *Given Assumption 1, consider heterogeneous MAS (1) and (2), Problem 1 is solved if $\lim_{t\to\infty} e(t) = 0$.*

3. Adaptive solutions to output regulator equations

The following technical result is needed in this section.

Lemma 3 (Huang, 2004). *Given Assumption 4, the following output regulator equations have unique solutions.*

$$\begin{cases} A_i\Pi_i + B_i\Gamma_i = \Pi_i S, \\ C_i\Pi_i = R. \end{cases} \quad \forall i \in \mathscr{F} \tag{8}$$

Unique solutions to (8) are needed for consensus of heterogeneous MAS. However, solving (8) requires knowledge of the leaders' dynamics (S, R). To solve (8) online without knowing the leaders' dynamics, let $Y = [S^T \quad R^T]^T$ and $\hat{Y}_i = [\hat{S}_i^T \quad \hat{R}_i^T]^T$. Define the local distributed observers for (S, R) as

$$\dot{\hat{Y}}_i = \kappa(\sum_{j=1}^{n} a_{ij}(\hat{Y}_j - \hat{Y}_i) + \sum_{k=n+1}^{n+m} g_i^k (Y - \hat{Y}_i)), \tag{9}$$

where the scalar coupling gain $\kappa > 0$.

Theorem 1. *Given Assumption 1, one obtains*

$$\tilde{Y}_i(t) \equiv Y - \hat{Y}_i(t) \to 0, \forall i \in \mathscr{F}. \tag{10}$$

Proof. The global estimation error dynamics of Y are

$$\dot{\tilde{Y}} = \mathbf{1}_n \otimes \dot{Y} - \dot{\hat{Y}} = -\kappa \sum_{k=n+1}^{n+m} (\Phi_k \otimes I_{(q+p)})\tilde{Y}. \tag{11}$$

From Lemma 1, $\sum_{k=n+1}^{n+m}(\Phi_k \otimes I_{(q+p)})$ is positive-definite. Therefore, $\tilde{Y}_i(t) \to 0$ asymptotically. ■

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