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An improved dynamic quantization scheme for uncertain linear networked control systems☆

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ABSTRACT

A Lyapunov–Krasovskii Functional (LKF)-based dynamic quantization strategy was innovated in the paper 'Dynamic quantization of uncertain linear networked control systems'. Though effective and comprehensive, it is conservative in terms of converging speed and the upper bound of the system states. This paper aims at improving the zooming-in algorithm to bring faster convergence of the closed-loop system. A more accurate upper bound for the system states is also obtained with smaller initial value and faster decay rate. The effectiveness of the improvements is illustrated by simulation.

1. Introduction

Networked control systems (NCSs) have become a popular field of study in recent years. The central concerns are time delay, quantization and packet dropout that exist in data transmission of NCSs, and linear matrix inequality (LMI)-based stability criteria for NCSs were derived in Han, Liu, and Yang (2015), Yang, Shi, Liu, and Gao (2011) and Yue, Tian, and Han (2013).

Since quantization errors prevent quantized systems from achieving asymptotical convergence, dynamic quantizers with zooming schemes were studied in Brockett and Liberzon (2000) and Liberzon (2003, 2006), where the zoom variables decrease along with the shrinking of the system states. The zooming scheme was applied to stabilize NCSs with packet losses in Yang, Xu, and Zhang (2016) and planar nonlinear systems with quantization in Yang, Xu, Xia, and Zhang (2017) respectively. The idea of dynamic quantization was extended to the case of Lyapunov–Krasovskii Functional (LKF) in Liu, Fridman, and Johansson (2015), where time delay and sampled data were considered. The constructed dynamic quantization strategy guarantees no saturation for the quantizers, and an exponentially decaying upper bound for the system states is also derived. This dynamic quantization strategy was further incorporated with round-robin scheduling in Liu, Fridman, Johansson, and Xia (2016).

Through careful examination of the derivation and the underlying logic of Liu et al. (2015), we found that the dynamic quantization algorithm and the upper bound for system states are conservative. This paper gives the following improvements: (1) A shorter time T_{new} is found to allow the LKF to decay to a certain proportion; a smaller time margin for the dynamic quantization algorithm is further proposed to be sufficient to tackle the variation of time delay. Combining these two improvements, we can achieve more frequent updates of the zoom variable, leading to faster convergence of the system states while guaranteeing that no quantizer saturation occurs. (2) A smaller upper bound for the system states in terms of both initial value and decay rate is found to enable a closer estimation on the convergence of the system states.

The rest of the paper is organized as follows. In Section 2 the improved zooming-in algorithm is designed to bring faster convergence. The smaller upper bound for the system states is obtained in Section 3. The effectiveness of the algorithm is illustrated in Section 4. Finally, conclusions are stated in Section 5.

2. Improved zooming-in algorithm

Consider the closed-loop model of the networked control system with delayed quantized feedback (6) in Liu et al. (2015):

$$\dot{x}(t) = Ax(t) + A_1 x(t_k - \eta_k) + \sum_{i=1}^{N} B_i \omega_i(t), \quad t \in [t_k, t_{k+1}) \tag{1}$$

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where $x \in \mathbb{R}^n$, $A, A_1 \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times n_i}$, $i = 1, \ldots, N$, η_k is the communication delay $\eta_k = t_k - s_k$, $0 \le \eta_m \le \eta_k \le \eta_M$, s_k is the sequence of sampling instants,

$$0 = s_0 < s_1 < \cdots < s_k < \cdots, \qquad s_{k+1} - s_k \le MATI.$$

MATI is the maximum allowable transmission interval. $\omega_i(t) = q_{i\mu}(C_i x(s_k)) - C_i x(s_k) \in \mathbb{R}^{n_i}$, $i = 1, \ldots, N$ represent the quantization errors. If $|C_i x(s_k)| \le \mu M_i$, then the i'th quantizer does not saturate and $|\omega_i(t)| \le \mu \Delta_i$, μ is the quantization parameter, Δ_i and M_i are the quantization error bounds and ranges, respectively. By applying the time-delay approach and constructing a Lyapunov–Krasovskii functional (LKF) $V(t, x_t, \dot{x}_t)$ (also written as V or $V(t)$ for simplicity later in this paper), it is obtained in Lemma 1 of Liu et al. (2015) that when certain LMIs are feasible, the following inequality holds:

$$\frac{d}{dt}V \le -2\alpha V + \sum_{i=1}^{N} b_i \mu^2 |\omega_i(t)|^2$$

then we have

$$\begin{aligned} V(t) &\le e^{-2\alpha(t-t_0)} V(t_k) + \mu^2 \sum_{i=1}^{N} b_i \Delta_i^2 \int_{t_0}^{t} e^{-2\alpha(t-s)} ds \\ &= e^{-2\alpha(t-t_0)} V(t_0) + \frac{\mu^2}{2\alpha} \sum_{i=1}^{N} b_i \Delta_i^2 \left(1 - e^{-2\alpha(t-t_0)}\right) \\ &= \left(V(t_0) - \frac{\mu^2}{2\alpha} \sum_{i=1}^{N} b_i \Delta_i^2\right) e^{-2\alpha(t-t_0)} + \frac{\mu^2}{2\alpha} \sum_{i=1}^{N} b_i \Delta_i^2. \end{aligned} \quad (2)$$

Remark 1. In (2), the integral is calculated instead of using the inequality

$$\int_{t_0}^{t} e^{-2\alpha(t-s)} ds < \int_{t_0}^{\infty} e^{-2\alpha(t-s)} ds = \frac{1}{2\alpha}$$

as in Liu et al. (2015), which leads to

$$V(t) \le V(t_0) e^{-2\alpha(t-t_0)} + \frac{\mu^2}{2\alpha} \sum_{i=1}^{N} b_i \Delta_i^2.$$

Hence, the result is smaller. Then by exploiting (2), we present the alternative for Lemma 2 of Liu et al. (2015):

Lemma 1. *Given $M_j > 0$, $j = 0, 1, \ldots, N$, $\Delta_i > 0$, $i = 0, 1, \ldots, N$, $0 < \eta_m < \tau_M$ and tuning parameters $\alpha > 0$, $0 < \nu < 1$, assume that there exist scalars $0 < \beta < 1$, $i = 1, \ldots, N$, $n \times n$ matrices $P > 0$, $S_0 > 0$, $R_0 > 0$, $S_1 > 0$, $R_1 > 0$, S_{12} such that the LMIs in Lemma 1 of Liu et al. (2015) and*

$$M_0^2 C_i^T C_i < P M_i^2, i = 1, \ldots, N \quad (3)$$

$$\frac{1}{2\alpha} \sum_{i=1}^{N} b_i \Delta_i^2 < \beta \nu^2 M_0^2 \quad (4)$$

hold. Let μ >0 be constant. Then the solutions of (1) with initial condition $x^T(t_0) P x(t_0) < \mu^2 M_0^2$ satisfies the followings:
(i) *The quantizers do not saturate at s_1, . i.e. $|C_i x(s_1)| = |y_i(s_1)| < \mu M_i$;*
(ii) *$V(t) < e^{-2\alpha(t-t_0)}\left(1 - \beta\nu^2\right)\mu^2 M_0^2 + \beta\nu^2\mu^2 M_0^2$ for $t > t_0$;*
(iii) *After finite time $T_{new} = -\frac{\ln \frac{\nu^2(1-\beta)}{1-\beta\nu^2}}{2\alpha}$, it is guaranteed that $V(t) < \nu^2\mu^2 M_0^2$.*

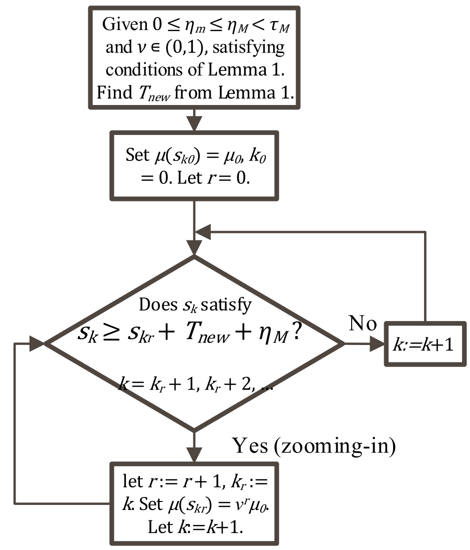

Fig. 1. The zooming-in algorithm for dynamic quantization.

Proof. (i) The same as Liu et al. (2015).
(ii) We know from (2) and (4) that the solution of (1) with initial condition $x^T(t_0) P x(t_0) < \mu^2 M_0^2$ satisfies

$$\begin{aligned} V(t) &< \left(\mu^2 M_0^2 - \beta\nu^2\mu^2 M_0^2\right) e^{-2\alpha(t-t_k)} + \beta\nu^2\mu^2 M_0^2 \\ &= e^{-2\alpha(t-t_i)}\left(1 - \beta\nu^2\right)\mu^2 M_0^2 + \beta\nu^2\mu^2 M_0^2. \end{aligned}$$

(iii) It is clear from point (ii) that when t is large enough such that $e^{-2\alpha(t-t_k)} = \frac{\nu^2(1-\beta)}{1-\beta\nu^2}$, then, it is held that $V(t) < \nu^2\mu^2 M_0^2$, i.e. the LKF evolves from $V(t_0) < \mu^2 M_0^2$ to $V(t) < \nu^2\mu^2 M_0^2$ within time $T_{new} = -\frac{\ln \frac{\nu^2(1-\beta)}{1-\beta\nu^2}}{2\alpha}$. □

Due to the adoption of a smaller bound of V, the value of T_{new} is smaller than that obtained in Liu et al. (2015), which is $T = -\frac{\ln \nu^2(1-\beta)}{2\alpha}$. Next we propose the modified dynamic quantization algorithm as shown in Fig. 1. The difference between the algorithm and that of Liu et al. (2015) is that the zooming-in condition becomes $s_k \ge s_{k_r} + T_{new} + \eta_M$ instead of $s_k \ge s_{k_r} + T + 2\eta_M - \eta_m$, thus leads to more frequent zooming-in. In the following proposition, we prove that our zooming-in algorithm can also, like that of Liu et al. (2015), prevent quantizer saturation.

Proposition 1. *Under the zooming-in algorithm in Fig. 1 and the conditions of Lemma 1 as well as*

$$M_0^2 C_i^T C_i < P M_i^2, \quad i = 1, \ldots, N \quad (5)$$

the solutions of (1) that start with $x^T(t_0) P x(t_0) < \mu^2 M_0^2$ satisfy $|C_i x(s_k)| = |y_i(s_k)| < \mu M_i$ for $i = 1, \ldots, N$ at every sampling instant, that is, there is no quantizer saturation.

Proof. We know from the proposed zooming mechanism that

$$\begin{aligned} s_{k_r} - t_{k_{r-1}} &= s_{k_r} - \left(s_{k_{r-1}} + \eta_{k_{r-1}}\right) \\ &\ge T_{new} + \eta_M - \eta_{k_{r-1}} \ge T_{new} \end{aligned} \quad (6)$$

which guarantees a time interval of length at least T_{new} for the LKF to evolve before the next sampling interval.

From (6), we get $s_{k_1} > t_0 + T_{new}$. Combining with Lemma 1, we have $V\left(s_{k_1}\right) < \nu^2\mu_0^2 M_0^2$. Then, according to (5), the following holds:

$$x^T(s_k) C_i^T C_i x(s_k) < \frac{x^T(s_k) P x(s_k) M_i^2}{M_0^2} < \nu^2 \mu_0^2 M_i^2$$

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