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Distributed adaptive consensus and synchronization in complex networks of dynamical systems ☆

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ABSTRACT

In this paper we propose novel distributed adaptive controllers for leaderless synchronization in networks of identical discrete-time dynamical systems. Separate algorithms are developed for the cases of known and unknown control directions. Assuming that the directed network graph is strongly connected, it is proved that all agent outputs converge toward an emerging, unknown in advance, synchronization trajectory. This trajectory is not available for use by the agent's controllers, and its pattern is determined by the internal model built-in in the distributed adaptation mechanism. It is also shown that for each agent the controller parameter estimates are convergent sequences, and the inputs are uniformly bounded signals. The key to obtaining presented results is to construct adaptive algorithms providing l_1-boundedness of certain error signals driving the local parameter estimators. The proposed estimators have a non-vanishing step-size.

1. Introduction

Consensus and synchronization phenomena represent a form of cooperative behavior characterized by a global coherent activity and emergence of spontaneous order in a population (system) of interacting members (agents). Synchronization processes play an important role in a variety of natural and man-made systems in biology, physics, chemistry, social environment, and engineering. The so called 'nearest neighbors' rule (Vicsek, Czirok, Jacob, Cohen, & Schochet, 1995) and its theoretical analysis (Jadbabaie, Lin, & Morse, 2003) were motivational catalyst for numerous results on distributed control for multi-agent synchronization. This paper considers the case where there is no leader among agents, and the synchronization trajectory emerges spontaneously as a result of inter-agent interactions. As it is pointed out in Ren (2009) leaderless consensus is at play in situations where the specifically predetermined "consensus equilibrium is not what is important, but rather that each system converges to an identical state" (see p.2138 in Ren (2009)). Flocking, rendezvous and attitude stabilization problems are examples of such instances (Ren, 2009). The existing results on *adaptive leaderless* synchronization prove that the error between the states of any two agents converges to zero. As it is discussed below (see the paragraph 'Context for the results of this paper') this assertion does not necessarily implies that the states of all agents simultaneously converge toward the same consensus trajectory. This paper offers novel adaptive controllers for leaderless synchronization, and it demonstrates that outputs of all agents converge to the same, unknown to agents in advance, consensus trajectory. Since the topic of this paper is the problem of adaptive consensus/synchronization based on internal model principle, we do not intend to comment in detail on the work related to non-adaptive, and non-internal model based methods. We make only brief mention of the results involving a fixed gain distributed controllers. A considerable body of work on multi-agent systems is related to the leader-following problem. This topic has been studied in a variety of cases including linear (Cao, Zhang, & Ren, 2015; Grip, Yang, Saberi, & Stoorvogel, 2012; Ni & Cheng, 2010; Su, Chen, Lam, & Lin, 2013; Su & Huang, 2012) and nonlinear (Meng, Lin, & Ren, 2013; Su & Huang, 2013, 2014; Xu, Wang, Hong, & Jiang, 2016) agent dynamics, identical (Cao et al., 2015; Ni & Cheng, 2010; Su et al., 2013) and heterogeneous agents (Grip et al., 2012; Meng et al., 2013; Su & Huang, 2012, 2013, 2014), presence of system uncertainties or uncertain systems (Su & Huang, 2013, 2014), as well as fixed and time-varying communication topologies. Similarly, the leaderless consensus problem has been explored for both linear and nonlinear agent dynamics under various scenarios of inter-agent information exchange, and diverse conditions related to graph topologies and system uncertainties (Burbano-L & di Bernardo, 2016; Scardovi & Sepulchre, 2009; Seo,

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Shim, & Back, 2009; Su & Huang, 2012; Trentelman, Takaba, & Monshizadeh, 2013; Zhu & Chen, 2014). Conditions sufficient for the existence of a synchronizing control law based on agent output exchange are derived in Ma and Zhang (2010), and Tuna (2009). In addition to the above, a large number of references on non-adaptive consensus can be found for example in the survey paper (Cao, Yu, Ren, & Chen, 2013) and recent monograph (Ren & Cao, 2011).

Internal model based non-adaptive synchronization: Important results establishing the connection of synchronization processes with the internal model principle are presented in Kim, Shim, and Seo (2011), Lunze (2012) and Wieland, Sepulchre, and Allgower (2011). It is shown that a synchronization of agents with linear dynamics implies the existence of an observable virtual exosystem (internal model) which defines the output trajectories on the agreement manifold, and is contained within each agent as an internal model. The internal model can be embedded in the controller dynamics without requiring for it to be a part of agent dynamics. Generalization of the results in Wieland et al. (2011) is presented in Grip, Saberi, & Stoorvogel (2013). Extension of the internal model principle to robust output synchronization of a nonlinear heterogeneous agents can be found in a number of interesting papers (Chen, 2014; Isidori, Marconi, & Casadei, 2014; Su & Huang, 2015; Xu et al., 2016; Zhu, Chen, & Middleton, 2016).

Recent work on adaptive consensus: The theory of adaptive consensus deals with two distinct issues: (1) the leader-following problem where each agent follows the leader's state, and (2) the problem of leaderless consensus where the aim is not to follow predetermined leader, but rather for all agents to achieve agreement on a common, unknown in advance value of their state. Because the subject of our work is a leaderless synchronization, we only briefly comment on the leader-following literature. This problem has been considered for various known and unknown, linear and nonlinear agent dynamics, fixed (undirected and directed) and time-varying graph topologies, identical and heterogeneous agents, as well as leaders with known and unknown inputs (see for example: Bai, Arcak, and Wen, 2009, Das and Lewis, 2010, Ding, 2015, Li, Wen, Duan, and Ren, 2015, Liu, 2015, Su, 2015, Sun, Geng, and Lv, 2016, Tang, Hong, and Wang, 2015, Wang, Wang, and Ji, 2016 Yu and Xia, 2012, Yu, Shen, and Xia, 2013).

Interesting results on leaderless consensus are presented in Li, Ren, Liu, and Fu (2013); Li, Ren, Liu, and Xie (2013) where it is proved that the difference between each agent state and the average of all agent states converges to zero. In Li, Ren, Liu and Xie (2013) the case of known identical linear agent dynamics and undirected graph topology is analyzed, whereas in Li, Ren, Liu and Fu (2013) identical linear and Lipschitz nonlinear dynamics on directed graphs is studied. Similar results are presented in Li & Ding (2015) where it is assumed that all agents have identical linear non-minimum phase SISO dynamics and strongly connected directed graph topology. There it is shown that the error between the outputs of any two agents tends to zero. All of the above mentioned references on adaptive consensus assume that the high-frequency gain or its sign (control direction) is known. Adaptive consensus in case of unknown control directions is treated in Chen, Li, Ren, and Wen (2014), Ding (2015), Junmin and Xudong (2014), Liu (2015), and Su (2015), based on the Nussbaum gain concept. For the first time this problem has been addressed in Chen et al. (2014) and Junmin and Xudong (2014) where the authors consider undirected graph topology, and show that the error between the states of any two agents converges to zero.

Context for the results of this paper: In the non-adaptive leaderless case it has been demonstrated that under certain conditions there exist distributed controllers such that all agent outputs, say $x_i(t)$, reach agreement on a common, bounded reference (synchronization) trajectory, say $x_c(t)$, that is $x_i(t) - x_c(t) \to 0$, as $t \to \infty$, $i = 1, \ldots, N$, where N is the number of agents (see for example Isidori et al. (2014), Kim et al. (2011), Lunze (2012), Wieland et al. (2011), Zhu et al. (2016)). To prove this claim in case of *adaptive leaderless* consensus is a rather challenging proposition and it is still an open problem. The existing results on this topic are limited to showing that the difference between agent outputs, or the error between any agent output and the average of all agent outputs, converges to zero, i.e., $x_i(t)-x_j(t) \to 0$, or $x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t) \to 0$, as $t \to \infty$ (see for example, Chen et al. (2014), Junmin and Xudong (2014), Li and Ding (2015), Li, Ren, Liu and Fu (2013)). Note that the above assertions do not imply that there exists a $\{x_c(t)\}$ (in case of the previously mentioned work, a constant x_c) so that $x_i(t)-x_c \to 0$, as $t \to \infty$, $i = 1, \ldots, N$. Take for example $x_i(t) = \log(t + i)$, or $x_i(t) = \cos(\log(t + i))$ for $i = 1, \ldots, N$.

Results of the paper: In this paper we show that it is possible for *adaptive leaderless* consensus to achieve the same synchronization results as those for a non-adaptive case. It is proved that the proposed distributed controllers guarantee emergence of a synchronization trajectory, say $x_c(t)$, $t = 0, 1, \ldots$, such that all agent outputs $x_i(t)$, $i = 1, \ldots, N$, satisfy $x_i(t) - x_c(t) \to 0$, as $t \to \infty$. Compared to Chen et al., (2014), Junmin & Xudong (2014), Li and Ding (2015) and Li, Ren, Liu and Fu (2013) where $x_c(t)$ is a constant, we allow arbitrary form for $x_c(t)$. The trajectory $\{x_c(t)\}$ is not physically measurable nor known in advance to agents, and it is a uniformly bounded sequence whose pattern is determined by the initial conditions and the internal model built-in in the agents' adaptive controllers. In order to derive the above proposition it is essential to show that the errors between the output of individual agents and certain averages of neighboring agent states are l_1 sequences. For this purpose, novel distributed adaptive controllers have been developed to generate such error signals. To our knowledge this is the first result providing l_1 performance in an adaptive synchronization setting. It is also proved that the parameter estimates of local controllers are convergent sequences. We consider networked discrete-time systems with identical, general order, linear dynamics, and separately analyze the algorithms for the cases of known and unknown control directions. In case of unknown control direction we present a novel algorithm, different than the existing discrete-time counterpart (Lee & Narendra, 1986). There the parameter estimator involves a discontinuous function termed "discrete-Nussbaum gain", whose values are either (+1) or (−1) depending on the output of an appropriately defined switching algorithm. Our solution for the unknown control directions entails a "smooth" controller without involving any switching logic.

The paper is organized as follows. The problem statement is given in Section 2. Section 3 presents the case of known control directions. The systems with unknown control directions are considered in Section 4.

Notation: The superscript T denotes the transpose of a matrix; $\|x\|$ is the Euclidean norm of vector x; $\operatorname{sgn}(a)$ is the sign function of a real number a; l_p, $p \in [1, \infty)$ denotes the normed infinite dimensional vector space of sequences $\{s(t)\}$, $t \geq 0$, satisfying $\sum_{t=0}^{\infty}|s(t)|^p < \infty$; $\rho(W)$ is the spectral radius of matrix W; α is used to denote vector of all ones, i.e., $\alpha = [1, 1\ldots, 1]$, and I is identity matrix. The dimensions of α and I are determined by the context. In this paper c_i ($i = 1, 2, \ldots,$) are used to denote nonnegative constants whose specific values are unimportant.

2. Problem formulation

Consider a networked system of N identical agents whose dynamics is given by:

$$A(q^{-1})x_i(t+1) = b_0 B(q^{-1})u_i(t), \quad b_0 \neq 0 \tag{1}$$