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Improved Razumikhin and Krasovskii approaches for discrete-time time-varying time-delay systems[☆]

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ABSTRACT

This paper studies stability of discrete-time time-varying time-delay systems. The existing Razumikhin and Krasovskii stability approaches for this class of systems are improved in the sense that the time-shifts of the Razumikhin functions and Krasovskii functionals are allowed to take both negative and positive values. Three kinds of stability concepts, say, uniform stability, uniformly asymptotic stability and uniformly exponential stability, are considered. The improvements of the Razumikhin and Krasovskii approaches are achieved by using the concept of uniformly asymptotically stable (UAS) function, the notion of overshoot associated with the UAS function and an improved comparison lemma. Both delay-dependent and delay-independent stability theorems are obtained for a class of discrete-time linear time-delay systems by using the improved Razumikhin and Krasovskii stability approaches. Numerical examples demonstrate the effectiveness of the proposed methods.

1. Introduction

Time-delay systems have received renewed attention in recent years because their important applications in engineering (Fridman, Seuret, & Richard, 2004; Lam, Xu, Ho, & Zou, 2012; Mazenc & Malisoff, 2010) and enormous challenge in theory (Chen, Zhong, & Zheng, 2016; Hale, 1971; He, Wang, Lin, & Wu, 2007; Sun, Liu, Wen, & Wang, 2016; Zhang & Han, 2015). Stability and stabilization are two fundamental problems that have extensively studied for time-delay systems (see, He et al., 2007; Kojima, Uchida, Shimemura, & Ishijima, 1994; Michiels & VyhliDal, 2005; Nam, Phat, Pathirana, & Trinh, 2016; Wu, Park, Su, & Chu, 2012; Zhang, Liu, Feng, & Zhang, 2013; Zhang, Xu, & Zou, 2008 and the references therein). The Lyapunov's second method as the most important approach in stability theory has been extended to time-delay systems in two ways. One is the Krasovskii approach, which replaces the Lyapunov function for delay-free systems with a Lyapunov functional (referred to as Krasovskii functional in this paper), which is a memory function containing the history information of the state vector. By this approach the stability is guaranteed if the time-derivative of the Krasovskii functional is negative definite (Gu, Chen, & Kharitonov, 2003; Hale, 1971). Another one is the Razumikhin approach, which also uses a Lyapunov function and by which the stability is asserted if the time-derivative of the Lyapunov function is negative definite under the so-called Razumikhin condition (Gu et al., 2003; Hale, 1971). In some cases, both the Razumikhin and the Krasovskii approaches are effective. However, in case of time-varying delays, the Razumikhin approach may be more easy to use as it generally does not need the exact information of the time-varying delay. Very recently, with the help of the Lyapunov's second method and by establishing some advanced inequalities (such as Wirtinger-based inequalities Nam, Pathirana, & Trinh, 2015 and summation inequalities Seuret, Gouaisbaut, & Fridman, 2015; Zhang, He, Jiang, & Wu, 2017), stability of time-delay systems has been re-examined carefully.

Both the Razumikhin and Krasovskii approaches have been developed for both continuous-time and discrete-time time-delay systems. However, as pointed in Elaydi and Zhang (1994) and Liu and Marquez (2007), since the solution of a discrete-time system is not a continuous or right-continuous function, which is different from the continuous-time system, the Razumikhin technique for discrete-time time-delay system is more challenging than the Krasovskii technique. As a result, there are two different types of Razumikhin methods for discrete-time time-delay systems in the literature, one is the so-called backward Razumikhin approach (Elaydi & Zhang, 1994; Zhang & Chen, 1998) and the other one is the so-called forward Razumikhin approach (Liu & Marquez, 2007). Formally speaking, the backward Razumikhin approach looks more like the corresponding discrete-time version of the Razumikhin approach for continuous-time time-delay systems. However, by this approach the time-shift (under the Razumikhin condition) of the Razumikhin function is given implicitly. The

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forward Razumikhin approach overcomes this shortcoming, while an additional (Razumikhin-type) condition should be imposed (Liu & Marquez, 2007). The Razumikhin approach has been extended in many aspects, for example, stochastic systems (Peng & Deng, 2017), hybrid systems (Liu & Sun, 2016; Liu & Teel, 2016), ISS analysis (Teel, 1998), practical stability analysis (Su, Zhang, Liu, & Su, 2010), and functional difference systems (Pepe, 2014). In addition to the Razumikhin and Krasovskii approaches, the Halanay-type inequalities based approach has also been built for discrete-time time-delay systems (see, for example, Mohamad and Gopalsamy (2000) and Xu, Feng, Zou, and Huang (2012)). It seems that the Razumikhin methods in Gielen, Teel, and Lazar (2013) and Pepe (2014) look more like Halanay-type inequalities based approaches.

For both continuous-time and discrete-time time-delay systems, the conventional Razumikhin and Krasovskii techniques require that the time-derivative (time-shift) of the corresponding Razumikhin function and Krasovskii functional are negative definite (under the Razumikhin condition in the former case). Motivated by the existing methods dealing with delay-free systems that the time-derivatives (time-shifts) are allowed to take both negative and positive values (with some additional requirements) (Zhou, 2016), we recently established generalized Razumikhin and Krasovskii approaches for continuous-time time-delay systems, in the sense that the time-derivatives of the corresponding Razumikhin function and Krasovskii functional are no longer required to be strictly negative definite (Zhou & Egorov, 2016). The generalization is achieved by using the uniformly stable function introduced in Zhou (2016) and a time-varying comparison lemma. It has been shown in Zhou and Egorov (2016) that the generalized Razumikhin and Krasovskii approaches can indeed provide much less conservative results than the conventional ones. The results in Zhou and Egorov (2016) were further extended to stochastic systems in Zhou and Luo (2018). A supplement to the Razumikhin-like theorem of Zhou and Egorov (2016) was later provided in Egorov and Zhou (2016). Relaxing the negative definite requirement in the conventional Razumikhin and Krasovskii methods was also investigated in Mazenc and Malisoff (2017), where a strictification technique for converting a non-strict Lyapunov function into a strict one was established. The technique in Mazenc and Malisoff (2017) is also applied on discrete-time time-delay systems, for which the well-known discrete-time Halanay lemma is improved.

The aim of the present paper is to extend the methods in Zhou and Egorov (2016) to the discrete-time setting. To be more specific, we will provide generalized Razumikhin and Krasovskii stability theorems in which the time-shifts of the Razumikhin function and the Krasovskii functional are no longer restricted to be negative definite (under the Razumikhin condition for the former case). Similar to the continuous-time setting, our improvements to the classical Razumikhin and Krasovskii approaches are based on our recent work on stability analysis of discrete-time linear time-varying systems (Zhou & Zhao, 2017), particularly, the uniformly stable (US) and uniformly asymptotically stable (UAS) functions introduced there. The results presented in this paper are not simple extensions of those for the continuous-time setting in Zhou and Egorov (2016). Firstly, different from the continuous-time setting considered in Zhou and Egorov (2016), we will establish theorems for three different stability concepts in this paper, say, uniformly stability (US), uniformly asymptotic stability (UAS) and uniformly exponential stability (UES) (for simplicity, we use US to denote both "uniformly stable" and "uniform stability" throughout this paper. The same explanation is applicable to UAS and UES). Secondly, different from the continuous-time setting, where the function $q(s)$ involved in the Razumikhin condition is linear, the corresponding function $q(s)$ is not so in this paper, which has been motivated by the existing Razumikhin stability theorems (Elaydi & Zhang, 1994; Zhang & Chen, 1998) and the supplement given in Egorov and Zhou (2016) to the Razumikhin theorem of Zhou and Egorov (2016). As a result, a different technique has been developed to establish the UAS theorem. Finally, different from the continuous-time setting considered in Zhou and Egorov (2016) where the overshoot (see Definition 3) of a UAS function dependents on a number in its uniformly convergence set (see Definition 2), motivated by Zhou and Luo (2018), we are able to show in this paper that the overshoot of a given UAS function is a constant depending only on the UAS function itself, which makes the stability theorems more easy to use. In this paper we also apply the improved Razumikhin and Krasovskii methods on the stability analysis of some discrete-time linear time-varying time-delay systems and both delay-dependent and delay-independent stability conditions are established. It is shown by numerical examples that the established approaches can indeed provide much less conservative results than the existing ones, and, in some cases, can even provide necessary and sufficient stability conditions.

The remainder of this paper is organized as follows. Problem formulation and preliminaries on US and UAS functions are given in Section 2 where a discrete-time time-varying comparison lemma is also provided. The improved Razumikhin and Krasovskii stability approaches are then established in Section 3. Applications of the improved Razumikhin and Krasovskii methods on stability analysis of discrete-time linear time-varying time-delay systems are provided in Section 4. Two numerical examples are given in Section 5 to illustrate the effectiveness of the proposed approaches. The paper is concluded in Section 6.

Notation: For any integers p and q with $p \le q$, we denote $\mathbf{I}[p,q] = \{p, p+1, \ldots, q\}$ and $\mathbf{I}[p,\infty) = \{p, p+1, \ldots\}$. Particularly, we denote $J = \mathbf{I}[0,\infty)$. Let $\mathscr{D}_{n,r}$, where $n \ge 1$ and $r \ge 1$ are two integers, denote the space of continuous vector functions mapping the set $\mathbf{I}[-r,0]$ into $\mathbf{R}^n$. For a real number x, the function $\lceil x \rceil$ rounds x to the nearest integer towards infinity. For any function f and two integers a, b with $a \le b$, we denote $|f|_{[a,b]} = \sup\{|f(s)|, s \in \mathbf{I}[a,b]\}$, where $|\cdot|$ denotes the usual Euclidean norm (or 2-norm) of its argument. For any $\varphi \in \mathscr{D}_{n,r}$, we denote $\|\varphi\| = \sup_{s\in\mathbf{I}[-r,0]}\{|\varphi(s)|\} = |\varphi|_{[-r,0]}$. Let $\mathcal{K}$ denote the set of continuous functions $\psi(\cdot) : [0,\infty) \to [0,\infty)$, which is strictly increasing and $\psi(0) = 0$. If $\psi \in \mathcal{K}$ and moreover $\lim_{s\to\infty}\psi(s) = \infty$, then it is denoted by $\psi \in \mathcal{K}_\infty$. For a positive definite matrix $P \in \mathbf{R}^{n\times n}$, we use $P^{\frac{1}{2}}$ to denote the unique positive definite matrix $X \in \mathbf{R}^{n\times n}$ satisfying $X^2 = P$. For a number $h > 0$, we denote $B_h = \{x \in \mathbf{R}^n : |x| \le h\}$ and $S_h = \{\xi : \xi \in \mathscr{D}_{n,r}, \|\xi\| \le h\}$. Finally, throughout this paper, if not specified, any function defined on the subset of J is bounded on any bounded interval.

2. Problem formulation and preliminaries

2.1. Problem formulation

In this paper we study the following discrete-time time-varying time-delay system

$$x(k+1) = f(k, x_k), \quad k \in J, \tag{1}$$

where $x_k = x(k+s), s \in \mathbf{I}[-r,0]$ and the function $f : J \times \mathscr{D}_{n,r}$ is such that the image by f of $J \times$ (a bounded subset of $\mathscr{D}_{n,r}$) is a bounded subset of $\mathbf{R}^n$ and $f(k,0) = 0$, where $r \ge 0$ is an integer denoting the delay in the system. Let the initial condition be $x_{k_0}(s) = \xi(s), s \in \mathbf{I}[-r,0], \xi \in \mathscr{D}_{n,r}, k_0 \in J$. Suppose that (1) has a unique solution for an arbitrary $\xi \in \mathscr{D}_{n,r}$ and denote the solution by $x(k) = x(k, k_0, \xi), k \ge k_0$.

We next introduce the stability concepts for system (1) (Elaydi & Zhang, 1994). The zero solution of system (1) is said to be uniformly stable (US) if, for any $\varepsilon > 0$ and any $k_0 \in J$, there exists a $\delta = \delta(\varepsilon)$ such that, $x_{k_0} = \xi$ satisfying $\|\xi\| \le \delta$