Automatica 91 (2018) 279–293

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Stochastic stabilization of slender beams in space: Modeling and boundary control☆

K.D. Do *, A.D. Lucey

Department of Mechanical Engineering, Curtin University, Kent Street, Bentley, WA 6102, Australia

ARTICLE INFO

Article history:
Received 1 February 2017
Received in revised form 27 September 2017
Accepted 30 November 2017

Keywords:
Slender beams
Large motions
Boundary control
Stochastic evolution system
Well-posedness
p-stability

ABSTRACT

This paper considers the problem of modeling and boundary feedback stabilization of extensible and shearable slender beams with large deformations and large rotations in space under both deterministic and stochastic loads induced by flows. Fully nonlinear equations of motion of the beams are first derived. Boundary feedback controllers are then designed for global practical exponential *p*-stabilization of the beams based on the Lyapunov direct method. A new Lyapunov-type theorem is developed to study well-posedness and stability of stochastic evolution systems (SESs) in Hilbert space.

1. Introduction

This paper focuses on relatively slender beams, for which the shear magnitude is smaller than that of the spatial gradient of the transverse displacements. Due to their large length-to-diameter ratio, extensibility and shearability, the relatively slender beams exhibit both large and small motions (both deflection and rotation) under external (both deterministic and stochastic) loads. Although large motions can cause a serious failure (loop formation or hockling) in beams, most of existing boundary control works (e.g., Cavallo, de Maria, & Pirozzi, 2010; Chentouf & Wang, 2015; Do, 2017b; Do & Pan, 2008; Fard & Sagatun, 2001; Guo & Jin, 2013; He & Ge, 2015; He, Ge, Voon, How, & Choo, 2014; He, Huang, & Li, 2017; He, Nie, Meng, & Liu, 2017; He, Sun, & Ge, 2015; Jin & Guo, 2015; Luo, Guo, & Morgul, 1999; Meurer, Thull, & Kugi, 2008; Miletíc, Stürzer, Arnold, & Kugi, 2016; Nguyen, Do, & Pan, 2013; Özer, 2017; Queiroz, Dawson, Nagarkatti, & Zhang, 2000; Zuyev, 2015 based on the Lyapunov direct and flatness methods, and Böhm, Krstíc, Küchler, and Sawodny (2014), Krstíc and Smyshlyaev (2008a) and Krstíc and Smyshlyaev (2008b) based on the backstepping method on single beams, and Endo, Matsuno, and Jia (2017), Henikl, Kemmetmüller, Meurer, and Kugi (2016), Kater and Meurer (2016) and Lagnese, Leugering, and Schmidt (1994) on multiple beams) on boundary control have considered only small deflection (vibration). None of the equations of motion in the above works can describe loop formation due to the fact that they are obtained by linearizing the axial stretch and rotational motions, and neglecting the shear strains, see Eringen (1952) and Love (1920), and therefore exclude large motions.

Boundary control of slender beams with large motions has received less attention. In Do (2011) and Do and Pan (2009) (see also Athisakul, Monprapussorn, and Chucheepsakul (2011) and Kokarakis and Bernitsas (1987) for models of slender beams, where only large deflection is considered), boundary control of unshearable risers/beams with large deflections was considered. Boundary control of extensible and shearable slender beams has been considered in Do (2016a) in three-dimensional space (3D). In these works, the external loads are assumed to be deterministic except for the work in Do (2017a), where stochastic external loads are initially considered for slender beams in two-dimensional space (2D). Control design and stability analysis for stochastic beams is much harder than for deterministic beams. For example, the stochastic component of flows, which enters to the beam system via the hydrodynamic/aerodynamic Centripetal matrix, potentially destabilizes the beam system under deterministic control designs.

The main contributions of this paper consist of three parts. First, fully nonlinear equations of motion of the beams and their properties are derived in an appropriate form for boundary control design by using deformation theory and sea loads on offshore structures. The unit quaternion is used for attitude representation of the

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Thomas Meurer under the direction of Editor Miroslav Krstic.

* Corresponding author.
E-mail addresses: duc@curtin.edu.au (K.D. Do), T.Lucey@curtin.edu.au (A.D. Lucey).

https://doi.org/10.1016/j.automatica.2018.01.017

beams to resolve singularities caused by Euler angles. Second, boundary feedback controllers are designed for global practical exponential stabilization of the beams based on the Lyapunov direct method. In the control design, various Young's and Hölder's inequalities and Sobolev embedding, a flexible combination of Earth-fixed and body-fixed coordinates, and cross vector products are used. Third, a new Lyapunov-type theorem is developed to study well-posedness and stability analysis of a class of nonlinear SESs in Hilbert space. This theorem does not require global monotonic and linear growth conditions as in e.g., Chow (2007), Liu (2006), Prato and Zabczyk (1992) and Prévôt and Röckner (2007). The Lyapunov function uses $\|.\|_V$ instead of $\|.\|_H$ as in Do (2017b). This allows to study well-posedness of SESs, for which it is difficult to apply multiple Gelfand triples because $V \subset H$.

Notations. The symbols $\wedge$ and $\vee$ denote the infimum and supremum operators, respectively. The symbols "col", "$\times$", "$\mathbb{E}$" denote the column operator, vector cross product operator, and the expected value, respectively.

2. Mathematical model

We assume that plane sections are rigid; and the beam material is elastic, homogeneous and isotropic. Equations of motion are briefly derived, see Do (2016a).

2.1. Kinematics

The reference configuration $\mathcal{B}^0$ is described by the position of the base straight line C^0 parameterized by its arclength coordinate s and the fixed basis $(\boldsymbol{b}_1^0, \boldsymbol{b}_2^0, \boldsymbol{b}_3^0)$, where $(\boldsymbol{b}_1^0, \boldsymbol{b}_2^0)$ are the principal axes of inertia of the cross section $S^0(s)$ through N^0, see Fig. 1A. Thus, C^0 is described by $\boldsymbol{r}^0(s) = 0\boldsymbol{e}_1 + 0\boldsymbol{e}_2 + s\boldsymbol{e}_3$. We denote by Γ the beam length in its reference state. The actual configuration $\mathcal{B}$ of the curved beam is described by the actual position of the base curve C and the actual configuration S of cross sections through N. The base curve is described by $\boldsymbol{r}$ while the material cross section is described by the unit vectors $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$ with $\boldsymbol{b}_3$ being aligned with $\boldsymbol{r}_s$ and $\boldsymbol{b}_3 = \boldsymbol{b}_1 \times \boldsymbol{b}_2$. The deformation from $\mathcal{B}^0$ to $\mathcal{B}$ is achieved by means of the vector $\boldsymbol{r} = r_1\boldsymbol{b}_1 + r_2\boldsymbol{b}_2 + r_3\boldsymbol{b}_3$, and the orthogonal tensor $\boldsymbol{R}_1(\boldsymbol{\theta})$ describing the incremental rigid rotation suffered by S^0 so that $\boldsymbol{b}_k = \boldsymbol{R}_1(\boldsymbol{\theta})\boldsymbol{b}_k^0, k = 1, 2, 3$ via the sequence $\theta_1 \to \theta_2 \to \theta_3$. This gives $\boldsymbol{b}_{ks} = \boldsymbol{R}_{1s}\boldsymbol{b}_k^0$, where $\boldsymbol{R}_{1s} = \boldsymbol{\mu} \times \boldsymbol{b}_k$ with $\boldsymbol{\mu}$ the axial vector of $\boldsymbol{R}_{1s}\boldsymbol{R}_1^T$. The generalized strains (the stretch ε and shear strains η_1 and η_2) are expressed by the stretch vector $\boldsymbol{\nu} = \eta_1\boldsymbol{b}_1 + \eta_2\boldsymbol{b}_2 + (1+\varepsilon)\boldsymbol{b}_3$ in its local basis: $\boldsymbol{\nu} = \boldsymbol{r}_s$. Thus,

$$\begin{aligned}&\boldsymbol{r}_s = \eta_1\boldsymbol{b}_1 + \eta_2\boldsymbol{b}_2 + (1+\varepsilon)\boldsymbol{b}_3\\&\boldsymbol{\mu}=\mu_1\boldsymbol{b}_1+\mu_2\boldsymbol{b}_2+\mu_3\boldsymbol{b}_3,\ \boldsymbol{\omega}=\omega_1\boldsymbol{b}_1+\omega_2\boldsymbol{b}_2+\omega_3\boldsymbol{b}_3,\\&\boldsymbol{b}_{ks}=\boldsymbol{\mu}\times\boldsymbol{b}_k,\ \boldsymbol{b}_{kt}=\boldsymbol{\omega}\times\boldsymbol{b}_k,\ (\boldsymbol{\mu}\times\boldsymbol{b}_k)_t=(\boldsymbol{\omega}\times\boldsymbol{b}_k)_s.\end{aligned} \tag{1}$$

From the above derivation, we have

$$\begin{aligned}&\mathrm{col}(\eta_1, \eta_2, \varepsilon) = \boldsymbol{R}_1^T(\boldsymbol{\theta})\boldsymbol{r}_s - \boldsymbol{r}_s^0,\\&\mathrm{col}(\mu_1, \mu_2, \mu_3)=\boldsymbol{R}_2^{-1}(\boldsymbol{\theta})\mathrm{col}(\theta_{1s}, \theta_{2s}, \theta_{3s}),\\&\mathrm{col}(\omega_1, \omega_2, \omega_3)=\boldsymbol{R}_2^{-1}(\boldsymbol{\theta})\mathrm{col}(\theta_{1t}, \theta_{2t}, \theta_{3t}),\end{aligned} \tag{2}$$

where $\boldsymbol{R}_2(\boldsymbol{\theta})$ is the transformation matrix that relates the angular vector $\boldsymbol{\omega}$ in the local basis to the Euler rate vector $\boldsymbol{\theta}_t$. When $\theta_2 = \pm\frac{\pi}{2}$, there are singularities in (2). Thus, we use the unit quaternion vector $\boldsymbol{q} = \mathrm{col}(q_1, q_2, q_3, q_4)$ for attitude representation with $\|\boldsymbol{q}\|^2 = 1$ relating to $(\theta_1, \theta_2, \theta_3)$ via the sequence $\theta_1 \to \theta_2 \to \theta_3$, see Kuipers (2002). Thus, $\boldsymbol{R}_1$ is given in terms of $\boldsymbol{q}$ as:

$$\boldsymbol{R}_1(\boldsymbol{q}) = \boldsymbol{I}_3 + 2q_1\boldsymbol{S}(\bar{\boldsymbol{q}}) + 2\boldsymbol{S}^2(\bar{\boldsymbol{q}}), \tag{3}$$

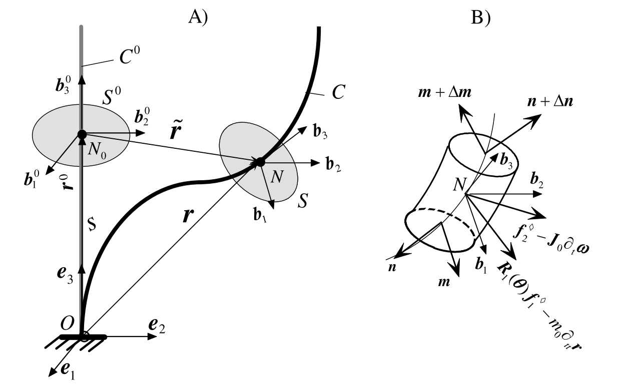

Fig. 1. (A) Deformation geometry of the beam; (B) Forces and moments acting on a beam element.

where $\bar{\boldsymbol{q}} := \mathrm{col}(q_2, q_3, q_4)$ and the matrix $\boldsymbol{S}(\boldsymbol{x})$ is defined as $\boldsymbol{S}(\boldsymbol{x})\boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$ for all $(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^3$. Let us also define:

$$\boldsymbol{K}(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix}-\bar{\boldsymbol{q}}^T\\ q_1\boldsymbol{I}_3 + \boldsymbol{S}(\bar{\boldsymbol{q}})\end{bmatrix}. \tag{4}$$

2.2. Kinetic

Balancing linear and angular momentum on a beam element, see Fig. 1B, gives the equations of motion:

$$\begin{aligned}&m_0\tilde{\boldsymbol{r}}_{tt} = \boldsymbol{n}_s + \boldsymbol{R}_1(\boldsymbol{\theta})\boldsymbol{f}_1^\diamond,\\&\boldsymbol{J}_0\boldsymbol{\omega}_t = \boldsymbol{m}_s + \boldsymbol{r}_s \times \boldsymbol{n} - \boldsymbol{\omega} \times \boldsymbol{J}_0\boldsymbol{\omega} + \boldsymbol{f}_2^\diamond,\end{aligned} \tag{5}$$

where m_0 is the beam mass per unit length; $\boldsymbol{J}_0$ is the mass moment matrix of inertia; $\boldsymbol{n}$ and $\boldsymbol{m}$ denote the contact force and moment vectors; and (see Fig. 1A)

$$\tilde{\boldsymbol{r}} = \boldsymbol{r} - \boldsymbol{r}^0. \tag{6}$$

(a) Nonconservative forces and moments $\boldsymbol{f}_1^\diamond$ and $\boldsymbol{f}_2^\diamond$: Let $\boldsymbol{v}$ be the linear velocity vector in the local basis, i.e.,

$$\boldsymbol{v} = \boldsymbol{R}_1^{-1}(\boldsymbol{\theta})\tilde{\boldsymbol{r}}_t. \tag{7}$$

Let $\boldsymbol{v}_f$ and $\boldsymbol{\omega}_f$ be the linear and angular velocity vectors of the fluid passing the beam at (s, t). The total generalized force vector induced by the fluid acting on the beam denoted by $\boldsymbol{f}^\diamond := \mathrm{col}(\boldsymbol{f}_1^\diamond, \boldsymbol{f}_2^\diamond)$ in the local basis is Fossen (2002):

$$\boldsymbol{f}^\diamond = \underbrace{\boldsymbol{M}_A^\diamond\boldsymbol{v}_{rt}^\diamond + \boldsymbol{C}_A^\diamond(\boldsymbol{v}_r^\diamond)\boldsymbol{v}_r^\diamond}_{\text{Added mass}} + \underbrace{\boldsymbol{D}_A^\diamond(\boldsymbol{v}_r^\diamond)\boldsymbol{v}_r^\diamond}_{\text{Damping}} + \underbrace{\boldsymbol{f}_0^\diamond(s, t)}_{\text{Other loads}}, \tag{8}$$

where

$$\begin{aligned}&\boldsymbol{v}_r^\diamond = \mathrm{col}(\boldsymbol{v}_r, \boldsymbol{\omega}_r),\ \boldsymbol{v}_r = \boldsymbol{v} - \boldsymbol{v}_f,\ \boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}_f,\\&\boldsymbol{f}_0^\diamond = \mathrm{col}(\boldsymbol{f}_{10}^\diamond, \boldsymbol{f}_{20}^\diamond),\ \boldsymbol{M}_A^\diamond = \mathrm{diag}(\boldsymbol{M}_A, \boldsymbol{J}_A),\\&\boldsymbol{D}_A^\diamond(\boldsymbol{v}_r^\diamond) = \mathrm{diag}(\boldsymbol{D}_{A1}(\boldsymbol{v}_r), \boldsymbol{D}_{A2}(\boldsymbol{\omega}_r)),\\&\boldsymbol{C}_A^\diamond(\boldsymbol{v}_r^\diamond)\boldsymbol{v}_r^\diamond = \mathrm{col}(\boldsymbol{\omega}_r \times (\boldsymbol{M}_A\boldsymbol{v}_r), \boldsymbol{\omega}_r \times (\boldsymbol{J}_A\boldsymbol{\omega}_r)\\&\qquad\qquad + \boldsymbol{v}_r \times (\boldsymbol{M}_A\boldsymbol{v}_r)),\\&\boldsymbol{D}_{A1}(\boldsymbol{v}_r) = \boldsymbol{D}_{A11} + \boldsymbol{D}_{A12}\boldsymbol{v}_r \otimes \boldsymbol{v}_r,\\&\boldsymbol{D}_{A2}(\boldsymbol{\omega}_r) = \boldsymbol{D}_{A21} + \boldsymbol{D}_{A22}\boldsymbol{\omega}_r \otimes \boldsymbol{\omega}_r,\end{aligned} \tag{9}$$

where the added mass and added inertia matrices $\boldsymbol{M}_A$ and $\boldsymbol{J}_A$ are diagonal and negative definite; $\boldsymbol{D}_{A1}(\boldsymbol{v}_r)$ and $\boldsymbol{D}_{A2}(\boldsymbol{\omega}_r)$ are damping matrices; $\boldsymbol{D}_{Aij}, (i, j) = 1, 2$ are diagonal and negative definite matrices; and the operator $\otimes$ is defined as $\boldsymbol{a} \otimes \boldsymbol{a} := \mathrm{diag}(a_1^2, a_2^2, a_3^2)$ with $\boldsymbol{a} = \mathrm{col}(a_1, a_2, a_3)$. In general, $\boldsymbol{v}_f$, $\boldsymbol{\omega}_f$ and $\boldsymbol{f}_{i0}^\diamond$ with $i = 1, 2$