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Brief paper

A nonlinear internal model design for heterogeneous second-order multi-agent systems with unknown leader[☆]

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ABSTRACT

This paper focuses on robust nonlinear coordination of heterogeneous uncertain second-order multi-agent systems subject to directed communication topologies. We develop a nonlinear internal model principle based approach for the problem in a framework of cooperative global robust output regulation, independent of the *a priori* of the leader dynamics information except its order. As a major consequence, this study assures, by means of establishing a strict-Lyapunov function for the closed-loop system, not only a specified exponential convergence rate but also tolerable bounds of unmodeled disturbances. Hence, the former guarantees an appealing performance and the latter an important robustness property.

1. Introduction

Feedback control of second-order systems has wide impacts on control theory developments and practice; to name but a few, see Bartolini, Ferrara, and Usai (1997), Fischer, Kan, Kamalapurkar, and Dixon (2014), Loría (2016) Roup and Bernstein (2001), Xian, de Queiroz, Dawson, and McIntyre (2004), and references thereof for an overview. One recent research focus is on coordination of multiple second-order systems in a distributed feedback control fashion, or namely synchronizing individual position/velocity in literature. For concrete studies, one may refer to Hong, Chen, and Bushnell (2008), Ren (2008) and Zhou, Zhang, Xiang, and Wu (2012) for linear second-order multi-agent systems, and to Fan, Chen, and Zhang (2014), Meng, Lin, and Ren (2013), Song, Cao, and Yu (2010), Su, Chen, Wang, and Lin (2011) and Su and Huang (2013a) for nonlinear extensions from interesting aspects. More specifically, to tackle such nonlinearity, Song et al. (2010) and Su et al. (2011) developed some linear controllers for nonlinear second-order multi-agent systems satisfying certain global Lipschitz-like conditions. Later, to relax such conditions, Fan et al. (2014) and Meng et al. (2013) further addressed some linear high-gain feedback controllers for the purpose of semi-global control and Su and Huang (2013a) explored a nonlinear controller for achieving global control.

One of the interesting research topics from the robust control viewpoint is to allow the control plant undergoing parametric uncertainties as well as external disturbances. In this direction, several interesting studies have been conducted. One attempt is to develop non-smooth feedback controllers following the sliding-mode control technique; see Wang and Ji (2015) and references thereof. In the leader-following scenario, this type of cooperative design is usually feedforward. That is, the dynamics of the leader system (more specifically, both the reference position and velocity information) has to be exactly known and utilized by the followers. The other attempt is to explore the internal model principle based controller leading to the so-called cooperative robust output regulation design, see Su and Huang (2013a) and references thereof. This approach can further allow that the dynamics of the leader system only partially known by the followers. For example, as shown in Su and Huang (2013a), when tracking a sinusoidal signal, the canonical internal model controller design relies only on its frequency, but not its amplitude and phase.

In practice, the problem with unknown leader dynamics is certainly of greater interest. Practically, this may occur if the leader

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dynamics contains unknown parameters. For example, the fundamental sinusoidal signals with unknown frequencies, amplitudes, and phases are exactly of this case to be modeled. One attempt in this direction is to combine canonical internal model with adaptive control tools, see Su and Huang (2013b). This approach, however, is restricted to plants with unity relative degree and topologies with bidirected communication. A preliminary study in Wang, Su, and Xu (2017) provided a non-adaptive internal model design but still worked on such bidirected communication topologies.

The main objective of this research is to explore a general investigation of robust coordination of nonlinear heterogeneous second-order multi-agent systems subject to unknown leaders and directed communication topologies. A nonlinear internal model based design is proposed in the framework of cooperative robust output regulation. Specifically, we first construct a set of nonlinear internal models that successfully convert the robust coordination problem into a robust non-adaptive cooperative stabilization problem of the augmented system. Then, we present an integral input-to-state stability (iISS) based two-step block backstepping synthesis for the resulting stabilization problem. The contribution of the present study is two-fold. On one hand, the developed method can give rise to a successful cooperative output regulator design for the general directed nonlinear networks with unknown leaders. On the other hand, a strict-Lyapunov function can be established, assuring an exponential convergence of the closed-loop system and providing an explicit bound of unmodeled actuator disturbances. It is noted that, the former guarantees an appealing performance and the latter an important robustness property.

The paper is organized as follows. Section 2 introduces the problem formulation of the cooperative robust output regulation for second-order nonlinear networks. Section 3 elaborates the nonlinear internal model design and Section 4 presents the main result. Section 5 presents an illustrative example to show the efficiency of the developed method. Section 6 closes the paper with a few remarks.

Throughout the paper, $\|\cdot\|$ is the Euclidean norm. I is an identity matrix of a compatible dimension. $\mathbb{R}_{\geq 0}$ denotes the set of nonnegative real numbers. A function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$, i.e., $f \in \mathcal{K}$ if, it is continuous and strictly increasing with $f(0) = 0$. $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}_\infty$ if, it is of class $\mathcal{K}$ and unbounded. The set of bounded $\mathcal{K}$ functions is denoted by $\mathcal{K}_o$, i.e., $\mathcal{K}_o = \mathcal{K} \setminus \mathcal{K}_\infty$. Id denotes the identical $\mathcal{K}_\infty$ function. The function $f : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if, for each fixed $s \geq 0$, $\beta(s, t)$ is continuous and decreases to zero as $t \to +\infty$, and for each fixed $t \geq 0$, $\beta(\cdot, t)$ is of class $\mathcal{K}$. For two continuous and positive definite functions $\kappa_1(s)$ and $\kappa_2(s)$, $\kappa_1 \in \mathcal{O}(\kappa_2)$ means $\limsup_{s\to 0^+} \frac{\kappa_1(s)}{\kappa_2(s)} < \infty$. For a pair of functions $f_1(s), f_2(s)$ of compatible dimensions, $f_1 \circ f_2(s) = f_1(f_2(s))$ denotes function composition.

2. Formulation and preliminary

Consider a dynamic network of nonlinear heterogeneous second-order systems described by

$$m_i\ddot{q}_i(t) = f_i(q_i(t), \dot{q}_i(t), v(t), w) + u_i(t), \quad 1 \leq i \leq N \tag{1}$$

where, for $1 \leq i \leq N$, $q_i \in \mathbb{R}$ is the position, $\dot{q}_i \in \mathbb{R}$ is the velocity, $u_i \in \mathbb{R}$ is the control input, $w \in \mathbb{W}$ represents some constant unknown parameter or parametric uncertainty in a specified compact set $\mathbb{W}$, and the parameter $m_i \triangleq m_i(w)$ is the uncertain inertia satisfying $0 < \underline{m}_i \leq m_i \leq \bar{m}_i$ for known constants $\underline{m}_i, \bar{m}_i$. The exogenous signal $v(t) \in \mathbb{R}^{n_v}$ is generated by the leader with agent index 0 as

$$\dot{v}(t) = S(\sigma)v(t), \quad q_0(t) = q_0(v(t), w) \tag{2}$$

where the output $q_0 \in \mathbb{R}$ is the reference to specify certain desired output reference, and $\sigma \in \mathbb{S}$ thereof is a constant unknown parameter in a known compact set $\mathbb{S}$.

Assume that for each $\sigma \in \mathbb{S}$, all eigenvalues of $S(\sigma)$ are distinct with zero real parts and the leader (2) is invariant in a compact set $\mathbb{V}$. In other words, the leader may generate any mixed sinusoidal and step signals and only the number of frequencies is known. All the involved amplitudes, frequencies, and phases are unknown. For technical simplicity, we further assume that the functions $f_i(q_i, \dot{q}_i, v, w)$ for $1 \leq i \leq N$ and $q_0(v, w)$ are smooth in their arguments.

2.1. Problem formulation

This study is concerned with the position tracking error given by for $1 \leq i \leq N$, $e(t) = [e_1(t), \ldots, e_N(t)]^\top$ with $e_i(t) = q_i(t) - q_0(t)$, and the relative position error given by $\hat{e}(t) = [\hat{e}_1(t), \ldots, \hat{e}_N(t)]^\top$ with $\hat{e}_i(t) = \sum_{j=0}^{N} a_{ij}\left(q_i(t) - q_j(t)\right)$, where coefficients a_{ij} for $0 \leq i, j \leq N$ are determined by a weighted adjacency matrix $\mathcal{A} \triangleq [a_{ij}]_{0\leq i,j\leq N}$ relating to a communication digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ for a node set $\mathcal{V} \triangleq \{0, 1, 2, \ldots, N\}$, an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$; see Godsil and Royle (2001) for details. Let $H = [h_{ij}]_{1\leq i,j\leq N}$ with $h_{ij} \triangleq \sum_{k=0}^{N} a_{ik}$ when $i = j$ and $h_{ij} \triangleq -a_{ij}$ when $i \neq j$. Since $\hat{e}_i = \sum_{j=0}^{N} a_{ij}(e_i - e_j)$ with $e_0 \equiv 0$, we have $\hat{e}(t) = He(t)$.

Problem 2.1. For the network (1) and (2), the control goal is to seek a smooth controller of the form $u_i = g_i^c(\chi_i, \hat{e}_i, \dot{q}_i)$, $\dot{\chi}_i = f_i^c(\chi_i, \hat{e}_i, \dot{q}_i)$, $1 \leq i \leq N$, such that, for any $[v(0)^\top, w^\top, \sigma^\top]^\top \in \mathbb{D} \triangleq \mathbb{V} \times \mathbb{W} \times \mathbb{S}$ and for any initial conditions $q_i(0)$, $\dot{q}_i(0)$, and $\chi_i(0)$ in their entire spaces, the trajectory of the closed-loop system is bounded over $[0, \infty)$, and the position tracking error satisfies $\lim_{t\to+\infty} e(t) = 0$.

Problem 2.1 is called cooperative robust output regulation that has been extensively studied in literature from various aspects; see Isidori, Marconi, and Casadei (2014), Meng, Yang, Dimarogonas, and Johansson (2015), Su and Huang (2013a), Wang, Hong, Huang, and Jiang (2010), Wieland, Sepulchre, and Allgöwer (2011) and references thereof.

2.2. Definition

The main tool for solving Problem 2.1 is the notion of iISS and its iISS-Lyapunov function characterization; see Angeli, Sontag, and Wang (2000) for details. Consider a general nonlinear system

$$\dot{x} = f(x, u, \mu) \tag{3}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the input, $\mu \triangleq \mu(t) \in \mathbb{D}$ for all $t \geq 0$ is locally essentially bounded disturbance varying in a compact set $\mathbb{D}$, and f is a smooth vector field in its arguments, satisfying $f(0, 0, \mu) = 0$ for all $\mu \in \mathbb{D}$. Assume the system (3) is forward complete.

Definition 2.1. A smooth function $V : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ is called an *iISS-Lyapunov function* (with state x and input u, robustly on μ) for the system (3) if there are comparison functions $\underline{\alpha}, \bar{\alpha} \in \mathcal{K}_\infty$, $\alpha \in \mathcal{K}$,[1] and $\gamma \in \mathcal{K}$ such that, along the trajectories of (3), for all $\mu \in \mathbb{D}$,

$$\underline{\alpha}(\|x\|) \leq V(t, x) \leq \bar{\alpha}(\|x\|), \quad \dot{V} \leq -\alpha \circ V(t, x) + \gamma(\|u\|) \tag{4}$$

where $\dot{V} \triangleq \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x, u, \mu)$. The system (3) is called iISS if it has an iISS-Lyapunov function. Moreover, if the above α of (4) can

[1] For the sake of technical simplicity, we use the iISS-Lyapunov function with a class $\mathcal{K}$ dissipation gain α instead of the general positive definite one, see Angeli et al. (2000).