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Enlarging the basin of attraction by a uniting output feedback controller[☆]

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ABSTRACT

We consider a system for which two predesigned stabilizing output feedback controllers with bounded domains of attraction are known. One renders the system asymptotically stable with some desired performance, and the other provides ultimate boundedness with larger domain of attraction. Assuming that two subsets of the domains of attraction are known, one larger than the other, this work states the problem of combining both controllers with the goal of guaranteeing asymptotic stability properties in the largest subset while the desired performance is locally achieved. We design a switching logic between the controllers that solves the problem, based on the existence of a local tunable observer. The resulting control law is defined by a hybrid output feedback controller. The effectiveness of the proposed solution is illustrated by a numerical example.

1. Introduction

There is a multitude of techniques to design asymptotically stabilizing control feedbacks laws. Nonetheless, most of the well known techniques (backstepping, feedback linearization, passivation, etc.) usually do not address the problem of system performance. It is of great importance to design control laws providing both asymptotic stability and guaranteed performance requirements. A simple local solution to this problem can be obtained, for instance, via linearization and linear control design techniques. This leads to the idea of uniting two predesigned local and global controllers as proposed in Teel and Kapoor (1997), so that local performance objectives are achieved while global asymptotic stability is guaranteed.

Different strategies have been proposed to tackle the problem of uniting local and global controllers. The first algorithm for patching two controllers was presented in Teel and Kapoor (1997) (see also Morin, Murray, & Praly, 1998; Pan, Ezal, Krener, & Kokotovic, 2001), and later applied to real experiments in Teel, Kaiser, and Murray (1997). The solution is given in the form of a continuous static time-invariant controller. However, considering general control systems, the uniting problem cannot be solved by only continuous feedback controllers as proved in Prieur (2001). In that reference (see also Efimov, 2006), a switching strategy based on hysteresis is also proposed, leading to the class of dynamic hybrid controllers. In Prieur and Teel (2011), the problem of uniting two output-feedback controllers is solved. The solution is provided as a hybrid controller, where the switching is performed with a norm observer. These results are extended in Sanfelice and Prieur (2013), considering the uniting of two hybrid output feedback controllers.

All these previous works focus on uniting local and global controllers. However, it is well-known that there are systems which cannot be globally stabilized, for instance unstable linear systems with bounded control (Sontag & Sussmann, 1990) and the examples in Mazenc and Praly (1994). For those systems, instead of requiring controllers with global stability properties, we can aim at designing controllers for semi-global asymptotic stabilization. The work (Teel & Praly, 1994) shows that stabilizability and observability are sufficient conditions for semi-global stabilization by dynamic output feedback. This leads to the variation of the uniting problem presented in this work, where only local stability and attractivity properties are required. A similar problem is studied in Efimov, Loria, and Panteley (2011) in an input–output sense and under a state-independent input-to-output stability assumption. We consider two output feedback controllers; one (referred to as *local* controller) renders the system locally asymptotically stable with some desired performance, and the other (referred to as *regional* controller) steers the trajectories starting from some

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given set to a neighborhood of the origin where the first controller applies. Clearly, the regional controller is assumed to have larger domain of attraction. The main goal of the proposed uniting problem is to enlarge the estimation of the domain of attraction of the local controller in a manner that the performance is not degraded on a neighborhood of the origin. The proposed solution consists of a switching logic between the controllers, which is implemented by a hybrid controller following the formalism for hybrid systems in Goebel, Sanfelice, and Teel (2012). As opposed to the results in Prieur and Teel (2011) and Sanfelice and Prieur (2013), our switching logic is based on a tunable observer. The assumption of a norm observer is weaker, but the rate of convergence of a norm observer is usually not tunable. As a consequence, the switching between the controllers may not be performed sufficiently fast to avoid a trajectory to leave the domain of attraction of the regional controller.

On the other hand, similarly to Prieur and Teel (2011) and Sanfelice and Prieur (2013), the robust local asymptotic stability of the proposed hybrid system is concluded by the hybrid basic conditions.

The outline of the paper is as follows. The uniting problem is introduced in Section 2. The main result follows in Section 3. First, a hybrid output feedback controller is designed, and second, the controller is proven to solve the uniting problem. Section 4 illustrates the effectiveness of the proposed solution by a numerical example.

Notation: Throughout this work, the following notation is used. The notation $\|x\|$ is the Euclidean norm for $x \in \mathbb{R}^n$. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_m(A)$ and $\lambda_M(A)$ stand for the minimum and maximum eigenvalues, respectively. A ball in $\mathbb{R}^n$ of radius ε is denoted by $\mathbb{B}(\varepsilon) := \{x \in \mathbb{R}^n : \|x\| \le \varepsilon\}$. The symbol $\ominus$ stands for the Minkowski difference. A function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}$ if it is continuous, strictly increasing, and $f(0) = 0$. The function f is of class $\mathcal{K}_\infty$ if $f \in \mathcal{K}$ and $\lim_{s\to\infty} f(s) = \infty$. A continuous function $f : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$ if for each fixed s, the function $r \mapsto f(r, s)$ belongs to class $\mathcal{K}$ and for each fixed r, the function $s \mapsto f(r, s)$ is nonincreasing and $\lim_{s\to\infty} f(r, s) = 0$. Given a set $S \subset \mathbb{R}^n$ and a point $x \in \mathbb{R}^n$, $\|x\|_S := \inf_{y\in S}\{\|x - y\|\}$. The reader is referred to Goebel et al. (2012) for the basic notation in hybrid systems.

2. Problem statement

Consider the following nonlinear systems defined by

$$\dot{x} = f(x, u),\ \ y = h(x) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a locally Lipschitz function with $f(0, 0) = 0$ and $h : \mathbb{R}^n \to \mathbb{R}^p$ is a continuously differentiable function with $h(0) = 0$. In addition, let us consider two dynamic output feedback controllers, leading to the following closed-loop systems:

$$\begin{aligned}\dot{x} &= f(x, \alpha_0(\zeta_0, h(x))),\\ \dot{\zeta}_0 &= \varphi_0(\zeta_0, h(x)),\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{x} &= f(x, \alpha_1(\zeta_1, h(x))),\\ \dot{\zeta}_1 &= \varphi_1(\zeta_1, h(x)),\end{aligned} \tag{3}$$

where $\zeta_q \in \mathbb{R}^{l_q}$ and $\varphi_q : \mathbb{R}^{l_q} \times \mathbb{R}^p \to \mathbb{R}^{l_q}$, $\alpha_q : \mathbb{R}^{l_q} \times \mathbb{R}^p \to \mathbb{R}^m$, $q \in \{0, 1\}$, are continuous functions vanishing at the origin. The local controller defined by α_0 and φ_0 is assumed to render the closed-loop system (2) locally asymptotically stable, while the regional controller given by α_1 and φ_1 guarantees ultimate boundedness of the closed-loop system (3) for a possibly bounded (but sufficiently large) set of initial conditions.

In this work, a hybrid output feedback controller is given by $(\mathcal{C}, \mathcal{D}, u, v, w)$, where $\mathcal{C} \subset \mathbb{R}^l$ and $\mathcal{D} \subset \mathbb{R}^l$ are closed sets and $u : \mathcal{C} \times \mathbb{R}^p \to \mathbb{R}^m$, $v : \mathcal{C} \times \mathbb{R}^p \to \mathbb{R}^l$, and $w : \mathcal{D} \times \mathbb{R}^p \to \mathbb{R}^l$ are continuous functions. The closed-loop system that results from the feedback interconnection of the system (1) and the hybrid controller, is described under the hybrid systems framework of Goebel et al. (2012) as follows:

$$\left.\begin{aligned}\dot{x} &= f(x, u(\zeta, h(x)))\\ \dot{\zeta} &= v(\zeta, h(x))\end{aligned}\right\},\ \ \zeta \in \mathcal{C},$$
$$\left.\begin{aligned}x^+ &= x\\ \zeta^+ &= w(\zeta, h(x))\end{aligned}\right\},\ \ \zeta \in \mathcal{D}. \tag{4}$$

A trajectory of the hybrid system (4) consists of a *hybrid time domain* $\mathrm{dom}(x, \zeta)$, and a *hybrid arc* $(x, \zeta) : \mathrm{dom}(x, \zeta) \to \mathbb{R}^n \times \mathbb{R}^l$. The trajectories are parameterized by (t, j), where t is the ordinary time and j corresponds to the number of jumps. Throughout this work, we will refer to the maximal trajectories of (4) simply as trajectories. The reader is referred to Goebel et al. (2012) for more details about the hybrid system framework.

Let us recall that a compact set $\mathcal{A} \subset \mathbb{R}^n \times \mathbb{R}^l$ is locally asymptotically stable for system (4) if

- (*stability*) for all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $(x^0, \zeta^0) \in \mathbb{R}^n \times (\mathcal{C} \cup \mathcal{D})$ satisfying $\|(x^0, \zeta^0)\|_{\mathcal{A}} \le \delta$, every trajectory of (4) starting at (x^0, ζ^0) satisfies $\|(x(t, j), \zeta(t, j))\|_{\mathcal{A}} \le \varepsilon$, for all $(t, j) \in \mathrm{dom}(x, \zeta)$;
- (*attractivity*) there exists $\delta_a > 0$ such that for all $(x^0, \zeta^0) \in \mathbb{R}^n \times (\mathcal{C} \cup \mathcal{D})$ satisfying $\|(x^0, \zeta^0)\|_{\mathcal{A}} \le \delta_a$, every trajectory of (4) starting at (x^0, ζ^0) is complete and satisfies $\lim_{t+j\to\infty} \|(x(t, j), \zeta(t, j))\|_{\mathcal{A}} = 0$.

Let us assume that the domain of attraction of the closed-loop system (2) contains some set $\mathcal{B} \subset \mathbb{R}^n \times \mathbb{R}^{l_0}$. Thus, this work focuses on the following uniting problem:

Uniting problem: The problem is to find a hybrid output feedback controller $(\mathcal{C}, \mathcal{D}, u, v, w)$ such that

- there exist a matrix $M \in \mathbb{R}^{l_0\times l}$ and a compact set $\mathcal{A} \subset \{0\} \times \ker(M)$, such that the set $\mathcal{A}$ is locally asymptotically stable for the system (4) with a domain of attraction containing $\mathcal{B}_\alpha := \{(x, \zeta) \in \mathbb{R}^n \times (\mathcal{C} \cup \mathcal{D}) : \frac{1}{\alpha}(x, M\zeta) \in \mathcal{B}\}$ for some $\alpha > 1$;
- there exists a continuous positive definite function $\rho \in \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}_{\ge 0}$, and $r > 0$ such that any trajectory of system (4) starting at (x^0, ζ^0), satisfying $\rho(x^0, \zeta^0) \le r$, has the hybrid time domain $[0, \infty) \times \{0\}$ and $(x(t, 0), M\zeta(t, 0)) = (\bar{x}(t), \bar{\zeta}_0(t))$ for some trajectory $(\bar{x}, \bar{\zeta}_0)$ of (2).

Roughly speaking, the uniting problem under study consists of two problems: first, by combining two different controllers, we look for an enlargement of $\mathcal{B}$, which is the estimation of the domain of attraction of system (2); second, there exists a projection from $\mathbb{R}^n \times \mathbb{R}^l$ to $\mathbb{R}^n \times \mathbb{R}^{l_0}$ such that the projected trajectories of the hybrid system (4) match the trajectories of the system (2) for small enough initial conditions.

Note that the set $\mathcal{B}_\alpha$ with $\alpha > 1$ is defined in such a way that its projection on the state–space of system (2) contains the estimation $\mathcal{B}$ of the domain of attraction of system (2). Therefore, if the domain of attraction of (4) contains $\mathcal{B}_\alpha$, then there are initial conditions for which the trajectories of (4) converge to $\mathcal{A}$, although the trajectories of system (2) are not guaranteed to converge. Finally, the compactness of the set $\mathcal{A}$ is required in order to conclude robust stability from the results in Goebel et al. (2012), as it will be commented in Remark 3.