Automatica 90 (2018) 81–90

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

On stability and convergence of optimal estimation for networked control systems with dual packet losses without acknowledgment☆

Hong Lin [a,b], Hongye Su [a,*], Michael Z.Q. Chen [c], Zhan Shu [d], Renquan Lu [e], Zheng-Guang Wu [a]

[a] State Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Yuquan Campus, Hangzhou, 310027, China
[b] Department of Mechanical Engineering, The University of Hong Kong, Pokfulam, Hong Kong
[c] School of Automation, Nanjing University of Science and Technology, Nanjing, Jiangsu 210094, China
[d] Electro-Mechanical Engineering Group, Faculty of Engineering and The Environment, University of Southampton, Southampton SO17 1BJ, UK
[e] School of Automation, Guangdong University of Technology, and Guangdong Key Laboratory of IoT Information Processing, Guangzhou 510006, China

ARTICLE INFO

Article history:
Received 3 December 2015
Received in revised form 12 September 2017
Accepted 26 November 2017
Available online 15 February 2018

Keywords:
Networked control systems
Optimal estimation
Stability
Packet losses
Packet acknowledgment

ABSTRACT

This paper studies the optimal state estimation problem for networked control systems with control and observation packet losses but without packet acknowledgment (ACK). The packet ACK is a signal sent by the actuator to inform the estimator whether control packets are lost or not. Systems with packet ACK are usually called transmission control protocol (TCP)-like systems, and those without ACK are named user datagram protocol (UDP)-like systems. For UDP-like systems, the optimal estimator is derived and it is consisted of an exponentially increasing number of terms. By developing an auxiliary estimator, it is shown that there exists a critical observation packet arrival rate determining the stability of the expected EC (EEC), and it is identical to its counterpart for TCP-like systems. It is revealed that whether there is packet ACK or not has no effect on the stability of the EEC. Furthermore, under some conditions the EEC converges exponentially.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Recently, significant attention has been paid to networked control systems (NCSs) as they bring numerous benefits, such as lower installation and maintenance costs, reduced network wiring, increased system flexibility, etc. However, the insertion of networks may make NCSs prone to network attacks (Liu, Xia, Tian, & Fei, 2018) and cause some network-induced constraints, such as limited communication (Che, Wang, & Yang, 2012), signal quantization (Che & Yang, 2013), and transmission packet losses (Yan, Zhang, Yang, Zhan, & Peng, 2017). There are two fundamental protocols in network communication for systems subject to packet losses. They are the transmission control protocol (TCP) and the user datagram protocol (UDP). For the TCP, the sending node retransmits lost data until it receives acknowledgment (ACK) from the receiving node. Such retransmission mechanism guarantees the success of data transmission, but for NCSs with unreliable network communication, it would be difficult to implement the TCP, as the packet ACK cannot be transmitted without delay and random loss (Garone, Sinopoli, & Casavola, 2010; Lin, Su, Shi, Lu, & Wu, 2017; Sinopoli, Schenato, Franceschetti, Poolla, & Sastry, 2008). For the UDP, the ACK scheme is not used and thus no retransmission of lost data is required. The UDP, with a less transmission reliability, is able to provide more timely communication, and thus turns out to be a favorable choice for real-time NCSs (Ploplys, Kawka, & Alleyne, 2004). The NCS without the packet ACK transmitted from the actuator to notice the estimator the status of control packet loss is usually called a UDP-like system, and the one with such packet ACK is called a TCP-like system (see Fig. 1). For the convenience of formulation, we denote by $\mathcal{S}^{u}_{UDP}$ a UDP-like system with only control packet losses, and by $\mathcal{S}^{uy}_{UDP}$ a UDP-like system with control and observation packet losses, i.e., dual packet losses. In this paper, we study the optimal estimator and its stability for the $\mathcal{S}^{uy}_{UDP}$ system.

For TCP-like systems, the optimal estimator is a time-varying Kalman filter, and the stability of the expected error covariance (EEC) has been studied in Sinopoli et al. (2004), in which it is

☆ The work is supported in part by the Fundamental Research Funds for the Central Universities (2017FZA5010), in part by Science Fund for Creative Research Groups of the National Natural Science Foundation of China (61621002), in part by National Natural Science Foundation of P.R. China (61374053, 61673339), in part by the China National Funds for Distinguished Young Scientists (61425009). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hideaki Ishii under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: linhongzju@163.com (H. Lin), hysu@iipc.zju.edu.cn (H. Su), mzqchen@njust.edu.cn (M.Z.Q. Chen), z.shu@soton.ac.uk (Z. Shu), rqlu@gdut.edu.cn (R. Lu), nashwzhg@126.com (Z.-G. Wu).

https://doi.org/10.1016/j.automatica.2017.12.059
0005-1098/© 2018 Elsevier Ltd. All rights reserved.

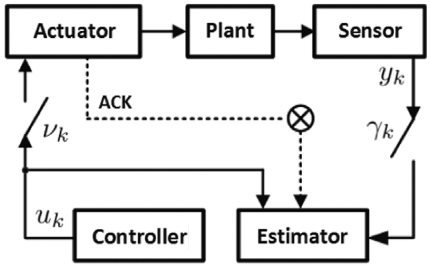

Fig. 1. The UDP-like system. The symbol $\otimes$ is used to emphasize that there is no acknowledgment signal from the actuator to the estimator.

pointed out that for an unstable system there exists a critical observation packet arrival rate, determining the boundedness of the EEC. NCSs with multiple packet losses were investigated in Liang, Chen, and Pan (2010) and Sun, Xie, Xiao, and Soh (2008). Thereafter, the critical value and its upper/lower bound have been investigated in Mo and Sinopoli (2012) and Plarre and Bullo (2009). For Markovian packet loss cases, significant results and techniques can be found in Censi (2011), Huang and Dey (2007) and You, Fu, and Xie (2011), and references therein.

For UDP-like systems, the literature on sub-optimal estimators is first reviewed as follows. For these systems, the linear minimum mean square error (MMSE) estimator was derived in Schenato, Sinopoli, Franceschetti, Poolla, and Sastry (2007), and other linear/non-linear estimators can be found in Lin, Su, Shi, Lu, and Wu (2015), Lin, Xu, Su, Xu, and Wu (2014), Moayedi, Foo, and Soh (2013) and Sinopoli et al. (2008). The UDP-like system can also be viewed as a Markovian jump system (MJS) with unknown jump modes (Lin, Su, Shu, Wu, & Xu, 2014, 2016). Various computationally efficient estimators designed for MJSs with unknown jump modes, such as the interacting multiple model (IMM) estimator (Li & Bar-Shalom, 1993) and the probability hypothesis density (PHD) filter (Clark & Bell, 2006) also apply to UDP-like systems. However, these estimators are in fact sub-optimal. Analytic characterization on the stability and performance of these estimators is usually unavailable, and numerical approaches such as the Monte Carlo method are often used (Li & Bar-Shalom, 1993).

For the optimal estimator, it is shown in Lin et al. (2016) that for the $\mathcal{S}^{u}_{UDP}$ system, it contains exponentially increasing terms, and the EEC is bounded under bounded control inputs if the $\mathcal{S}^{u}_{UDP}$ system is detectable. However, the condition for the stability of the optimal estimator for the $\mathcal{S}^{uy}_{UDP}$ system, to our best knowledge, is still unknown, due to some challenging issues: (1) A random variable γ_k, not presented in the $\mathcal{S}^{u}_{UDP}$ system, occurs not only in the Riccati equation (2e) but also in the power term (15b). As a result, equations for EC are more complicated than that in Lin et al. (2016), and the summation part with exponentially increasing terms in (17d) will become unbounded, making analysis of the stability difficult. (2) Existing methods are not applicable to the $\mathcal{S}^{uy}_{UDP}$ system. The sequential Monte Carlo method, a simulation method, has known to be a practical tool to evaluate the EEC (Doucet, De Freitas, & Gordon, 2001). The hybrid approach developed in Li and Bar-Shalom (1993) is an efficient off-line algorithm for approximately computing EEC with finite mixing terms. The techniques proposed in Clark and Bell (2006) and Seah and Hwang (2008) to analyze the stability of the IMM and the PHD estimators, the good approximations for the optimal estimator, can be employed to approximately study the stability of the optimal estimator for UDP-like systems. However, these aforementioned methods merely render the stability and convergence results in an experimental or approximate way, and cannot determine the stability of EEC in the desired theoretical view. In Lin et al. (2016), an auxiliary estimator method was developed to study stability of EECs for the $\mathcal{S}^{u}_{UDP}$ system. However, all the observations $\{y_1, \dots, y_k\}$ are required in constructing this auxiliary estimator, and thus this method is not applicable due to random losses of observations in the $\mathcal{S}^{uy}_{UDP}$ system. In Lin, Su, et al. (2014), another type of auxiliary estimator was constructed for the $\mathcal{S}^{u}_{UDP}$ system, but the relationship between the optimal and the auxiliary estimators is not established.

For UDP-like systems, the optimal estimator and its stability and convergence are studied in this paper. Main results and contributions are summarized as follows:

(1) We obtain the optimal estimator for UDP-like system with dual packet losses, which is consisted of an exponentially increasing number of terms.
(2) We show that the stability of the EEC is only determined by the observation packet arrival rate, and is independent of the control packet arrival rate. That is, there is a critical value for a given UDP-like system, and the EEC is stable if the observation packet arrival rate is greater than this critical value. Moreover, this critical value is identical to its counterpart for the TCP-like system corresponding this UDP-like system. It reveals the fact whether there is packet ACK or not does not affect the stability of the optimal estimator.
(3) We show that the EEC, although containing exponentially increasing terms, converges if there is no observation packet loss and control inputs eventually tend to zero.

The paper is organized as follows. The system and problems are formulated in Section 2. The optimal estimator for UDP-like systems is obtained in Section 3. An auxiliary estimator is constructed in Section 4. The conditions on the stability and convergence of the optimal estimator are established in Section 5. In Section 6, numerical examples are given to illustrate the obtained results. Conclusions are presented in Section 7. The proofs of lemmas are presented in Appendix.

Notations:

- $\mathbb{P}(\cdot)$ denotes the probability measure.
- $p(\cdot)$ and $p(\cdot|\cdot)$ denote the probability density function (pdf) and the conditional pdf, respectively.
- $\mathcal{N}(\mu, P)$ denotes a Gaussian pdf with mean μ and covariance P. Both $x \sim \mathcal{N}(\mu, P)$ and $p(x) = \mathcal{N}(\mu, P)$ mean the pdf of the random variable x is a Gaussian pdf with mean μ and covariance P. $\mathcal{N}_x(\mu, P)$ is used to emphasize that the random variable of the pdf $\mathcal{N}(\mu, P)$ is x.
- $\underset{x}{\mathbb{E}}[\cdot]$ and $\underset{x}{\mathrm{cov}}(\cdot)$ denote the probability expectation and the covariance with respect to x, respectively.
- $\|\cdot\|$ denotes the norm. Specifically, for a vector, $\|\cdot\|$ denotes the 2-norm; For a matrix, $\|\cdot\|$ denotes the spectral norm, i.e., the maximum singular value.
- $(\cdot)'$ denotes the transpose of a matrix or vector.
- $(\cdot)^2_I$ with the identity matrix I means $(\cdot)(\cdot)'$.
- $(\cdot)_2$ denotes the binary representation, e.g., $(101)_2 = 5$.
- $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$ denote the natural number, the integer, and the real number, respectively.

2. System setup and problem formulation

Consider the following system:

$$x_{k+1} = Ax_k + \nu_k Bu_k + \omega_k$$
$$y_k = \begin{cases} Cx_k + \upsilon_k, & \text{for } \gamma_k = 1 \\ \emptyset, & \text{for } \gamma_k = 0 \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^q$ is the control input, and $y_k \in \mathbb{R}^p$ is the observation. ω_k and υ_k are Gaussian noises with zero means and covariances $Q \geq 0$ and $R > 0$, respectively.

Download English Version:

https://daneshyari.com/en/article/7108901

Download Persian Version:

https://daneshyari.com/article/7108901

Daneshyari.com