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Brief paper

Distributed predictor-based stabilization of continuous interconnected systems with input delays ☆

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ABSTRACT

In this paper, we address the stabilization problem of interconnected systems with input delays. The considered system consists of two coupled subsystems with input delays. Distributed predictor-based controllers are proposed to stabilize such interconnected systems and each controller is independent of the others and does not utilize the state information of other subsystems to predict the state of the corresponding subsystem. We also present a low-pass filter version of the predictor-based controller. In order to analyze the stability of the closed-loop systems, such systems are transformed into partial differential equations (PDE). Under the numerical implementations of the predictor-based controller, exponential stability is asserted for the closed-loop systems and explicit Lyapunov functionals are constructed. Finally, an example is given to show the effectiveness of the distributed predictor-based controller.

1. Introduction

Stabilization of control systems with input delays has attracted a lot of attention over the past years due to its extreme importance in theory and practice (Artstein, 1982; Karafyllis & Jiang, 2011; Karafyllis & Krstic, 2012; Krstic, 2008, 2009, 2010; Liu, Xia, Rees, & Hu, 2007; Mazenc, Niculescu, & Krstic, 2012; Sun, Liu, Wen, & Wang, 2016). These stabilization approaches include predictor-based stabilization (Bekiaris-Liberis & Krstic, 2011; Karafyllis & Krstic, 2012; Krstic, 2008; Mazenc et al., 2012) and various control designs based on Lyapunov–Krasovskii functional and Lyapunov–Razumikhin function (Fridman & Shaked, 2002, 2003; Liu, Sun, Liu, & Teel, 2016). Among these stabilization approaches, predictor-based stabilization plays a dominant role especially for the case that the input delays are very long. Substantial results have been reported for such stabilization approaches in recent years (Karafyllis & Krstic, 2012; Krstic, 2008; Mazenc et al., 2012; Mondié & Michiels, 2003; Zhong, 2004; Zhou, Lin, & Duan, 2012).

In the literature, there are mainly three kinds of predictor-based stabilization for control systems with input delays. The first one is based on Smith-predictor (Artstein, 1982; Krstic, 2008, 2009, 2010). Such predictor-based controller utilizes the past input and the current state to predict the future state and then the control value is sent to the plant. Without any disturbances such as computation errors and model uncertainties, such controller will stabilize the control systems with arbitrarily long input delays. The papers (Mondié & Michiels, 2003; Zhong, 2004) propose some approaches to suppress the influence of numerical implementations on stability. The author of the paper (Krstic, 2008) firstly constructs a Lyapunov function for predictor-based control systems and therefore, the robustness against delay uncertainty and model uncertainty can be analyzed with the proposed Lyapunov function. The key technique in Krstic (2008) is to transform the closed-loop system into a PDE and then an infinite-dimensional backstepping technique can be adopted. Such a result is then extended to control systems with time-varying input delays in Krstic (2009). Predictor-based stabilization for control systems with distributed delays is also investigated in Bekiaris-Liberis and Krstic (2011). In Bekiaris-Liberis and Krstic (2011), there are multiple control inputs with delays and each controller predicts the future state by a centralized manner. The second prediction approach abandons the integral term based on a low-gain principle (Zhou et al., 2012) and various extensions of such predictors have been investigated such as control systems with input delays and state delays (Zhou, Li, Zheng, & Duan, 2012). The third approach is given in Mazenc

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and Malisoff (2017) and Najafi, Hosseinnia, Sheikholeslam, and Karimadini (2013) and distributed term is not involved. The controllers in Mazenc and Malisoff (2017) and Najafi et al. (2013) consist of a series of dynamical subsystems and the state of each subsystem tracks the future state of the plant dynamically. As is known, interconnected systems can be found in many important situations such as networked control systems and complex systems (Borgers & Heemels, 2014; Liu, Wang, & Liu, 2016). The considered systems consist of multiple coupled subsystems with input delays and each subsystem has a local controller which is independent of the other subsystems. Moreover, each controller can access the state information of the corresponding subsystem and cannot utilize the state information of the other subsystems. Compensation for such control systems with input delays keeps a challenging problem since only part of the state and input information can be used to predict the future state for each controller. Besides, numerical implementation for predictor-based controllers is an important factor that may influence the stability of the closed-loop systems (Mondié & Michiels, 2003; Zhong, 2004) and Lyapunov functionals play a dominant role in robustness analysis with respect to various kinds of disturbances (Hale & Lunel, 1993; Khalil, 2002). Therefore, finding an explicit Lyapunov functional for predictor-based controllers under numerical implementations becomes an urgent work. This motivates the investigation of this paper.

The main contribution of this paper can be concluded as the following points. The first one is that distributed predictor-based controller is proposed to stabilize the interconnected systems with input delays. Each controller predicts the state of the corresponding subsystem without using the state information of other subsystems. Such kinds of controllers can also be used to stabilize a complex system with input delays which can be decomposed into interconnected systems with input delays (Borgers & Heemels, 2014; Liu et al., 2016). The second one is that low-pass filter versions of the distributed controllers are also presented. It is asserted that under numerical implementations, the low-pass filter versions will permit an explicit Lyapunov functional and exponential stability of the whole closed-loop system can be concluded provided that the numerical implementations have enough accuracy. The construction of such Lyapunov functionals depend on the backstepping transformation (Krstic, 2008) by introducing a transport PDE and the construction of Lyapunov functionals for delayed systems (Fridman, 2014; Liu et al., 2016).

Throughout this paper, $\mathbb{R}$ denotes the set of real and $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ dimensional matrices. $|\cdot|$ denotes the matrix 2-norm. For a positive definite matrix P, $\lambda_{max}(P)$ denotes the largest eigenvalue of P. I denotes the identity matrix.

2. Distributed controllers

Consider the following interconnected systems consisting of two linear plants with input delays

$$\begin{cases} \dot{z}_1 = A_{11}z_1(t) + A_{12}z_2(t) + B_1U_1(t-d_1) \\ \dot{z}_2 = A_{21}z_1(t) + A_{22}z_2(t) + B_2U_2(t-d_2) \end{cases} \tag{1}$$

where $z_1 \in \mathbb{R}^{n_1}$ and $z_2 \in \mathbb{R}^{n_2}$ are respectively the states of two different plants $\mathcal{P}_1$ and $\mathcal{P}_2$, $U_1 \in \mathbb{R}^{m_1}$ and $U_2 \in \mathbb{R}^{m_2}$ are respectively the inputs of the two plants, $d_1 > 0$ and $d_2 > 0$ are respectively two delay constants.

Now, we design a distributed predictor-based controller to stabilize the system (1) as follows

$$\begin{aligned} U_1(t) &= K_1(e^{A_{11}d_1}z_1(t) + \int_{t-d_1}^{t} e^{A_{11}(t-\theta)}B_1U_1(\theta)d\theta) \\ U_2(t) &= K_2(e^{A_{22}d_2}z_2(t) + \int_{t-d_2}^{t} e^{A_{22}(t-\theta)}B_2U_2(\theta)d\theta). \end{aligned} \tag{2}$$

A stability theorem is presented as follows.

Theorem 1. *Consider the system consisting of the plant (1) and the controller (2). The closed-loop system is exponentially stable if for a given constant $\delta > 0$, there exist symmetric matrices $P_j > 0\,(j = 1, 2)$, constants $c_j > 0\,(j = 1, 2)$ and $a_j > 0\,(j = 1, 2)$ such that the following conditions hold for $j = 1, 2$*

$$\begin{bmatrix} \Theta_j & P_jB_j & P_jA_{j(3-j)} \\ B_j^TP_j & -\frac{a_j}{2}I & 0 \\ A_{j(3-j)}^TP_j & 0 & -c_jI \end{bmatrix} < 0 \tag{3}$$

where

$$\begin{aligned} \Theta_j &= A_j^TP_j + P_jA_j + (\delta + c_{3-j} + \frac{1}{2}a_{3-j}\gamma_{3-j})I \\ \gamma_j &= \frac{4}{3}d_je^{(1+\delta+\rho_j)d_j}\lambda_{max}(A_{j(3-j)}A_{j(3-j)}^T)\lambda_{max}(K_jK_j^T) \\ \rho_j &= |A_{jj} + A_{jj}^T|, j = 1, 2. \end{aligned} \tag{4}$$

The proof of Theorem 1 is trivial based on Lemma 7 given below.

Remark 2. We note that the paper (Bekiaris-Liberis & Krstic, 2011) considers distributed delays. Each controller in the paper (Bekiaris-Liberis & Krstic, 2011) utilizes the state information of the whole systems. If a complex system can be decomposed as an interconnected system in the form of (1), then we offer an alternative controller design compared with that of Bekiaris-Liberis and Krstic (2011). Since each distributed controller does not utilize all the input information and state information, arbitrarily long input delays may not be permitted.

Next, we will give explicit design of controller gains. The controller gains are designed as follows

$$K_1 = X_1P_1, K_2 = X_2P_2, \tag{5}$$

where $X_j \in \mathbb{R}^{m_j\times n_j}$ and $P_j \in \mathbb{R}^{n_j\times n_j}$. With Theorem 1 at hand, we are ready to give the stabilization result.

Theorem 3. *The controller (2) stabilizes the plant (1) exponentially with K_1 and K_2 specified in (5) if for a given constant $\delta^* > 0$, there exist symmetric matrices $Q_j = P_j^{-1} > 0\,(j = 1, 2)$, matrices $X_j \in \mathbb{R}^{m_j\times n_j}(j = 1, 2)$, positive numbers $a_j^* > 0\,(j = 1, 2)$, $c_j^* > 0\,(j = 1, 2)$, $\alpha_j > 0\,(j = 1, 2)$ and $\beta_j > 0\,(j = 1, 2)$ such that the following matrix inequalities hold for $j = 1, 2$*

$$\begin{bmatrix} \Omega_{jj} & Q_j & Q_j & Q_j \\ Q_j & -\delta^*I & 0 & 0 \\ Q_j & 0 & -c_{3-j}^*I & 0 \\ Q_j & 0 & 0 & -2\gamma_{3-j}^*a_{3-j}^*I \end{bmatrix} < 0 \tag{6}$$

$$\alpha_jQ_j > I, X_jX_j^T < \beta_j \tag{7}$$

where

$$\begin{aligned} \gamma_j^*\lambda_{max}(A_{j(3-j)}A_{j(3-j)}^T) &= \frac{3}{4\alpha_j^2\beta_jd_je^{(1+\delta+\rho_j)d_j}} \\ \Omega_{jj} &= Q_jA_{jj}^T + A_{jj}Q_j + 2a_j^*B_jB_j^T + B_1X_1 + X_1^TB_1^T \\ &\quad + c_j^*A_{j(3-j)}A_{j(3-j)}^T \\ \rho_j &= |A_{jj} + A_{jj}^T|, j = 1, 2. \end{aligned} \tag{8}$$

The proof of Theorem 3 is a combination of Theorem 1 and Schur complement lemma and thus is omitted.

Remark 4. α_j and $\beta_j\,(j = 1, 2)$ are tuning parameters. If we firstly give a set of positive numbers $\alpha_j\,(j = 1, 2)$ and $\beta_j\,(j = 1, 2)$, then the linear matrix inequality conditions can be obtained by combining Theorem 3 and the Schur complement lemma to design