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Stability analysis of a general class of singularly perturbed linear hybrid systems☆

Jihene Ben Rejeb [a], Irinel-Constantin Morărescu [a,*], Antoine Girard [b], Jamal Daafouz [a]

[a] Université de Lorraine, CRAN, UMR 7039 and CNRS, CRAN, UMR 7039, 2 av. Forêt de Haye, Vandœuvre-lès-Nancy, France
[b] Laboratoire des signaux et systèmes (L2S), CNRS, CentraleSupélec, Université Paris-Sud, Université Paris-Saclay, 3, rue Joliot-Curie, 91192 Gif-sur-Yvette, cedex, France

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ABSTRACT

We introduce and analyze a general class of singularly perturbed linear hybrid systems with both switches and impulses, in which the slow or fast nature of the variables can be mode-dependent. This means that, at switching instants, some of the slow variables can become fast and vice-versa. Firstly, we show that using a mode-dependent variable reordering we can rewrite this class of systems in a form in which the variables preserve their slow or fast nature over time. Secondly, we establish, through singular perturbation techniques, an upper bound on the minimum dwell-time ensuring the overall system's stability. Remarkably, this bound is the sum of two terms. The first term, which can be equal to zero, only depends on the matrices of the reduced order linear hybrid system describing the slow dynamics and corresponds to an upper bound on the minimum dwell time ensuring the stability of that system. The order of magnitude of the second term is determined by that of the parameter defining the ratio between the two time-scales of the singularly perturbed system. We show that the proposed framework can also take into account the change of dimension of the state vector at switching instants. Numerical illustrations complete our study.

1. Introduction

Systems characterized by processes that evolve on different time-scales are often encountered in biology (Chen & Aihara, 2002; Hodgkin & Huxley, 1952) but are also present in engineering (Malloci, 2009; Sanfelice & Teel, 2011). In this case, the standard stability analysis becomes more difficult and singular perturbation theory (Khalil, 2001; Kokotović, Khalil, & O'Reilly, 1999) has to be used. This theory is based on Tikhonov approach that proposes to approximate the dynamics by decoupling the slow dynamical processes from the faster ones. The stability analysis is done separately for each time scale and under appropriate assumptions one can conclude on the stability of the overall system. Significant results related to stability analysis and approximation of solutions of singularly perturbed systems can be found in Balachandra and Sethna (1975), Nesic and Teel (2001) and Teel, Moreau, and Nesic (2003). We also note that singular perturbation theory was used to study the behavior of piecewise smooth systems with state triggered switches (Fiore, Hogan, & di Bernardo, 2016; Llibre, da Silva, & Teixeira, 2009).

Another feature that characterizes many physical systems is the presence of discrete events that occur during the continuous evolution. These events include abrupt changes of dynamics or instantaneous state jumps, which lead to the classes of switched systems or impulsive systems, respectively. Stability analysis and stabilization of singularly perturbed linear switched systems are considered in Alwan, Liu, and Ingalls (2008) and Malloci, Daafouz, and Iung (2009). Interestingly, it is shown in Malloci, Daafouz, and Iung (2009) that even though the switched dynamics on each time scale are stable for all switching signals, the overall system may be destabilized by fast switching signals. Clearly, this is in contrast with classical results on continuous singularly perturbed linear systems (Kokotović et al., 1999) and is a motivation for developing dedicated techniques for stability analysis of singularly perturbed hybrid systems. Stability analysis of singularly perturbed impulsive systems is considered in Abdelrahim, Postoyan, and Daafouz (2015) and Simeonov and Bainov (1988). More general singularly perturbed hybrid systems can involve both switches and impulses.

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* Corresponding author.
E-mail addresses: jihene.ben-rejeb@univ-lorraine.fr (J.B. Rejeb), constantin.morarescu@univ-lorraine.fr (I.-C. Morărescu), antoine.girard@l2s.centralesupelec.fr (A. Girard), jamal.daafouz@univ-lorraine.fr (J. Daafouz).

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A stability result for this class of systems can be found in Sanfelice and Teel (2011). In these works, the slow or fast nature of the state variable does not change when an event (switch or impulse) occurs. In this paper we introduce and analyze a class of singularly perturbed linear hybrid systems in which, at switching instants, slow variables can become fast and vice-versa. Our framework also includes the analysis of singularly perturbed linear systems with or without switches and/or impulses. Moreover, taking advantage of the linear dynamics under study, we go beyond the results in Sanfelice and Teel (2011) by characterizing the required dwell-time in terms of the parameter defining the ratio between the two time-scales. Although the technique in Sanfelice and Teel (2011) can be adapted to take into account the change of the slow or fast nature of the variables, our results are intrinsically different due to the different way to obtain the reduced order system. Indeed, for the linear switching system presented in Malloci, Daafouz, and Iung (2009) we obtain a reduced order system which is stable for any switching rule while using the method in Sanfelice and Teel (2011) the reduced order system is stable only for switching rules satisfying a dwell-time condition which is independent of the ratio between the two time-scales. Consequently, we are able to characterize more precisely the size of the dwell-time guaranteeing overall system stability.

The class of dynamical systems discussed in this paper is motivated by an industrial application in steel production. The objective in rolling mills is to reduce the thickness of a strip and this goal is reached by maintaining the strip in a straight line and close to the mill axis. When each stand is linked to the others by the strip traction, there is no discontinuity in the model. The system has a two time scale nature as there is a slow dynamics corresponding to the lateral displacement of the strip after each stand and a fast dynamics corresponding to the angle between the strip and the mill axis. The corresponding control problem can be treated using classical linear techniques as it is enough from a practical point of view to consider small deviations around an ideal operating point (see Malloci, Daafouz, Iung, Bonidal, and Szczepanski, 2010 and references therein). The situation is different in the last phase of the rolling process called the tail end phase and where the strip leaves the stands one after the other. Traction is lost each time the strip leaves a stand and this increases the difficulty to guide the strip as it is free to move in all directions. There are several difficulties in this phase. The first one is related to model discontinuities. Each time the strip leaves a stand the system dynamics changes and switching occurs. Moreover, the tail end phase is very short, the switchings are very fast and stability of all subsystems is not a sufficient condition to guarantee the stability of the whole system. The second difficulty is related to the changes in the nature of the dynamics after switching. The angle which was a fast variable becomes a slow variable and this change occurs at each time the strip leaves a stand. A system with this behavior can be defined as a switched system with multiple time scales, changes in the nature of the state variables and changes in the dimension of the state vector (Malloci, Daafouz, Iung, Bonidal, & Szczepanski, 2009).

Starting from the above motivation, we introduce and analyze a general class of singularly perturbed linear hybrid systems with mode-dependent nature of the state variable in which the sequence of discrete events is time-dependent. Although some preliminary results have been presented in Rejeb, Morărescu, Girard, and Daafouz (2016), the main contributions of the current work are:

- a new class of singularly perturbed hybrid systems and a procedure to rewrite such systems as linear hybrid singularly perturbed systems where the nature of variables does not change at switching instants, both cases of fixed and variable dimensions of the slow and fast state vectors are considered;
- a new approach for stability analysis of singularly perturbed linear hybrid systems with both switches and impulses;
- the derivation of an upper bound on the minimal dwell-time between two events that ensures the stability of the singularly perturbed linear hybrid system.

It is noteworthy that, this bound is given as the sum of two terms. The first one corresponds to an upper bound on the minimum dwell-time ensuring the stability of the reduced order linear hybrid system describing the slow dynamics. The order of magnitude of the second term is determined by that of the parameter ε defining the ratio between the two time-scales of the singularly perturbed system. In particular, it follows that when the reduced order system has a common quadratic Lyapunov function, the first term is zero and the minimum dwell-time ensuring the stability of the overall system goes to zero as fast as ε or $-\varepsilon \ln(\varepsilon)$ when the time scale parameter ε goes to zero.

Basically, we combine the classical singular perturbation theory (Kokotović et al., 1999) with Lyapunov function arguments for hybrid systems (see Goebel, Sanfelice, & Teel, 2012 for details). Our results clearly differ from existing ones on singularly perturbed linear hybrid systems that we mentioned previously: Malloci, Daafouz, and Iung (2009) deals with the existence of common quadratic Lyapunov functions and thus characterizes systems that are stable without dwell-time assumption; the condition on the dwell-time established in Alwan et al. (2008) does not present a clear separation between the slow and fast dynamics of the system; and in Abdelrahim et al. (2015), Sanfelice and Teel (2011) and Simeonov and Bainov (1988) the stability is established under a dwell-time condition where the dwell-time does not explicitly depend on the time-scale parameter.

The paper is organized as follows : Section 2 describes the hybrid system model in the singular perturbation form and introduces the relevant notations. In this section, we also introduce a mode-dependent reordering of the state components allowing to rewrite the system in a form in which the variables preserve their slow or fast nature over time. Section 3 is devoted to new preliminary results concerning the stability analysis of singularly perturbed linear systems without switches or jumps. Section 4.1 presents the main results along with their Lyapunov-based proofs. These results give stability conditions and establish an upper-bound on the minimum dwell-time ensuring the stability of the system. An extension to the case of mode-dependent dimension of the state-vector is provided in Section 4.3. To illustrate the results, we provide in Section 5 a dwell-time analysis and a numerical example in the particular case of scalar fast and slow dynamics with only two switching modes. Some concluding remarks end the paper.

Notation

Throughout this paper, $\mathbb{R}_+$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote respectively, the set of nonnegative real numbers, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The identity matrix of dimension n is denoted by $\mathbf{I}_n$. We also denote by $\mathbf{0}_{n,m} \in \mathbb{R}^{n\times m}$ the matrix whose components are all 0. For a matrix $A \in \mathbb{R}^{n\times n}$, $\|A\|$ denotes the spectral norm *i.e.* induced 2 norm. $A \geq \mathbf{0}$ ($A \leq \mathbf{0}$) means that A is positive semidefinite (negative semidefinite). We write $A^\top$ and A^{-1} to respectively denote the transpose and the inverse of A. For a symmetric matrix $A \geq \mathbf{0}$, $A^{\frac{1}{2}}$ is the unique symmetric matrix $B \geq 0$ such that $B^2 = A$. The matrix A is said to be Hurwitz if all its eigenvalues have negative real parts. A is said to be Schur if all its eigenvalues have modulus smaller than one. The matrix A is said to be positive if all its coefficients are positive. We also use $x(t^-) = \lim_{\delta\to 0,\ \delta>0} x(t-\delta)$. Given a function $\eta : (0, \varepsilon^*) \to \mathbb{R}$, we say that $\eta(\varepsilon) = \mathcal{O}(\varepsilon)$ if and only if there exist $\varepsilon_0 \in (0, \varepsilon^*)$ and $c > 0$, such that for all $\varepsilon \in (0, \varepsilon_0)$, $|\eta(\varepsilon)| \leq c\varepsilon$.