Automatica 91 (2018) 79–88

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Disturbance estimator based output feedback exponential stabilization for Euler–Bernoulli beam equation with boundary control ☆

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ARTICLE INFO

Article history:
Received 21 December 2016
Received in revised form 6 October 2017
Accepted 2 December 2017

Keywords:
Disturbance rejection
Output feedback
Exponential stabilization
Euler–Bernoulli beam equation

ABSTRACT

In this paper, we are concerned with the output feedback exponential stabilization problem for a system (plant) described by a one-dimensional Euler–Bernoulli beam equation. The measurements are only the displacement and the angular velocity at the right end. An infinite dimensional estimator is designed to estimate the disturbance. With the estimated disturbance, we propose a state observer that is exponentially convergent to the original system, then design two different kinds of stabilizing controllers: one is based on the velocity feedback, the other is based on the angular velocity feedback. In both cases, by adopting the Riesz basis approach, the exponential stability of the closed-loop systems is built with guaranteeing that all internal systems are uniformly bounded. The numerical experiments are carried out to illustrate the theoretical results.

1. Introduction

In this paper, we are concerned with the output feedback exponential stabilization for an Euler–Bernoulli beam equation with shear force boundary control. The system is governed by the following partial differential equation:

$$\begin{cases} w_{tt}(x,t)+w_{xxxx}(x,t)=0,\ 0<x<1,\ t\geq 0,\\ w(0,t)=w_x(0,t)=0,\ t\geq 0,\\ w_{xx}(1,t)=0,\ t\geq 0,\\ w_{xxx}(1,t)=u(t)+f(w(\cdot,t),w_t(\cdot,t))+d(t),\\ w(x,0)=w_0(x),\ w_t(x,0)=w_1(x),\ 0\leq x\leq 1,\\ y_m(t)=\{w(1,t),w_{xt}(1,t)\},\ t\geq 0,\end{cases} \tag{1}$$

where $w(x,t)$ is the transverse displacement of the beam at time t and position x. u is the input (control) through shear force, y_m is the output signal, that is, the boundary pointwise signals $w(1,t)$ and $w_{xt}(1,t)$ are measured. $f: H_e^2(0,1)\times L^2(0,1)\to\mathbb{R}$ is an unknown possible nonlinear mapping that reflects the internal uncertainty, and d represents the unknown external disturbance which is only supposed to satisfy $d\in L^\infty(0,\infty)$. For the sake of simplicity, we use the notation

$$F(t):=f(w(\cdot,t),w_t(\cdot,t))+d(t)$$

as the "*total disturbance*". Beam (1) is clamped at one end and free at another end, which models typically the vibration control of a single link flexible robot arm with the total disturbance in the free (working) end. The objective of this paper is to design a feedback controller which generates the control signal u (using the measurements y_m) such that the state (w,w_t) of the system depicted in Fig. 1 converges to zero, exponentially.

The vibration controls for Euler–Bernoulli beam equation have received considerable attention since 1980s and numerous interesting results on the feedback stabilization have been derived by the backstepping approach (Smyshlyaev, Guo, & Krstic, 2009), by the Lyapunov approach integrated with energy multipliers (Chen, Delfour, Krall, & Payre, 1987), by the proportional derivative and strain control algorithm (Matsuno, Ohno, & Orlov, 2002) and by the frequency domain approach (Rebarber, 1995). For the engineering interpretation of the Euler–Bernoulli beam equation, we refer to Han, Benaroya, and Wei (1999). Most of these works, among many others, focus however, on those beams that have no uncertainty. When the disturbance flows into system, the stabilization problem raises a new challenge for the design of the control. To suppress the vibration and attenuate external distributed and boundary disturbance for a beam equation, the Lyapunov approach incorporated with a disturbance observer is adopted in Ge, Zhang,

☆ This work was supported by the Israel Science Foundation under grant 800/14 and the National Natural Science Foundation of China (No. 61403239). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Fouad Giri under the direction of Editor Miroslav Krstic.

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https://doi.org/10.1016/j.automatica.2018.01.031

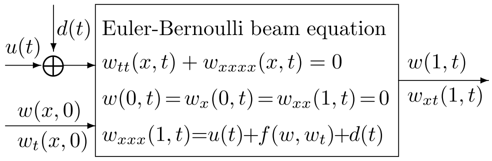

Fig. 1. Block diagram of the open-loop system (1).

and He (2011). The stabilization problem of system (1) without the internal uncertainty is first considered in Guo and Jin (2013), where two types of full state feedback controllers are constructed via both active disturbance rejection control (ADRC) and sliding mode control (SMC), respectively. The recent result on the stabilization for beam equation is discussed in Jin and Guo (2015), where Lyapunov redesign approach based the output feedback law is proposed. It is worth mentioning that many control methods have been developed to cope with internal uncertainty and external disturbance in PDEs in the literature. The adaptive control approach deals with parabolic PDEs subject to parameter uncertainties in Ahmed-Ali, Giri, Krstic, Burlion, and Lamnabhi-Lagarrigue (2016), and with hyperbolic PDEs with parameter uncertainties in Anfinsen and Aamo (2015) and Anfinsen, Diagne, Aamo, and Krstic (2016). The state feedback adaptive controls in Krstic (2010) and output feedback adaptive controls in Bresch-Pietri and Krstic (2014) are designed for one-dimensional wave equations in which the uncertainties are the unknown parameters. A recent result on output feedback stabilization for wave equation with disturbance is in Feng and Guo (2017b), where a new disturbance estimator for handling the unknown disturbance input is introduced. In Zhou and Weiss (2017), by using two signals only, the exponential stability for one-dimensional unstable/anti-stable wave equations is established. Very recently, the output feedback stabilization for multi-dimensional wave equation considered in Guo and Zhou (2015) is investigated in Feng and Guo (2017a) and Zhou and Guo (2017). However, both results on the stability in Feng and Guo (2017b), Guo and Zhou (2015) and Zhou and Guo (2017) are asymptotically stable. In this paper, we will achieve the exponential stability for the controlled beam equation.

To derive the exponential stability for a system with uncertainties, SMC that is inherently robust is the most popular approach that can achieve the exponential stability, but most often, the state feedback controllers (that are usually discontinuous) are designed in Cheng, Radisavljevic, and Su (2011), Guo and Jin (2013), Guo, Zhou, AL-Fhaid, Younas, and Asiri (2014) and Pisano, Orlov, and Usai (2011), which is not the case of output feedback. The interested reader may refer to (Orlov & Utkin, 1998) for achieving the finite time stability to heat processes by unit SMC with the state feedback. However, there are only few works on output feedback exponential stabilization for PDEs (such as (Jin & Guo, 2015)). In our work, inspired by Feng and Guo (2017b), we mainly deal with the output feedback exponential stabilization of Euler–Bernoulli beam equation by relaxing the assumptions required in Jin and Guo (2015), where the disturbance is supposed to satisfy $d \in H^1_{loc}(0,\infty) \cap L^\infty(0,\infty)$ and the initial state of system is assumed to be smooth. It is worth noting that the so-called Lyapunov redesign used in Jin and Guo (2015) seems not applicable because there is an internal nonlinear uncertainty.

It is well-known that when there is no disturbance, system (1) can be exponentially stabilized by the collocated feedback control (Guo & Yu, 2001):

$$u(t) = k w_t(1,t), \quad k > 0, \tag{2}$$

or by the collocated feedback control (Guo, Wang, & Yung, 2005):

$$u(t) = k w_{xt}(1,t), \quad k > 0, \ k \neq 1. \tag{3}$$

However, both (2) and (3) are not robust to external disturbance, for instance, when $f \equiv 0$, d is a constant, system (1) under (2) or (3) has a nonzero solution $(w, w_t) = ((x^3 - x^2)d/6, 0)$. When no disturbance flows into the system, the classical stabilizing control law is (2), whereas (3) is untraditional but very efficient. This efficiency has been explained numerically in Luo and Guo (1997) and proved theoretically in Guo et al. (2005). In this paper, based on the above facts, we will design two different types of exponential stabilizing controllers.

We consider system (1) in the energy Hilbert state space defined by $\mathcal{H} = H^2_e(0,1) \times L^2(0,1)$, $H^2_e(0,1) = \{\phi \in H^2(0,1) | \ \phi(0) = \phi'(0) = 0\}$, with the inner product induced norm given by

$$\|(\phi,\psi)\|^2_{\mathcal{H}} = \int_0^1 [|\phi''(x)|^2 + |\psi(x)|^2]dx, \ \forall (\phi,\psi) \in \mathcal{H}.$$

Define the operator $\mathcal{A}$ as follows:

$$\begin{cases} \mathcal{A}(\phi,\psi) = (\psi, -\phi^{(4)}), \ \forall (\phi,\psi) \in D(\mathcal{A}), \\ D(\mathcal{A}) = \{(\phi,\psi) \in \mathcal{H} \cap (H^4(0,1) \times H^2_e(0,1)) | \\ \qquad \phi''(1) = \phi'''(1) = 0\}, \end{cases} \tag{4}$$

and $\mathcal{B} = (0, \delta(x-1))$. By the boundary control system theory (Tucsnak & Weiss, 2009, Chapter 10), we can write system (1) as

$$\frac{d}{dt}\begin{pmatrix} w(\cdot,t) \\ w_t(\cdot,t) \end{pmatrix}^\top = \mathcal{A}\begin{pmatrix} w(\cdot,t) \\ w_t(\cdot,t) \end{pmatrix}^\top + \mathcal{B}[F(t) + u(t)]. \tag{5}$$

Proposition 1.1. *The operator $\mathcal{A}$ defined by (4) generates a C_0-group $e^{\mathcal{A}t}$ on $\mathcal{H}$ and $\mathcal{B}$ is admissible to $e^{\mathcal{A}t}$. Suppose that $f : \mathcal{H} \to \mathbb{R}$ is continuous and satisfies global Lipschitz condition in $\mathcal{H}$. Then, for any $(w_0, w_1) \in \mathcal{H}$, $u \in L^2_{loc}(0,\infty)$, and $d \in L^2_{loc}(0,\infty)$, there exists a unique global solution (mild solution) to (5) such that $(w(\cdot,t), w_t(\cdot,t)) \in C(0,\infty;\mathcal{H})$.*

Proof. We keep in mind that the proof on the admissibility of $\mathcal{B}$ for $e^{A_1 t}$ in Lemma 2.1 is independent of this proposition and $\mathcal{A}$ is the special case of A_1 with $c_1 = 0$, then the admissibility of $\mathcal{B}$ for $e^{\mathcal{A}t}$ can directly follow from Lemma 2.1 by letting $c_1 = 0$ in (10). Therefore, it follows from Lemma A.2 that (5) admits a unique global solution such that $(w(\cdot,t), w_t(\cdot,t)) \in C(0,\infty;\mathcal{H})$. □

The main contributions of this paper are twofold: (a) design a disturbance estimator and an estimator-based state observer for Euler–Bernoulli beam equation; (b) achieve the exponential stability for the closed-loop systems for both the velocity feedback and the angular velocity feedback by rejecting the disturbance that consists of not only the external disturbance but also the internal possibly nonlinear uncertainties.

The paper is organized as follows: In Section 2, we propose an untraditional disturbance estimator without invoking high gain to estimate the total disturbance. Section 3 is devoted to the design of the state observer. With the estimated state, an observer based stabilizing control law is presented. The exponential stability of the closed-loop system is concluded. In Section 4, we present an alternative method to design the stabilizing control law that maybe has the fast convergence rate. The numerical simulation result is given in Section 5. Finally, some concluding remarks are presented in Section 6.

2. Disturbance estimator design

In this section, we propose an infinite dimensional estimator to estimate the total disturbance, in terms of input and output