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Brief paper

An approach to quantized consensus of continuous-time linear multi-agent systems[☆]

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ABSTRACT

This paper investigates the consensus problem of continuous-time linear multi-agent systems (MASs), under the communication constraint of limited bandwidth. By constructing a novel dynamic quantizer, a distributed protocol via sampled and quantized data is designed to solve this problem. It is shown that the required number of the quantization levels of the new quantizer remains to be small even if the number of the agents in the MAS is large. A simulation example is given to illustrate the effectiveness of the proposed consensus protocol.

1. Introduction

Multi-agent systems (MASs) have been widely studied in the past decade due to their great potential in various engineering applications (Jadbabaie, Lin, & Morse, 2003; Meng, Ren, & You, 2010; Ren, 2008; Ren & Beard, 2008; Su, Wang, & Lin, 2009). The consensus problem with the objective for all agents to reach a common goal is a fundamental problem in study of MASs, and has received considerable attention (Chen, Lewis, & Xie, 2011; Dibaji, Ishii, & Tempo, 2016; Dimarogonas & Johansson, 2010; Frasca, 2012; Frasca, Carli, Fagnani, & Zampieri, 2009; Kashyap, Başar, & Srikant, 2007; Li, Fu, Xie, & Zhang, 2011; Li, Liu, Wang, & Yin, 2014; Qiu, Xie, & Hong, 2016; Su, Chen, Wang, & Lin, 2011; You & Xie, 2011; Zhang, Lewis, & Das, 2011; Zhu, Jiang, & Feng, 2014).

Quantized consensus, as one of consensus problems, has attracted increasing attention from researchers in the field (Ceragioli, De Persis, & Frasca, 2011; Chen et al., 2011; Dibaji et al., 2016; El Chamie, Liu, & Başar, 2016; Frasca, 2012; Frasca et al., 2009; Kashyap et al., 2007; Lavaei & Murray, 2012; Li et al., 2011, 2014; Qiu et al., 2016; Thanou, Kokiopoulou, & Frossard, 2012; You & Xie, 2011). In early works, most researchers focus on the quantized consensus problem for discrete-time MASs with single integrator agent dynamics, by designing static quantizers (El Chamie et al., 2016; Frasca et al., 2009; Kashyap et al., 2007) and dynamic quantization schemes (Li et al., 2011, 2014; Qiu et al., 2016; Rego, Pu, Alessandretti, Aguiar, & Jones, 2015; You & Xie, 2011), respectively. More recently, the quantized consensus problem of continuous-time MASs starts to attract more attention. Under the assumption that the communication graph is connected, the authors in Dimarogonas and Johansson (2010) presented a distributed protocol by quantizing the differences between the states of each pair of agents. In Chen et al. (2011), the authors designed a distributed protocol based on binary quantizers to solve the consensus problem of leader-following MASs. Using Krasovskii differential inclusions, the authors in Ceragioli et al. (2011) designed a distributed protocol to achieve consensus for MASs over balanced graph with quantized information of the states, while the authors in Frasca (2012) discussed the quantized consensus problem for MASs over directed graphs. However, to our best knowledge, there is no results in open literature on quantized consensus with sampled data for general linear continuous-time MASs, especially with consideration of limited communication bandwidth.

In this paper, by constructing a ISS-Lyapunov function, a distributed protocol is developed to solve the quantized consensus problem of continuous-time MASs. The main contributions of this work can be summarized as follows.

(**I**) Compared with the papers (Ceragioli et al., 2011; Chen et al., 2011; Dimarogonas & Johansson, 2010; Frasca, 2012) where quantized consensus of continuous-time single-integrator MASs is considered, this paper considers MASs with more general agent

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dynamics. It is worth pointing out that to our best knowledge, the quantized consensus problem of general linear continuous-time MAS have not been discussed in the literature to date.

(**II**) The constraint of limited bandwidth on the communication channel between each pair of agents is also considered in this paper. To satisfy the constraint of limited communication bandwidth, each agent is only required to communicate with its neighbors at fixed time instants, and only finite-bit data are required to be transmitted between each pair of agents at each time instant. It is worth pointing out that in most existing works on quantized consensus of continuous-time MASs, each agent is required to continuously communicate with its neighbors.

(**III**) A novel dynamic quantizer with finite quantization levels is constructed, and the required number of the quantization levels remains to be small even though the number of agents in the MAS is large.

The remainder of this paper is organized as follows. In Section 2, some relevant preliminaries on graphs are provided and the problem under consideration is formulated. In Section 3, a distributed protocol and the corresponding state estimator are designed. The consensus analysis of the closed-loop system is given in Section 4. The simulation example is given in Section 5. Section 6 gives the final conclusion.

Notations: $\|.\|$ and $\|.\|_\infty$ denotes 2-norm and infinity-norm, respectively. $|a|$ denotes the absolute value of $a \in \mathbb{C}$. $\lfloor a \rfloor$ denotes the maximum integer not greater than $a \in \mathbb{R}$, while $\lceil a \rceil$ denotes the minimum integer not smaller than $a \in \mathbb{R}$. $\mathrm{diag}(A_1, \dots, A_n)$ denotes a diagonal matrix with $A_1, \dots, A_n$ as its diagonal elements. $\otimes$ is the Kronecker product.

2. Preliminaries

2.1. Graph theory

The communications between agents are described by an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Each agent can be denoted as a node and $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $(i, j) \in \mathcal{E}$ represents that the information of agent i is accessible to agent j. $\mathcal{N}_i$ represents the set of all agents whose information is available to agent i. $\mathcal{A} \triangleq [a_{ij}]_{N\times N}$ is the adjacency matrix of the graph $\mathcal{G}$, where $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. A sequence of edges $(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)$ with $(i_{j-1}, i_j) \in \mathcal{E}$ for all $j \in \{2, \dots, k\}$ is called a path from agent i_1 to i_k. The graph $\mathcal{G}$ is called connected if there always exists a path between any two different nodes of the graph. Furthermore, we define the Laplacian matrix $\mathcal{L}_\mathcal{G} \triangleq [l_{ij}]_{N\times N}$, where if $i \neq j$, $l_{ij} = -a_{ij}$; otherwise $l_{ii} = \sum_{j\neq i}^{N} a_{ij}$. Denote $\lambda_j \in \mathbb{R}$, $1 \le j \le N$ as the eigenvalues of $\mathcal{L}_\mathcal{G}$, and rank all eigenvalues in an ascending order $0 = \lambda_1 \le \dots \le \lambda_N$.

2.2. Problem formulation

Consider a group of agents which can be described as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \dots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}$ represent the state and control input of agent i, respectively. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^n$ are the state and input matrices, respectively.

A typical encoder/decoder scheme is shown in Fig. 1, where $\{t_k^j\}$ is the communication time sequence. $\widehat{x}_j^i(t)$ represents the estimation of the state of agent j by agent i based on the quantized information that it has received up to time $t_k^j \le t$. For simplicity, a fixed sampling time is adopted in this work, $t_k^j = kT$, $k \in \mathbb{N}$, where the time interval $T > 0$.

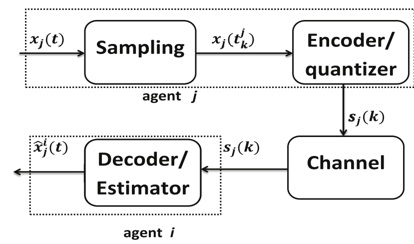

Fig. 1. Encoder/decoder scheme.

This paper aims to develop a distributed protocol $u_i(t) = u_i(\widehat{x}_i^i(t), \widehat{x}_j^i(t))$, $j \in \mathcal{N}_i$ and the corresponding state estimator $\widehat{x}_j^i(t)$ under the constraint of limited bandwidth on the communication network, such that all agents reach consensus for any initial state, that is,

$$\lim_{t\to\infty} \|x_j(t) - x_i(t)\| = 0, \forall\, i, j \in \mathcal{V}. \tag{2}$$

It is noted that the trivial case of A being Hurwitz will be excluded in this study. To proceed further, the following assumptions are needed.

Assumption 1. The pair (A, B) is stabilizable.

Assumption 2. The graph $\mathcal{G}$ is connected.

Assumption 3. $\|x_i(0)\|_\infty \le M_0$, $\forall\, i \in \mathcal{V}$ and M_0 is known by all agents.

Assumption 4. N_1 is an upper bound of N, which is known by all agents.

Remark 1. It is worth pointing out that Assumption 3 is required in most existing works (Li et al., 2011; You & Xie, 2011) to ensure that the quantizer is unsaturated at the initial time, when the quantized consensus problem of MASs is considered.

3. Design of distributed protocol

In this section, a novel distributed protocol including the dynamic quantizer and state estimators is presented.

It is noted that the state estimators of all agents are designed to be of the same form, and thus $\widehat{x}_j^i(t) = \widehat{x}_j^l(t)$, $\forall j \in \mathcal{V}$, $i \in \mathcal{N}_j$. For simplicity of notation, $\widehat{x}_j^i(t)$ and $\widehat{x}_j^l(t)$ will be denoted as $\widehat{x}_j(t)$. Hence, similar to Zhang et al. (2011), the following distributed protocol is used:

$$u_i(t) = K \sum_{j=1}^{N} a_{ij}\Big(\widehat{x}_j(t) - \widehat{x}_i(t)\Big), \tag{3}$$

where $\widehat{x}_i(t) \in \mathbb{R}^n$ and $\widehat{x}_j(t) \in \mathbb{R}^n$ are the estimations of the state $x_i(t)$ and $x_j(t)$, respectively, $K = cB^TP$, $c \ge \frac{1}{4\left(1-\cos\left(\frac{\pi}{N_1}\right)\right)}$ is a constant, N_1 is an upper bound of N, $P > 0$ is the solution to the following Riccati equation:

$$A^TP + PA - PBB^TP + \epsilon I_n = 0 \tag{4}$$

with $\epsilon > 0$ and I_n being a unit matrix with n dimensions.

Remark 2. Let $\bar{x}(t) = 1/N\sum_{i=1}^{N} x_i(t)$. Then by applying the distributed protocol (3) to the MAS (1), one has

$$\begin{aligned}\dot{\bar{x}}(t) = {} & 1/N(\mathbf{1}_N^T \otimes A - \mathbf{1}_N^T\mathcal{L}_\mathcal{G} \otimes BK)x(t) \\ & + 1/N(\mathbf{1}_N^T\mathcal{L}_\mathcal{G} \otimes BK)e(t) = A\bar{x}(t)\end{aligned} \tag{5}$$