Automatica 90 (2018) 172–184

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Robust MPC for tracking constrained unicycle robots with additive disturbances ☆

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ARTICLE INFO

Article history:
Received 18 November 2016
Received in revised form 21 June 2017
Accepted 28 September 2017
Available online 15 February 2018

Keywords:
Robust control
Model predictive control (MPC)
Unicycle robots
Bounded disturbances

ABSTRACT

Two robust model predictive control (MPC) schemes are proposed for tracking unicycle robots with input constraint and bounded disturbances: tube-MPC and nominal robust MPC (NRMPC). In tube-MPC, the control signal consists of a control action and a nonlinear feedback law based on the deviation of the actual states from the states of a nominal system. It renders the actual trajectory within a tube centered along the optimal trajectory of the nominal system. Recursive feasibility and input-to-state stability are established and the constraints are ensured by tightening the input domain and the terminal region. In NRMPC, an optimal control sequence is obtained by solving an optimization problem based on the current state, and then the first portion of this sequence is applied to the real system in an open-loop manner during each sampling period. The state of the nominal system model is updated by the actual state at each step, which provides additional feedback. By introducing a robust state constraint and tightening the terminal region, recursive feasibility and input-to-state stability are guaranteed. Simulation results demonstrate the effectiveness of both strategies proposed.

1. Introduction

Tracking control of nonholonomic systems is a fundamental motion control problem and has broad applications in many important fields such as unmanned ground vehicle navigation (Simanek, Reinstein, & Kubelka, 2015), multi-vehicle cooperative control (Wang & Ding, 2014) and formation control (Lafferriere, Williams, Caughman, & Veerman, 2005). So far, many techniques have been developed for control of nonholonomic robots (Ghommam, Mehrjerdi, Saad, & Mnif, 2010; Jiang & Nijmeijer, 1997; Lee, Song, Lee, & Teng, 2001; Marshall, Broucke, & Francis, 2006; Yang & Kim, 1999). However, these techniques either ignore the mechanical constraints, or require the persistent excitation of the reference trajectory, i.e., the linear and angular velocity must not converge to zero (Gu & Hu, 2006). Model predictive control (MPC) is widely used for constrained systems. By solving a finite horizon open-loop optimization problem on-line based on the current system state at each sampling instant, an optimal control sequence is obtained. The first portion of the sequence is applied to the system at each actuator update (Mayne, Rawlings, Rao, & Scokaert, 2000). MPC of four-wheel vehicles was studied in Frasch et al. (2013), Shakouri and Ordys (2011, 2014) and Tashiro (2013), in which real-time control for application was emphasized. MPC for tracking of noholonomic systems was studied in Chen, Sun, Yang, and Chen (2010), Gu and Hu (2006), Sun and Xia (2016) and Wang and Ding (2014), where the robots were considered to be perfectly modeled. However, when the system is uncertain or perturbed, stability and feasibility of such MPC may be lost. Stochastic MPC and robust MPC are two main approaches to deal with uncertainty (Mayne, 2016). In stochastic MPC, it usually "soften" the state and terminal constraints to obtain a meaningful optimal control problem (see Dai, Xia, Gao, Kouvaritakis, & Cannon, 2015; Grammatico, Subbaraman, & Teel, 2013; Hokayem, Cinquemani, Chatterjee, Ramponi, & Lygeros, 2012; Zhang, Georghiou, & Lygeros, 2015). This paper focuses on robust MPC and will present two robust MPC schemes for a classical unicycle robot tracking problem.

There are several design methods for robust MPC. One of the simplest approaches is to ignore the uncertainties and rely on the inherent robustness of deterministic MPC, in which an open-loop control action computed on-line is applied recursively to the system (Marruedo, Alamo, & Camacho, 2002b; Scokaert & Rawlings,

☆ This work was supported in part by the Beijing Natural Science Foundation under Grant 4161001, in part by the National Natural Science Foundation of China under Grant 61720106010, Grant 61422102, Grant 61603041 and Grant 61503026, and in part by the Foundation for Innovative Research Groups of the National Natural Science Foundation of China under Grant 61621063. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Riccardo Scattolini under the direction of Editor Ian R. Petersen.

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https://doi.org/10.1016/j.automatica.2017.12.048
0005-1098/

1995). However, the open-loop control and the uncertainty may degrade the control performance, or even render the system unstable. Hence, feedback MPC was proposed in Kothare, Balakrishnan, and Morari (1996), Lee and Yu (1997) and Wan and Kothare (2002), in which a sequence of feedback control laws is obtained by solving an optimization problem. The determination of a feedback policy is usually prohibitively difficult. To overcome this difficulty, it is intuitive to focus on simplifying approximations by, for instance, solving a min–max optimization problem on-line. Min–max MPC provides a conservative robust solution for systems with bounded disturbances by considering all possible disturbances realizations (Lee & Yu, 1997; Limón, Alamo, Salas, & Camacho, 2006; Wan & Kothare, 2002). It is in most cases computationally intractable to achieve such feedback laws, since the computational complexity of min–max MPC grows exponentially with the increase of the prediction horizon.

Tube-MPC taking advantage of both open-loop and feedback MPC approaches was reported in Fleming, Kouvaritakis, and Cannon (2015), Langson, Chryssochoos, Raković, and Mayne (2004), Mayne, Kerrigan, Van Wyk, and Falugi (2011), Mayne and Langson (2001), Mayne, Seron, and Raković (2005) and Yu, Maier, Chen, and Allgöwer (2013). Here the controller consists of an optimal control action and a feedback control law. The optimal control action steers the state to the origin asymptotically, and the feedback control law maintains the actual state within a "tube" centered along the optimal state trajectory. Tube-MPC for linear systems was advocated in Langson et al. (2004) and Mayne and Langson (2001), where the center of the tube was provided by employing a nominal system and the actual trajectory was restricted by an affine feedback law. It was shown that the computational complexity is linear rather than exponential with the increase of prediction horizon. The authors of Mayne et al. (2005) took the initial state of the nominal system employed in the optimization problem as a decision variable in addition to the traditional control sequence, and proved several potential advantages of such an approach. Tube-MPC for nonlinear systems with additive disturbances was studied in Mayne et al. (2011) and Yu et al. (2013), where the controller possessed a similar structure as in the linear case but the feedback law was replaced by another MPC to attenuate the effect of disturbances. Two optimization problems have to be solved on-line, which increases the computation burden.

In fact, tube-MPC provides a suboptimal solution because it has to tighten the input domain in the optimization problem, which may degrade the control performance. It is natural to inquire if nominal MPC is sufficiently robust to disturbances. A robust MPC via constraint restriction was developed in Chisci, Rossiter, and Zappa (2001) for discrete-time linear systems, in which asymptotic state regulation and feasibility of the optimization problem were guaranteed. In Marruedo, Alamo, and Camacho (2002a), a robust MPC for discrete-time nonlinear systems using nominal predictions was presented. By tightening the state constraints and choosing a suitable terminal region, robust feasibility and input-to-state stability were guaranteed. In Richards and How (2006), the authors designed a constraint tightened in a monotonic sequence in the optimization problem such that the solution is feasible for all admissible disturbances. A novel robust dual-mode MPC scheme for a class of nonlinear systems was proposed in Li and Shi (2014b), the system of which is assumed to be linearizable. Since the procedure of this class of robust MPC is almost the same as nominal MPC, we call this class nominal robust MPC (NRMPC) in this paper.

Robust MPC for linear systems is well studied but for nonlinear systems is still challenging since it is usually intractable to design a feedback law yielding a corresponding robust invariant set. Especially, the study of robust MPC for nonholonomic systems remains open. Consequently, this paper focuses on the design of robust MPC for the tracking of unicycle robots with coupled input constraint and bounded additive disturbance, which represents a particular class of nonholonomic systems. We discuss the two robust MPC schemes introduced above. First, a tube-MPC strategy with two degrees of freedom is developed, in which the nominal system is employed to generate a central trajectory and a nonlinear feedback is designed to steer the system trajectory within the tube for all admissible disturbances. Recursive feasibility and input-to-state stability are guaranteed by tightening the input domain and terminal constraint via affine transformation and all the constraints are ensured. Since tube-MPC sacrifices optimality for simplicity, an NRMPC strategy is presented, in which the state of the nominal system is updated by the actual one in each step. In such a way, the control action applied to the real system is optimal with respect to the current state. Input-to-state stability is also established in this case by utilizing the recursive feasibility and the tightened terminal region.

The remainder of this paper is organized as follows. In Section 2, we outline the control problem and some preliminaries. Tube-MPC and NRMPC are developed in Sections 3 and 4, respectively. In Section 5, simulation results are given. Finally, we summarize the paper in Section 6.

Notation: $\mathbb{R}$ denotes the real space and $\mathbb{N}$ denotes the collection of all nonnegative integers. For a given matrix M, $\|M\|$ denotes its 2-norm. $\text{diag}\{x_1, x_2, \ldots, x_n\}$ denotes the diagonal matrix with entries $x_1, x_2, \ldots, x_n \in \mathbb{R}$. For two vectors $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ and $y = [y_1, y_2, \ldots, y_n]^{\mathrm{T}}$, $x < y$ means $\{x_1 < y_1, x_2 < y_2, \ldots, x_n < y_n\}$ and $|x| \triangleq [|x_1|, |x_2|, \ldots, |x_n|]^{\mathrm{T}}$ denotes its absolute value. $\|x\| \triangleq \sqrt{x^{\mathrm{T}}x}$ is the Euclidean norm. P-weighted norm is denoted as $\|x\|_P \triangleq \sqrt{x^{\mathrm{T}}Px}$, where P is a positive definite matrix with appropriate dimension. Given two sets $\mathbb{A}$ and $\mathbb{B}$, $\mathbb{A} \oplus \mathbb{B} \triangleq \{a + b | a \in \mathbb{A}, b \in \mathbb{B}\}$, $\mathbb{A} \ominus \mathbb{B} \triangleq \{a | \{a\} \oplus \mathbb{B} \subset \mathbb{A}\}$ and $M\mathbb{A} \triangleq \{Ma | a \in \mathbb{A}\}$, where M is a matrix with appropriate dimensions.

2. Problem formulation and preliminaries

In this section, we first introduce the kinematics of the nonholonomic robot and deduce the coupled input constraint from its mechanical model. Then, we formulate the tracking problem as our control objective, and finally give some preliminaries for facilitating the development of our main results.

2.1. Kinematics of the unicycle robot

Consider a nonholonomic robot described by the following unicycle-modeled kinematics:

$$\dot{\xi}(t) = f(\xi(t), u(t)) = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} u(t), \tag{1}$$

where $\xi(t) = [p^{\mathrm{T}}(t), \theta(t)]^{\mathrm{T}} \in \mathbb{R}^2 \times (-\pi, \pi]$ is the state, consisting of position $p(t) = [x(t), y(t)]^{\mathrm{T}}$ and orientation $\theta(t)$, and $u(t) = [v(t), \omega(t)]^{\mathrm{T}}$ is the control input with the linear velocity $v(t)$ and the angular velocity $\omega(t)$.

The unicycle robot is shown in Fig. 1, where $\rho > 0$ is half of the wheelbase, v^L and v^R are the velocities of the left and the right driving wheels, respectively. It is assumed that the two wheels possess the same mechanical properties and the magnitudes of their velocities are bounded by $|v^L| \le a$ and $|v^R| \le a$, where $a \in \mathbb{R}$ is a known positive constant. The linear and angular velocities of the robot are then

$$\begin{aligned} v &= (v^L + v^R)/2, \\ \omega &= (v^R - v^L)/2\rho. \end{aligned} \tag{2}$$