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Brief paper

Mode discernibility and bounded-error state estimation for nonlinear hybrid systems ☆

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ABSTRACT

State estimation is a key engineering problem when addressing control or diagnosis issues for complex dynamical systems. The issue is still challenging when the latter systems must be modelled as hybrid discrete–continuous dynamics, which is true for many complex and safety-critical systems. In this paper, we investigate nonlinear hybrid state estimation in a bounded-error framework using reliable and robust methods. We first establish a testable condition for current mode location discernibility. Then we build our hybrid state estimator which relies on a prediction–correction approach. An illustrative example is presented.

1. Introduction

State estimation is a key engineering problem when addressing control or diagnosis issues with complex dynamical systems. Many systems exhibit both smooth continuous dynamics and abrupt switches, hence can be efficiently modelled using hybrid automata, which combine discrete and continuous variables (Alur et al., 1995). Hybrid state estimation aims at reconstructing both the discrete mode, hence the switching sequence, and the associated continuous state variables, based on a set of possibly discrete-time measurements, the knowledge of the hybrid model, and assumptions about the uncertainties and perturbations acting on the system. For instance, Wang, Li, Zhou, and Liu (2007) developed a robust exponentially ultimately bounded hybrid state observer using the unknown input extended Kalman observer for hybrid systems with discrete-time nonlinear dynamics, while Guo and Huang (2013) developed a moving horizon estimation scheme for switched systems and analysed its stability under the uniform observability property. Balluchi, Benvenuti, Di Benedetto, and Sangiovanni-Vincentelli (2013) addressed exponentially ultimately bounded observer design for hybrid systems with linear continuous-time dynamics, and Barhoumi, Msahli, Djemaï, and Busawon (2012) addressed the synthesis of high gain observers for uniformly observable nonlinear hybrid systems.

In this paper, we address hybrid state estimation in the *unknown-but-bounded-error* (UBBE) framework, where one assumes that all uncertain quantities, not only measurement noise but model uncertainty and modelling errors belong to a known bounded set with no other assumption about the distribution within the set (Milanese, Norton, Piet-Lahanier, & Walter, 1996; Schweppe, 1968). In many cases, the UBBE assumption is natural and straightforward, and it requires less data than any statistical assumptions. In the UBBE framework, the estimation problem no longer has a unique solution, but there exists a set of state vectors that are consistent with measured data, the model structure and the prior error bounds. Then, *set-membership estimation* (SME) techniques allow the derivation of a conservative outer-approximation of the set of *consistent* state vectors at each time instant. There has been a significant research effort related to SME with *continuous* systems and the developed approaches may be sorted in two main types. One type of methods focus on the design of Luenberger-like *interval* observers, which assume the

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availability of continuous measurements (a.o. Efimov, Raïssi, Chebotarev, & Zolghadri, 2013, Gouzé, Rapaport, & Hadj-Sadok, 2000, Raïssi, Efimov, & Zolghadri, 2012, Mazenc, Andrieu, & Malisoff, 2015; Mazenc & Dinh, 2014, Thabet, Raïssi, Combastel, Efimov, & Zolghadri, 2014). Another type of methods use and extend the predictor–corrector estimation scheme as encountered in the Kalman filter (Xiong, Jauberthie, Travé-Massuyès, & Gall, 2013). For nonlinear systems, Kieffer, Jaulin, and Walter (2002) developed the first predictor–corrector based SME approach for discrete-time systems using interval analysis, then Jaulin (2002), Meslem and Ramdani (2011), Meslem, Ramdani, and Candau (2010) and Raïssi, Ramdani, and Candau (2004) extended the approach to handle state estimation for continuous-time systems with discrete measurements by combining interval analysis and *reachability* computation capabilities as obtained using guaranteed solving tools for interval initial value problems (IVP) for nonlinear ordinary differential equations (ODE). This paper is in line with the second set of methods and aims at extending the predictor–corrector-based SME approach to truly nonlinear hybrid continuous–discrete dynamical systems with discrete measurements, thus developing an SME technique to *simultaneously* reconstruct, at each time instant, the set of consistent system's switching sequence and the corresponding set of consistent continuous state vectors.

SME for truly nonlinear hybrid systems is a challenging issue that has attracted only few researchers. To the best of our knowledge, the only works addressing this issue are by Benazera and Travé-Massuyès (2009), who addressed hybrid systems with discrete-time only nonlinear continuous dynamics, and Eggers, Ramdani, Nedialkov, and Fränzle (2012) who investigated the feasibility of using satisfiability checkers. Clearly, if one knew in which mode the hybrid system is operating, the estimation of the continuous component of the hybrid system would merely make use of the existing SME algorithms for continuous systems. Therefore, the main ingredient of our SME for hybrid systems is the ability to distinguish the current active location mode from the observation of the input–output behaviour. To the best of our knowledge, the observability and detectability of hybrid systems have been studied only for linear switching systems (Babaali & Pappas, 2005; De Santis, 2011; De Santis & Di Benedetto, 2017; De Santis, Di Benedetto, & Pola, 2003; De Santis, Di Benedetto, & Pola, 2009; Fliess, Join, & Perruquetti, 2008; Lou & Yang, 2011). In this paper we introduce a new computable condition for analysing mode discernibility for the general class of nonlinear hybrid systems. We say that two location modes are discernible if there exists a control making it possible to distinguish them by their outputs. In the case of autonomous systems, the output trajectories must differ at some point in time. Then, using an one-parameter-tuned composite continuous model, we show that the identifiability of the tuning parameter implies current mode discernibility. The contribution of this paper is twofold. First, we give a computable condition for current mode discernibility, then we build a predictor–corrector-type scheme for SME of the complete state of general class of hybrid systems, in the UBBE framework.

The paper is structured as follows: Section 2 defines hybrid dynamical systems, while Section 3 formulates the estimation problem. Section 4 introduces our approach for current mode discernibility analysis, while Section 5 describes the complete state set-membership estimation. Section 6 discusses method complexity and convergence. Section 7 reports the numerical evaluation on a realistic example, before conclusions.

2. Hybrid dynamical systems

Hybrid dynamical systems (HDS) can be represented by a hybrid automaton (Alur et al., 1995) given by

$$HA = (\mathbb{Q}, \mathbb{Z}, \mathbb{U}, \mathbb{F}, \mathrm{Inv}, \Sigma, \Psi, \mathbb{G}, \mathbb{A}), \tag{1}$$

where: $\mathbb{Q} = \{q\}$ is a set of locations, i.e. discrete state or modes; domain $\mathbb{Z} \subseteq \mathbb{R}^n$ is the definition domain of the continuous component with dimension n that may depend on q; domain $\mathbb{U} \subseteq \mathbb{R}^{n_u}$ is the set of admissible control inputs; $\mathbb{F} = \{f_q\}$ is the set of non-autonomous differential equations characterizing flow transition in mode q, of the form

$$\mathrm{flow}(q): \quad \dot{z}(t) = f_q(z(t), u(t)), \tag{2}$$

where $f_q : \mathbb{Z} \times \mathbb{U} \mapsto \mathbb{Z}$ is a nonlinear function assumed sufficiently smooth over $\mathbb{D} \subseteq \mathbb{R}^n$; Inv is an optional invariant, which assigns a domain to the continuous state space of each location:

$$\mathrm{Inv}(q): \quad \nu_q(z(t)) < 0, \tag{3}$$

where inequalities are taken componentwise, $\nu_q : \mathbb{Z} \mapsto \mathbb{R}^m$ is also nonlinear, and the number m of inequalities may also depend on q; Σ is a set of exogenous events; $\Psi = \{\rho_e\}_{e\in\mathbb{A}}$ is the set of reset maps, taken as continuous nonlinear functions; $\mathbb{G} = \{\gamma_e\}_{e\in\mathbb{A}}$ is the set of guard conditions of the form:

$$\mathrm{guard}(e): \quad \gamma_e(z(t)) = 0; \tag{4}$$

where $\gamma_e(.) : \mathbb{Z} \mapsto \mathbb{R}^{m'}$ is a nonlinear continuous function; $\mathbb{A} \subseteq \mathbb{Q} \times \mathbb{Q}$ is the set of discrete transitions $\{e = (q \to q')\}$ given by the 5-uple $(q, \mathrm{guard}, sq, \rho_e, q')$, where q and q' represent upstream and downstream locations respectively, $sq \in \Sigma$, $\rho_e \in \Psi$, and guard $\in \mathbb{G}$. A transition $q \to q'$ occurs when the continuous state flow reaches the guard set, i.e. when the continuous state satisfies condition (4).

Let us also consider the following measurement equation

$$\mathrm{output}(q): \quad y(t) = \mu_q^{\top} z(t), \tag{5}$$

where $\mu_q \in \mathbb{R}^{n\times n_y}$, depends on mode q.

Let us now recall the concept of hybrid trajectory (or hybrid solution). Let us consider a finite time horizon $[t_0,\ t_N]$ and denote $\chi(t_0) = (q_0,\ z_{q_0}(t_0))$ the initial hybrid state. We can define as in continuous dynamics,

$$z_{q_0}\,(t;\ t_0,\ \chi(t_0)) \tag{6}$$

the continuous state vector solution of the initial value problem (IVP) for the continuous ordinary differential equation (ODE) (2) starting from the initial state vector $z_{q_0}(t_0)$ at time t_0 in mode q_0. A discrete transition $e = q_0 \to q_1$ occurs when the continuous flow trajectory intersects the guard set at time t_e, i.e. $\exists t_e \geq t_0,\ \gamma_e(z_{q_0}(t_e)) = 0$. Then, the continuous state vector is reset as $z_{q_1}(t_e^{+}) = \rho_e(z_{q_1}(t_e^{-}))$. The switching sequence for HDS (2)–(5) may be written in the general case of M discrete transitions as

$$seq = \{(t_0,\ q_0), (t_{e_1},\ q_1),\ (t_{e_2},\ q_2), \ldots, (t_{e_M},\ q_M)\}. \tag{7}$$

In fact, at each time instant $t \in [t_0,\ t_N]$, we can define the hybrid solution trajectory of the hybrid system (2)–(5) starting from the continuous state vector $z_{q_0}(t_0)$ at t_0 in the discrete mode q_0 as

$$\chi(t;\ t_0,\ \chi(t_0)) = \left(q_i(t),\ z_{q_i(t)}(t;\ t_{e_i},\ \chi(t_{e_i};\ t_0,\ \chi(t_0)))\right), \tag{8}$$

where t_{e_i} is a switching time instant such that $t_{e_i} \leq t \leq t_{e_{i+1}}, e_i \in \mathbb{A}$. We can also define the HDS output by

$$y_{q_i(t)}(t;\ t_0,\ \chi(t_0)) = \mu_{q_i(t)}^{\top} z_{q_i(t)}(t;\ t_0,\ \chi(t_0)) \tag{9}$$

where $z_{q_i(t)}(t)$ is the continuous component of $\chi(t)$. Now, let us consider the set $\boldsymbol{\chi}_0 = \mathbb{Q}_0 \times \mathbb{Z}_0$ of possible initial hybrid state $\chi(t_0)$, cartesian product of the set of possible initial discrete modes $\mathbb{Q}_0$ and the bounded initial domain $\mathbb{Z}_0$ of $z_{q_0}(t_0)$ when in any mode