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Sliding mode control of hybrid switched systems via an event-triggered mechanism☆

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ABSTRACT

This paper is devoted to solving the problem of sliding mode control for discrete-time switched systems via an event-triggered strategy. First, a new linear switching function combined with corresponding networked sliding mode dynamics is constructed using a time-delay system design method and event-triggering scheme. Then, on the basis of the Lyapunov functional technique and the average dwell time approach, sufficient conditions for the existence of the concerned networked sliding mode control are established in terms of linear matrix inequalities. Furthermore, an event-triggered sliding mode control law is developed to drive the resultant closed-loop system trajectories into a bounded switched region and maintain them therein for subsequent periods. Finally, a verification example is given to show the effectiveness of the proposed new design techniques.

1. Introduction

Switched systems are a typical class of hybrid jump systems and consist of several subsystems with either continuous-time or discrete-time dynamics and a switch signal to administe the corresponding activation (Eguchi, Do, Kittipanyangam, Abe, & Oota, 2016). Physical systems with switching features can be regarded as hybrid switched systems, such as power and electronics systems, automated highway systems, flight control systems, and network control systems. Generally, there are two kinds of hybrid switched systems: restricted switched systems and random switched systems. The different system characteristics result from various switching signals and have a marked impact on stability analysis and system performance synthesis. Furthermore, the piecewise Lyapunov stability theories and the average dwell time (ADT) techniques have been proven to be effective methods for hybrid switched systems (Kundu & Chatterjee, 2015). Due to the broad theory and application prospects, switched systems have been extensively integrated with various control methods, such as fuzzy logic control (Su, Shi, Wu, & Song, 2016), sliding mode control (Wu, Gao, Liu, & Li, 2017), and model predictive control (Hariprasad & Bhartiya, 2017). The development of hybrid switched systems has led to extensive investigation into the problems of stability analysis (Kundu & Chatterjee, 2015), controller design (Yang, Liu, Shi, Thomas, & Basin, 2014), model reduction (Wu & Zheng, 2009), and filtering (Su et al., 2016).

Networked control systems (NCSs), with their characteristics of simple maintenance (Xiao, Sha, Yuan, & Wang, 2017; Zhu, Xiao, Sun, Wang, & Yang, 2017), easy installation (Mahmoud & Rahman, 2016; Selivanov & Fridman, 2016a), and low cost (Selivanov & Fridman, 2016b), have undergone rapid development recently (Kumar, Kaarthik, Gopakumar, Leon, & Franquelo, 2015; Premaratne, Halgamuge, & Mareels, 2017). However, the sharing of limited network bandwidth in NCSs often results in delayed transmission, the stochastic process, jitter, and packet loss, which may deteriorate system performance (Basin & Rodriguez-Gonzalez, 2005, 2006; Chadli, Abdo, & Ding, 2013). Therefore, owing to the capability to reduce sampling number while maintaining a satisfactory system performance, the event-triggered technique (Mahmoud & Memon, 2015; Zou, Su, & Niu, 2017a, b) has attracted a large amount of attention in the past five years. The event-triggered control is an effective control strategy in which the

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control task is implemented only when a pre-specified triggering criterion is satisfied, which means that transmission signals are not always triggered and transmitted at every sampling period during the control process. Thus, communication and computation resources can be greatly reduced. There have been many attempts to study the event-triggered approach. For instance, a solution to $\mathcal{L}_p$ event-triggered control problems was presented in Dolk, Borgers, and Heemels (2017), $\mathcal{H}_\infty$ event-triggered filtering issues were proposed in Davoodi, Meskin, and Khorasani (2017), maximum-likelihood state-estimation approaches were discussed in Shi, Chen, and Shi (2014), and $\mathcal{H}_\infty$ stabilization problems were solved in Ahn, Wu, and Shi (2016). Moreover, the event-triggered strategy has been widely investigated in many practical situations, such as multi-agent systems (Garcia, Cao, & Casbeer, 2017) and power systems (Wen, Yu, Zeng, & Wang, 2016). Additionally, networked switched systems are considered to be a class of hybrid systems with a finite number of subsystems, sampled state measurement, and a switching signal. These systems are characterized by sampling, network-induced delay, and switching (Fiacchinia & Tarbouriech, 2017). Although some studies exist on networked switched systems (Deaecto, Souza, & Geromel, 2015; Kruszewski, Jiang, Fridman, Richard, & Toguyeni, 2012), control synthesis issues have not been fully investigated. Moreover, the network communication environment significantly complicates the analysis and design of switched systems. In particular, taking switching rule and network-induced delays into account is of great importance, which motivates the present work.

Another active research front is the necessity of finding solutions for uncertainty disturbances (Kumar & Ganapathy, 2015; Lam, Wu, & Lam, 2015; Zhuk, Polyakov, & Nakonechnyi, 2017), modeling errors (Lam & Li, 2013), or system parameter variations (Poznyak, 2017) in practical systems. Due to its superior capability to guarantee a pre-specified system performance despite the existence of disturbances/uncertainties, sliding mode control (SMC) has been widely adopted for system controls. Considerable attention has been given to the SMC strategy. In particular, SMC solutions have been developed for many dynamic complex systems, including singular systems (Chadli & Darouach, 2014; Song, Niu, & Zou, 2016), stochastic systems (Wu et al., 2017), switched systems (Pisanoa, Tanellib, & Ferrarac, 2016), nonlinear polynomial systems (Basin & Rodriguez-Ramirez, 2014), and fuzzy systems (Khanesar, Kaynak, Yin, & Gao, 2015) . Note that engineering systems are ultimately converted into digital control systems for implementation (Liu, Laghrouche, Harmouche, & Wack, 2014; Liu, Luo, Yang, & Wu, 2016). Thus, numerous works are focused on SMC techniques for discrete-time systems (Su, Liu, Shi, & Yang, 2017). The discrete-time sliding mode control (DSMC) approaches are designed to force system trajectories into a small bounded region with a desirable system performance. By virtue of its fast response, easy implementation, and robustness, DSMC can be viewed as an effective solution in the instance of the aforementioned NCSs. Note that a few studies have focused on event-triggered sliding mode control schemes, such as linear time invariant (LTI) systems (Behera & Bandyopadhyay, 2017), and stochastic systems (Wu et al., 2017). However, to the best of our knowledge, few results on event-triggered DSMC design have been published for networked switched systems using the linear matrix inequality technique and the delay system analysis approach. There are many open problems related to the event-triggered DSMC design for discrete-time hybrid switched systems, such as how to deal with the network-induced delay, how to design the discrete-time sliding mode surface and sliding mode controller, regarding the event-triggered scheme, and how to effectively handle the system performance level for the resultant networked sliding mode dynamics with time-delay. Such issues form the motivation of our works due to their theoretical and practical importance.

In this paper, we focus on the event-triggered SMC strategy design for switched systems with matched disturbances. Specifically, the delay system analysis technique is adopted for the networked switched system and to handle the DSMC issue. The average dwell time analysis technique and the Lyapunov theory are applied to handle switching and triggering conditions. The main contributions of the paper are as follows.

- With the delay system analysis technique and the designed linear sliding mode surface, a networked sliding mode dynamics with time-delay is constructed.
- Based on the average dwell time analysis technique and the piecewise Lyapunov theory, sufficient conditions of exponential stability are proposed for the networked sliding mode dynamics in linear matrix inequality form.
- In consideration of the event-triggered technique, an improved discrete-time sliding mode controller is designed to guarantee the closed-loop system to be driven into the sliding region.

Notations. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P > 0$ ($\geqslant 0$) means that P is real symmetric and positive definite (semi-definite); diag{. . .} stands for a block-diagonal matrix; In symmetric block matrices or long matrix expressions, we use an asterisk ($\star$) to represent a term that is induced by symmetry.

2. Problem formulation and preliminaries

2.1. System description

The discrete-time switched system is described by:

$$x(k+1) = A\big(\alpha(k)\big)x(k) + B\Big[u(k) + F\big(\alpha(k)\big)f(x, k)\Big], \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the control input; $\alpha(k) : \mathbf{Z}^+ \rightarrow \mathcal{N}$ is the switching signal, where $\mathcal{N} = 1, 2, \ldots, N$. At any discrete sampling time k, the value of $\alpha(k)$ may be built by some hybrid schemes, for example, may rely on $x(k)$, k, or both k and $x(k)$. Here, define α as $\alpha(k)$ for simplicity. Assume that the sequence of sub-modes in switching signal α is unknown *a priori*, but its instantaneous value is accessible in real time. For the arbitrary switching time consecutiveness

$$k_0 < k_1 < k_2 < \cdots < k_l < \cdots$$

of switching signal α, the duration of switching time interval between $[k_l, k_{l+1}]$ is called the dwell time, where

$$\Big\{A(\alpha),\ F(\alpha) : \alpha \in \mathcal{N}\Big\}$$

are a family of system matrices. The relationship between the resultant system matrices and submode i is denoted by

$$A(i) = A(\alpha),\ F(i) = F(\alpha),$$

and $\Big\{A(i),\ F(i)\Big\}$ are known real matrices. Moreover, $f(x, k)$ denotes an unknown nonlinear function, which satisfies

$$\Big\|F(i)f(x, k)\Big\| \leqslant \eta(i),\ i \in \mathcal{N},$$

where $\eta(i) > 0$ are positive scalars.

Definition 1. For any $T_2 > T_1 \geqslant 0$, let $N_\alpha(T_1, T_2)$ denote the number of the switchings of $\alpha(k)$ over (T_1, T_2). If $N_\alpha(T_1, T_2) \leqslant N_0 + \frac{T_2 - T_1}{T_a}$ holds for $T_a > 0$, $N_0 \leqslant 0$, then, T_a is called an average dwell time.