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Brief paper

Admissibility analysis of discrete linear time-varying descriptor systems[☆]

Karina A. Barbosa[a,*], Carlos E. de Souza[b], Daniel Coutinho[c]

[a] Department of Electrical Engineering, Universidad de Santiago de Chile, Av. Ecuador 3519, Santiago, Chile
[b] Department of Mathematical and Computational Methods, Laboratório Nacional de Computação Científica (LNCC/MCTIC), Av. Getúlio Vargas 333, Petrópolis, RJ 25651-075, Brazil
[c] Department of Automation and Systems, Universidade Federal de Santa Catarina (UFSC), PO BOX 476, Florianópolis, SC 88040-900, Brazil

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ABSTRACT

This paper deals with the problem of admissibility analysis (i.e. regularity, causality and exponential stability) of discrete-time linear descriptor systems with uncertain time-varying parameters in the system state-space model matrices. The parameters enter affinely into all the matrices, and both their admissible values and variations are assumed to belong to given intervals. First, necessary and sufficient admissibility conditions for uncertainty-free discrete linear time-varying descriptor systems are presented. Next, strict linear matrix inequality conditions based on parameter-dependent Lyapunov functions are proposed to ensure robust admissibility of uncertain linear descriptor systems. Both the cases of Lyapunov functions with affine and quadratic dependence on the system uncertain parameters are considered. The robust admissibility analysis methods incorporate information on available bounds on both the admissible values and variation of the uncertain parameters. Numerical examples are presented to demonstrate the potentials of the proposed methods.

1. Introduction

Descriptor system is an important class of dynamical system which can model practical processes with both static and dynamical behavior, such as in economics (Luenbergar & Arbel, 1977), biological systems (Zhang, Liu, & Zhang, 2012), power systems (Chakrabortty & Ilic, 2012) and mechanical systems (Duan, 2010). Due to the relevance of descriptor systems, in the last decades substantial efforts have been made to extend classical control results to descriptor system, see, for example, Dai (1989), Duan (2010) and Xu and Lam (2006), and references therein. Note that the analysis of descriptor systems dynamics is more complex than that of standard systems (i.e. system with only dynamical behavior), as it is necessary to ensure that the system has a unique solution and is free of impulsive modes (in the continuous-time case) or causal (in the case of discrete-time systems). A stable and impulsive-free, or causal, descriptor system is commonly referred to as an admissible system.

In the case of uncertainty-free discrete-time linear time-invariant descriptor systems, necessary and sufficient conditions of admissibility have been proposed, as for instance, in Ishihara and Terra (2003), Ishihara, Terra, Cerri, and Manfrim (2010) and Xu and Lam (2006), whereas linear matrix inequality (LMI) based approaches of admissibility analysis of descriptor systems with polytopic-type uncertain constant parameters have been presented in Fang (2002) and Kuo and Fang (2003). Despite the large number of admissibility results for linear descriptor systems few works have dealt with systems subject to uncertain time-varying parameters. In this setting, we can cite, e.g., Bara (2011a, b) which have presented LMI conditions of robust admissibility (i.e. admissibility holds for all admissible uncertain parameters) and control methods for respectively continuous- and discrete-time uncertain polytopic linear descriptor systems based on affine parameter-dependent Lyapunov functions but considering the system in a differential–algebraic form (or difference-algebraic, in the discrete-time case). On the other hand, considering the system in a general form, robust admissibility conditions based on LMIs have been proposed in Barbosa, de Souza, and Coutinho (2012, 2013) for respectively the discrete- and continuous-time cases using affine and quadratic parameter-dependent Lyapunov functions.

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* Corresponding author.
E-mail addresses: karina.barbosa@usach.cl (K.A. Barbosa), csouza@lncc.br (C.E. de Souza), daniel.coutinho@ufsc.br (D. Coutinho).

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In the aforementioned works, it was assumed that the matrix coefficient of the one-step-ahead state, referred to as *the one-step-ahead state matrix*, is uncertainty-free and constant. Recently, admissibility and control of descriptor systems with an uncertain one-step-ahead state matrix has been investigated in Sari, Bachelier, and Mehdi (2012) considering constant polytopic parameter uncertainties and an affine parameter-dependent Lyapunov function, whereas Mao (2012) has focused on systems subject to norm-bounded parameter uncertainties and considering the notion of quadratic stability. To the best of the authors' knowledge, to-date the problem of admissibility analysis for discrete-time linear descriptor systems subject to convex bounded time-varying parameter uncertainty in the one-step-ahead state matrix has not been yet fully resolved, and will be addressed in this paper.

This paper firstly focuses on the admissibility of general discrete linear time-varying (DLTV) descriptor systems. Three equivalent necessary and sufficient admissibility conditions are developed, where the first one is given in terms of a new generalized strict Lyapunov difference inequality and the other two are tailored via strict linear matrix difference inequalities. Next, we address the problem of robust admissibility analysis for discrete-time linear descriptor systems subject to convex-bounded uncertain time-varying parameters. The parameters appear affinely in all the matrices of the system state-space model (including the one-step-ahead state matrix) and their values and variations are unknown, but they are assumed to belong to given intervals. This class of systems appears in many applications such as in economics and population dynamic studies, as for instance, the Leontief model of a multisector economy and the Leslie population model with uncertain parameters; see, e.g., Campbell (1980) and references therein. Based on the proposed admissibility results for DLTV descriptor systems, we derive methods of robust admissibility analysis based on parameter-dependent Lyapunov functions. Strict LMI conditions are proposed considering Lyapunov functions with either affine or quadratic dependence on the system uncertain parameters. The potentials of the proposed robust admissibility analysis methods are illustrated via numerical examples.

Notation. $\mathbb{Z}_i^+$ is the set of integers equal or larger than i, $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of $m\times n$ real matrices, I_n is the $n\times n$ identity matrix, 0_n and $0_{m\times n}$ are the $n\times n$ and $m\times n$ matrices of zeros, respectively, and diag$\{\cdots\}$ stands for a block-diagonal matrix. For a real matrix S, S^T denotes its transpose, He$\{S\}$ stands for $S+S^T$ and $S>0$ means that S is symmetric and positive-definite. The symbol $\star$ in a symmetric block matrix stands for the transpose of the blocks outside the main diagonal block and $\mathcal{V}_{\mathcal{B}}$ denotes the set of all the vertices of a polytope $\mathcal{B}$. The one-step time delayed $x(k)$ will be denoted by $x(k_-) := x(k-1)$.

2. DLTV descriptor systems

Consider the following DLTV descriptor system:

$$E(k)x(k+1) = A(k)x(k) + u(k), \quad x(0) = x_0, \tag{1}$$

for all $k \in \mathbb{Z}_0^+$, where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^n$ is the system input, $A(k), E(k) \in \mathbb{R}^{n\times n}$ are bounded time-varying matrices, and x_0 is a consistent initial condition in a sense to be defined later in this paper. In addition, the matrix $E(k)$ is allowed to be singular and satisfies

$$\text{rank}\{E(k)\} = r \le n, \quad \forall\, k \in \mathbb{Z}_0^+. \tag{2}$$

In the following, some definitions regarding the solution of time-varying descriptor systems are introduced.

Definition 1. Consider the DLTV descriptor system in (1), with x_0 being a consistent initial condition. Then:

(a) The system is said to be regular if for any x_0 and $u(k) \in \mathbb{R}^n$ there exists a solution $x(k)$ for all $k \in \mathbb{Z}_0^+$ and it is unique.

(b) The system is said to be causal if it is regular and the solution $x(k)$ for any x_0 and $u(k) \in \mathbb{R}^n$ is a function of x_0 and $u(0), \dots, u(k)$ for all $k \in \mathbb{Z}_0^+$.

(c) The system is said to be exponentially stable if it is regular and for any x_0 and $u(k) \equiv 0$, there exist real scalars $\alpha > 0$ and $\beta \in (0, 1)$ such that

$$\|x(k)\| \le \alpha\, \beta^k \, \|x_0\|, \quad \forall\, k \in \mathbb{Z}_1^+.$$

(d) The system is said to be admissible if it is causal and exponentially stable.

This paper is concerned in assessing the admissibility of the DLTV descriptor system in (1). To this end, two equivalent state-space representations will be introduced in the sequel, which will be instrumental to derive the main results of this paper. Firstly, performing a singular value decomposition (SVD) of $E(k)$, for any $k \in \mathbb{Z}_0^+$, followed by a normalization, we can obtain:

$$\bar{E} := M(k)E(k)N(k+1) = \text{diag}\{I_r, 0_{n-r}\}, \tag{3}$$

where $M(k)$ and $N(k)$ are bounded nonsingular $n\times n$ real matrices for all $k \in \mathbb{Z}_0^+$. Thus, defining the time-varying coordinate transformation

$$\begin{cases} \xi(k) = \begin{bmatrix} \xi_1^T(k) & \xi_2^T(k) \end{bmatrix}^T = N^{-1}(k)x(k), \\ \xi_1(k) \in \mathbb{R}^r, \quad \xi_2(k) \in \mathbb{R}^{n-r}, \end{cases} \tag{4}$$

and considering (3) yield the following equivalent state-space representation of system (1):

$$\begin{aligned} \bar{E}\xi(k+1) &= \bar{A}(k)\xi(k) + \bar{B}(k)u(k), \\ &\quad \xi(0) = \xi_0 \in \mathbb{R}^n, \ \forall\, k \in \mathbb{Z}_0^+, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \bar{A}(k) &:= M(k)A(k)N(k) = \begin{bmatrix} A_1(k) & A_2(k) \\ A_3(k) & A_4(k) \end{bmatrix}, \\ \bar{B}(k) &:= M(k) = \begin{bmatrix} B_1^T(k) & B_2^T(k) \end{bmatrix}^T, \end{aligned} \tag{6}$$

and the partitions of $\bar{A}(k)$ and $\bar{B}(k)$ are compatible with $\xi_1(k)$ and $\xi_2(k)$. Moreover, we denote $E(-1) := E(0)$, $M(-1) := M(0)$ and $N(0) := N(1)$. Note that due to the nonuniqueness of the SVD, the representation in (5) is nonunique and will hereafter be referred to as an SVD form of system (1).

Remark 1. Given a decomposition of the matrix $E(k)$ as in (3), it can be easily shown that any bounded nonsingular matrices $\widehat{M}(k)$ and $\widehat{N}(k)$ that decomposes $E(k)$ as $\widehat{M}(k)E(k)\widehat{N}(k+1) = \text{diag}\{I_r, 0_{n-r}\}$ can be expressed as $\widehat{M}(k) = T(k)M(k)$ and $\widehat{N}(k) = N(k)U(k)$, with

$$T(k) = \begin{bmatrix} T_1(k) & T_2(k) \\ 0 & T_3(k) \end{bmatrix}, \quad U(k) = \begin{bmatrix} T_1^{-1}(k) & 0 \\ U_2(k) & U_3(k) \end{bmatrix}, \tag{7}$$

where $T_1(k) \in \mathbb{R}^{r\times r}$, $T_3(k), U_3(k) \in \mathbb{R}^{(n-r)\times(n-r)}$, $T_2(k)$ and $U_2(k)$ are any bounded matrices with $T_1(k)$, $T_3(k)$ and $U_3(k)$ being nonsingular for all $k \in \mathbb{Z}_0^+$. □

Observe that taking the definitions of $\bar{E}$, $\bar{A}(k)$ and $\bar{B}(k)$ into account, the initial condition ξ_0 of (5) has to satisfy the following constraint:

$$\begin{bmatrix} A_3(0) & A_4(0) \end{bmatrix}\xi_0 + B_2(0)u(0) = 0. \tag{8}$$

In light of (8), the notion of a consistent initial condition for system (1) is characterized by the following result.