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Brief paper

Stability analysis of linear stochastic neutral-type time-delay systems with two delays☆

Zhao-Yan Li [a,*], James Lam [b], Yong Wang [a]

[a] Department of Mathematics, Harbin Institute of Technology, Harbin, 150001, China
[b] Department of Mechanical Engineering, University of Hong Kong, Hong Kong

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ABSTRACT

This paper is concerned with stability analysis of stochastic neutral-type time-delay systems with two different delays in both the state variables and the retarded derivatives of state variables. Sufficient conditions in terms of linear matrix inequalities (LMIs) are obtained by applying the Lyapunov–Krasovskii functional (LKF) based approach. A novel spectral radius based condition is imposed to guarantee the stability of the associated difference operator and a novel method is also proposed to analyze the stability of the overall system. Moreover, by carefully analyzing the relationship between these two different delays, a method is provided to construct all possible yet the minimal number of simple quadratic integral functions in the LKF, which helps to reduce both the conservatism and complexity of the resulting stability conditions. The effectiveness of the proposed approaches is illustrated by two numerical examples.

1. Introduction

Stochastic perturbations exist in many real-world systems. Dynamic systems with stochastic perturbations are referred to as stochastic systems, which play an important role in engineering fields. Therefore, increasing efforts have been devoted to study stochastic systems in recent years (see Che, Guan, & Wang, 2013, Chen, Xiang, & Mahmoud, 2013, Mao, 2007, Wu, Shi, Su, & Chu, 2011, Wu, Shi, Su, & Chu, 2013, Zhao & Deng, 2014 and references cited there). On the other hand, time-delay systems of neutral-type are encountered in a wide range of engineering problems such as heat exchangers, distributed networks, and population ecology (Kolmanovskii & Myshkis, 1992). The main feature of neutral-type time-delay systems is that they incorporate retarded derivatives of state variables, which makes the analysis and design of this class of systems very challenging. Therefore, the analysis and design of neutral-type delay systems has been a hot topic for a long time (see Hale, 1971, Han, 2009 and the references therein). Neutral-type time-delay systems with stochastic perturbation (referred to as stochastic neutral time-delay systems) are thus even challenging. During the past several decades, there have been many references explored plenty of problems for stochastic neutral-type time-delay systems, especially, in the linear case (see, for example, Li, Lin, & Zhou, 2015, Mao, 2007). For example, stability and stabilization of stochastic neutral-type time-delay systems with Markovian jumping parameters was studied in Chen, Xu, Zhang, and Qi (2016), and delayed feedback stabilization of stochastic neutral delay systems was investigated in Chen, Xu, and Zou (2016). For more related work, see Li et al. (2015), Zhao and Deng (2014) and the references cited there.

Let us consider a stochastic neutral-type time-delay system described by

$$\mathrm{d}\left(x(t)-\varphi_0\left(X\left(t\right)\right)\right)=f_0\left(x(t),X\left(t\right)\right)\mathrm{d}t+g_0\left(x(t),X(t)\right)\mathrm{d}\omega\left(t\right),$$

where φ_0, f_0, g_0 are some functions and $X(t)=(x(t-\tau_1),\ldots,x(t-\tau_2),x(t-\tau_m))$ with $\tau_i, i=1,2,\ldots,m$ being positive constants. This paper is concerned with the problem of mean square exponential stability analysis of this class of stochastic neutral-type time-delay systems when φ_0, f_0 and g_0 are linear functions of its arguments. In this case this system will be referred to as linear stochastic neutral-type time-delay system.

- If $m=1$, namely, there is only delay in the system, stability analysis of this type of systems has been extensively investigated in the literature (see, for example, Chen et al., 2013, Deng, Mao, & Wan, 2013, Li et al., 2015, Chen, Zheng, & Xue, 2010, Mahmoud, 2000, Mahmoud & Ismail, 2010, Song, Park, Wu, & Zhang, 2013, Xiang, Liu, & Mahmoud, 2013 and Xu, Lam, Shi, Boukas, & Zou, 2013).

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* Corresponding author.
E-mail addresses: lizhaoyan@hit.edu.cn (Z.-Y. Li), james.lam@hku.hk (J. Lam), mathwy@hit.edu.cn (Y. Wang).

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- If $m = 2$, φ_0 is only a function of τ_1, and f_0 and g_0 are only functions of τ_2, (this case of delay is referred to as mixed delay), stability analysis of this type of systems has also been widely studied in the literature, for example, Ding, He, Wu, and Ning (2015) and Zhou and Zhou (2010) (the corresponding deterministic case was studied in Han, 2009, He, Wu, She, & Liu, 2004, Lu, Wu, & Bai, 2014, Shu, Lam, & Xu, 2009 and Xu, Lam, Yang, & Verriest, 2003).
- If $m = 2$ and φ_0 is only a function of τ_1, namely, there is only one retarded derivative of state variable in the system, stability analysis of this type of systems was studied in Huang and Mao (2009).
- When m is a general integer and φ_0 is only a function of τ_1, namely, there is only one retarded derivative of state variable in the system, exponential stability analysis of the system was investigated in Chen, Hu, and Wang (2014).
- When m is a general integer and φ_0 contains more retarded derivatives of the state variable, to the best of our knowledge, stability analysis of this class of systems was not investigated in the literature. However, there are some results available in the literature for the corresponding deterministic systems (see, for instance, Chen, Jin, Hu, & Luo, 2008, Fridman, 2001, Fan, Lien, & Hsieh, 2002, Kharitonov, Collado, & Mondie, 2006, Park & Won, 1999, Zhang, Wu, & He, 2004).

In this paper, we are interested in the stability analysis of stochastic neutral-type time-delay system with two different delays, in both the state variables and the retarded derivatives of state variables. Sufficient conditions in terms of linear matrix inequalities (LMIs) are obtained by applying the Lyapunov–Krasovskii functional (LKF) based approach. The contribution of this paper is twofold. On the one hand, we have studied carefully the exponential stability of the associated $\mathcal{D}$ operator defined as $\mathcal{D}(x_t) = x(t) - \varphi_0(X(t))$, which is well known to be essential in the stability analysis of the overall system. In the literature, if there are more than one delay in the $\mathcal{D}$ operator, it is generally needed that the sum of the norms of the coefficient matrices are smaller than 1. In this paper, this requirement has been relaxed by a spectral radius condition, and a novel analysis is proposed to analysis of the stability of the overall system in terms of the state variable $x(t)$. On the other hand, by carefully analyzing the relationship between these two different delays, we show how to construct *all possible* simple quadratic integral functions in the LKF, which helps to reduce the conservatism of the resulting stability conditions. We also show how to obtain the *simplest* quadratic integral functions in the sense that none of them can be absorbed by the others, which helps to reduce the number of decision variables. The proposed method is also applied to deterministic neutral-type time-delays systems with two delays and a stability condition with less decision variables is derived. The effectiveness of the proposed approaches is illustrated by two examples borrowed from the literature.

The remaining part of this paper is organized as follows. Problem formulation and some preliminaries are given in Section 2. Mean square exponential stability of the $\mathcal{D}$ operator and the construction of the LKF are respectively studied in Sections 3 and 4. A sufficient condition for exponential stability of the considered system is then proposed in Section 5. Two examples are worked out in Section 6, and Section 7 concludes the paper.

Notation. The notation used in this paper is standard. For a matrix $A \in \mathbf{R}^{n\times m}$, we use A^{T}, $\|A\|$ and $\rho(A)$ to denote its transpose, 2-induced norm and spectral radius. We use $A \otimes B$ to denote the Kronecker product of matrices A and B, and $\mathbf{N}$ to denote the set of natural numbers. The symbol $|\cdot|$ refers to the Euclidean norm. For a matrix $B \in \mathbf{R}^{n\times m}$ with $\mathrm{rank}(B) = r < m$, where $\mathrm{rank}(B)$ is the rank of B, the right orthogonal complement of B is denoted by $B^{\perp} \in \mathbf{R}^{m\times(m-r)}$ which is of rank $m-r$. We use $0_{n\times m}$ to denote the zero matrix with dimensions $n \times m$. The symbol $\mathbf{E}\{\cdot\}$ denotes the expectation operator. For two integers p and q with $q \geq p$, we denote $\mathbf{I}[p, q] = \{p, p+1, \ldots, q\}$. For a positive scalar h, let $\mathbf{C}([-h, 0], \mathbf{R}^n)$ denote the family of continuous vector functions mapping the interval $[-h, 0]$ into $\mathbf{R}^n$ with norm $\|\varphi\| = \sup_{-h\leq\theta\leq0}\{|\varphi(\theta)|\}$. If $x(t)$ is a $\mathbf{R}^n$-valued stochastic process defined on $t \in [-h, \infty)$, let $x_t \in \mathbf{C}([-h, 0], \mathbf{R}^n)$ denote the restriction of $x(t)$ to the interval $[t-h, t]$ translated to $[-h, 0]$, that is, $x_t(\theta) = x(t+\theta), \theta \in [-h, 0]$, which is regarded as a $\mathbf{C}([-h, 0], \mathbf{R}^n)$-valued stochastic process.

2. Problem formulation

Consider the following linear stochastic neutral-type time-delay system

$$\mathrm{d}\varphi(t) = f(t)\,\mathrm{d}t + g(t)\,\mathrm{d}\omega(t), \tag{1}$$

where $\omega(t)$ represents the scalar Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with the filtration $\{\mathcal{F}_t\}_{t\geq0}$ satisfying the usual conditions, and

$$\begin{cases} \varphi(t) = \varphi(x_t) = x(t) - C_1x(t-\tau_1) - C_2x(t-\tau_2), \\ f(t) = f(x_t) = A_0x(t) + A_1x(t-\tau_1) + A_2x(t-\tau_2), \\ g(t) = g(x_t) = H_0x(t) + H_1x(t-\tau_1) + H_2x(t-\tau_2), \end{cases}$$

in which $C_i, A_j, H_j, i = 1, 2, j = 0, 1, 2$ are $n \times n$ constant matrices, and τ_1 and τ_2 are positive constants denoting time delays in the system. Hereafter, we assume without loss of generality that $\tau_2 > \tau_1$ (otherwise, we change the order of them). We let the initial value be $x_0 = x(\theta), \theta \in [-\tau_2, 0]$, which is an $\mathcal{H}_0$-measurable $\mathbf{C}([-\tau_2, 0], \mathbf{R}^n)$-valued random variable such that $\sup_{\theta\in[-\tau_2,0]}\mathbf{E}\{|x(\theta)|^2\} < \infty$. Denote the solution of system (1) with the initial data x_0 by $x(t), t \geq 0$.

Definition 1 (*Mao, 2007*)**.** The stochastic neutral-type time-delay system (1) is said to be mean square exponentially stable if there exist positive constants λ and k such that

$$\mathbf{E}\left\{|x(t)|^2\right\} \leq k \sup_{\theta\in[-\tau_2,0]} \mathbf{E}\{|x(\theta)|^2\}\mathrm{e}^{-\lambda t}, \ \forall t \geq 0.$$

In the present paper, we are interested in studying the mean square exponential stability of system (1) by using Lyapunov–Krasovskii functional (LKF) based approach. We will provide new methods for analyzing the stability of the associated $\mathcal{D}$ operator for system (1) and show how to construct a suitable LKF.

3. Exponential stability of the $\mathcal{D}$ operator

To study the mean square exponential stability of the stochastic system (1), we need to investigate the mean square exponential stability of the associated $\mathcal{D}$ operator defined as

$$\mathcal{D}(x_t) = C_1x(t-\tau_1) + C_2x(t-\tau_2), \tag{2}$$

or, the mean square exponential stability of the following stochastic difference equation

$$x(t) = C_1x(t-\tau_1) + C_2x(t-\tau_2) + \varphi(t). \tag{3}$$

When $\mathcal{D}(x_t)$ is a general function of x_t, it is generally required that (see Hale, 1971 and Chapter 6 in Mao, 2007) $|\mathcal{D}(x_t)| \leq \nu\|x_t\|$ is satisfied for some constant $\nu \in (0, 1)$. Applying this requirement on the particular operator (2) gives the following assumption.

Assumption 1. The matrices C_1 and C_2 are such that

$$\|C_1\| + \|C_2\| < 1. \tag{4}$$