Automatica 91 (2018) 225–232

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Global finite-time attitude consensus of leader-following spacecraft systems based on distributed observers☆

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ARTICLE INFO

Article history:
Received 24 September 2016
Received in revised form 25 July 2017
Accepted 11 December 2017

Keywords:
Attitude consensus
Distributed observer
Finite-time
Leader-following
Second-order sliding modes

ABSTRACT

This paper addresses the leader-following attitude consensus problem for a group of spacecraft when at least one follower can access the leader's attitude and velocity relative to the inertial space. A nonlinear distributed observer is designed to estimate the leader's states for each follower. The observer possesses one important and novel feature of keeping attitude and angular velocity estimation errors on second-order sliding modes, and thus provides finite-time convergent estimates for each follower. Further, quaternion-based hybrid homogeneous controllers recently developed for single spacecraft are extended and then applied, by establishing a separation principle with the proposed observer, to track the leader's attitude motion. As a result, global finite-time attitude consensus is achieved on the entire attitude manifold, with either full-state measurements or attitude-only measurements, as long as the network topology among the followers is undirected and connected. Numerical simulations are presented to demonstrate the performance of the proposed methods.

1. Introduction

Distributed attitude consensus of multiple cooperative spacecraft has drawn increasing attention due to its applications in formation flying, space-based interferometry, in-orbit assembly, etc. It can be classified as two types, namely, leaderless consensus that requires all spacecraft to reach an arbitrary yet probably *a priori* unknown synchronized state (Thunberg, Song, Montijano, Hong, & Hu, 2014), and leader-following consensus that requires each follower to track a prescribed group attitude trajectory provided by a real or virtual leader (see Abdessameud & Tayebi, 2009; Dimarogonas, Tsiotras, & Kyriakopoulos, 2009; VanDyke & Hall, 2006). This paper mainly focuses on the leader-following type.

The leader-following attitude consensus issue was first addressed by assuming that the leader's trajectory is available to all followers Abdessameud and Tayebi (2009), Dimarogonas et al. (2009), Mayhew, Sanfelice, Sheng, Arcak, and Teel (2012), Ren (2010) and VanDyke and Hall (2006). In practice, a more common yet challenging case is that only a portion of the followers can access the state of the leader. To deal with this problem, first-order sliding mode estimators were derived in Meng, Ren, and You (2010), Zou (2014) and Zou, de Ruiter, and Kumar (2016) to estimate the reference attitude and/or velocity in finite time. These designs can be traced back to the work of Cao, Ren, and Meng (2010) for single/double-integrator systems. Asymptotic distributed estimators were also proposed when the reference angular velocity is linearly parameterized (Bai, Arcak, & Wen, 2008) or generated by a known, stable, linear system (Cai & Huang, 2014, 2016). The methods of Du, Li, and Qian (2011) and Ren (2007, 2010) involve no estimators but require the spacecraft to transmit their accelerations apart from their attitudes and velocities. Among the above methods, those of Du et al. (2011) and Zou et al. (2016), further guarantee finite-time stability, which implies that the consensus behavior can be achieved in finite time instead of infinite time as for asymptotic or exponential stability. In addition, angular velocity measurements are not needed for the consensus algorithms of Abdessameud and Tayebi (2009), Cai and Huang (2016), Lawton and Beard (2002), Ren (2010), Zou (2014) and Zou et al. (2016).

Another important issue is the complex nonlinearity intrinsic in attitude control. More precisely, the attitude configuration, the set of 3×3 rotation matrices SO(3), is a nonlinear manifold not diffeomorphic to any Euclidean space and precludes the existence of continuous, globally stabilizing, state-feedback laws on SO(3) (Bhat & Bernstein, 2000; Chaturvedi, Sanyal, & McClamroch, 2011).

☆ This work was supported in part by the National Natural Science Foundation of China under Project 11702010 and Project 11432001, and in part by the Natural Sciences and Engineering Research Council of Canada through a Discovery Accelerator Supplement. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Daniel Liberzon.

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https://doi.org/10.1016/j.automatica.2018.01.037
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In addition, the attitude kinematics and dynamics are both nonlinear. Due to these features, the attitude consensus laws extended from algorithms for linear systems ensure merely local or at most almost global stability (Igarashi, Hatanaka, Fujita, & Spong, 2009; Thunberg et al., 2014) while the methods of Zou (2014) and Zou et al. (2016) result in semi-global stability. In addition, some quaternion-based control schemes can cause the undesirable unwinding phenomenon due to neglecting that the unit-quaternion space is a double-covering of SO(3) (Bai et al., 2008; Cai & Huang, 2014, 2016; Ren, 2007). To overcome this problem, Mayhew et al. (2012) developed a hybrid feedback scheme with network-based hysteretic switching logics, resulting in global attitude consensus and simultaneous robustness to measurement noise. This method, however, relies on the availability of the leader's state to all followers and, similarly to Abdessameud and Tayebi (2009) and Bai et al. (2008), does not allow cycles in the communication graph. Otherwise, undesirable equilibria other than the consensus state can arise and fail the control objective.

This paper investigates the global attitude consensus of a leader-following spacecraft network in terms of the quaternion parameterization. The communication graph between followers is assumed to be an undirected connected graph and only a subset of the followers has access to the dynamic leader. In order to estimate the leader's states for each follower, a novel nonlinear distributed observer is designed such that finite-time convergence is guaranteed only if at least one follower connects to the leader. Following this, the hybrid homogeneous attitude controllers developed in Gui and Vukovich (2016) are extended and then applied together with the distributed observer to perform consensus control by establishing a separation principle. More precisely, the resultant consensus laws can restore the uniformly globally finite-time stable systems of Gui and Vukovich (2016), in both the full-state measurement case and attitude-only measurement case, where the latter relies on a quaternion filter to inject the necessary damping instead of velocity feedback. As a result, the proposed control schemes avoid the unwinding problem and achieve global finite-time attitude consensus which, to the best of the our knowledge, has not been reported in existing cooperative attitude control literature. As another contribution, the proposed observer requires only the boundedness of the leader's angular velocity and its derivatives for finite-time convergence and hence possesses better robustness and allows more generic reference trajectories than the distributed observers in Cai and Huang (2014, 2016) which are limited to stationary or periodic reference trajectories. In addition, it keeps the attitude and angular velocity estimation errors on second-order sliding modes, indicating higher accuracy during digital implementation than the distributed estimator derived in Cao et al. (2010) and its variants in Meng et al. (2010), Zou (2014) and Zou et al. (2016) that all attain first-order sliding modes.

2. Preliminaries

Throughout this paper, denote by I_n the $n \times n$ identity matrix, $\mathbf{1}_n = [1, \ldots, 1]^T$, and $\mathbb{I}_n = \{1, \ldots, n\}$. For all $x \in \mathbb{R}$ and $\alpha \geq 0$, let $\text{sgn}^\alpha(x) = \text{sgn}(x)|x|^\alpha$ and $\text{sat}_\alpha(x) = \text{sgn}(x)\min\{|x|^\alpha, 1\}$, where $\text{sgn}(\cdot)$ is the standard sign function. Clearly, $\text{sgn}^\alpha(x)$ is a continuous nonsmooth function if $0 < \alpha < 1$, while $\text{sat}_\alpha(x)$ becomes the standard saturation function $\text{sat}(x)$ if $\alpha = 1$. For all $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ and $\alpha \geq 0$, let $\text{sgn}^\alpha(x) = [\text{sgn}^\alpha(x_1), \ldots, \text{sgn}^\alpha(x_n)]$ and $\text{sat}_\alpha(x) = [\text{sat}_\alpha(x_1), \ldots, \text{sat}_\alpha(x_n)]$. Denote by $\|\cdot\|_p$ the p-norm of a vector, respectively, for $p = 1, 2, \infty$. For all $A \in \mathbb{R}^{m\times n}$, let $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$ be its maximum and minimum singular values, respectively. Note that $\bar{\sigma}(A)$ equals to its induced 2-norm $\|A\|_2 = \max_{\{x\in\mathbb{R}^n:\|x\|_2=1\}}\|Ax\|_2$. Given $x \in \mathbb{R}^3$, $x^\times$ is the skew-symmetric matrix satisfying $x^\times y = x \times y, \forall y \in \mathbb{R}^3$, where $\times$ is the cross product on $\mathbb{R}^3$.

A quaternion $Q = [\eta, q^T]^T \in \mathbb{R}^4$ consists of a scalar part $\eta \in \mathbb{R}$ and a vector part $q \in \mathbb{R}^3$. Let $\text{vec}(Q)$ give the vector part of Q, i.e., $\text{vec}(Q) = q$. The quaternion multiplication is defined as

$$Q \circ Q' = \begin{bmatrix} \eta\eta' - q^T q' \\ \eta q' + \eta' q + q \times q' \end{bmatrix}, Q' = [\eta', q'^T]^T \in \mathbb{R}^4$$

which is associative and distributive but is not commutative. In addition, the conjugation of Q is given by $Q^* = [\eta, -q^T]^T \in \mathbb{R}^4$. Note that $(Q \circ Q')^* = (Q')^* \circ Q^*$. A 3-D vector is treated as a quaternion with zero scalar part when operating with a quaternion. With the identity element $\mathbf{1} = [1, 0, 0, 0]^T$, the set of unit quaternions is defined as $\mathbb{S}^3 = \{Q \in \mathbb{R}^4 : Q \circ Q^* = \mathbf{1}\}$.

2.1. System equations

Consider a system of n rigid spacecraft (agents). Denote by $Q_i = [\eta_i, q_i^T]^T \in \mathbb{S}^3, \forall i \in \mathbb{I}_n$, the attitude quaternion of the body-fixed frame of the ith agent, $\mathcal{F}_i$, relative to the inertial frame $\mathcal{F}_I$. The equations of motion of the ith agent are

$$\dot{Q}_i = \frac{1}{2}Q_i \circ \omega_i = \frac{1}{2}E(Q_i)\omega_i,\ \ E(Q_i) = \begin{bmatrix} -q_i^T \\ q_i^\times + \eta_i I_3 \end{bmatrix} \tag{1}$$

$$J_i\dot{\omega}_i = -\omega_i \times J_i\omega_i + u_i, \tag{2}$$

where $\omega_i \in \mathbb{R}^3$ and $J_i = J_i^T$ are the angular velocity and inertia tensor of agent i expressed in $\mathcal{F}_i$. u_i is the corresponding control torque. The rotation matrix from $\mathcal{F}_I$ to $\mathcal{F}_i$ can be computed from Q_i by

$$R(Q_i) = (\eta_i^2 - q_i^T q_i)I_3 - 2\eta_i q_i^\times + 2q_i q_i^T. \tag{3}$$

Assume that the desired trajectory is generated by a leader spacecraft with body-fixed frame $\mathcal{F}_0$. Denote by $Q_0 \in \mathbb{S}^3$ and $\omega_0 \in \mathbb{R}^3$ the attitude quaternion and angular velocity of $\mathcal{F}_0$ relative to $\mathcal{F}_I$. In addition, (Q_0, ω_0) obeys the same kinematics as (1). The attitude and angular velocity error of the ith follower relative to the leader is defined as $Q_{i0} = Q_0^* \circ Q_i$ and $\omega_{i0} = \omega_i - R(Q_{i0})\omega_0$. Letting $\bar{\omega}_{i0} = R(Q_{i0})\omega_0$, the system equations in terms of Q_{i0} and ω_{i0} are then written as

$$\dot{Q}_{i0} = \frac{1}{2}Q_{i0} \circ \omega_{i0} = \frac{1}{2}E(Q_{i0})\omega_{i0}, \tag{4}$$

$$J_i\dot{\omega}_{i0} = \Xi(\omega_{i0}, \bar{\omega}_{i0})\omega_{i0} - u_{fi} + u_i, \tag{5}$$

where $\Xi(\omega_{i0}, \bar{\omega}_{i0}) = (J_i\omega_{i0} + J_i\bar{\omega}_{i0})^\times - \bar{\omega}_{i0}^\times J_i - J_i\bar{\omega}_{i0}^\times$ is skew-symmetric and

$$u_{fi} = J_i R(Q_{i0})\dot{\omega}_0 + \bar{\omega}_{i0}^\times J_i\bar{\omega}_{i0}, \tag{6}$$

represents the torque to be compensated for perfect tracking of the desired trajectory. When every follower has access to the leader's trajectory, attitude consensus can be achieved by applying the controllers of Gui and Vukovich (2016) and Mayhew, Sanfelice, and Teel (2011) to globally stabilize $(Q_{i0}, \omega_{i0}) = (\pm\mathbf{1}, 0)$, $i \in \mathbb{I}_n$. These methods, however, cannot be applied if the leader's trajectory is available to only one or some of the followers.

As many studies on coordinated attitude control of formation flying spacecraft, the above dynamics models assume that all spacecraft share the same inertial frame $\mathcal{F}_I$. This is true in practice because the inertial frame is usually set as the Earth-centered inertial frame for Earth spacecraft systems, and the heliocentric inertial frame for deep-space spacecraft systems.

2.2. Communication graph

The information flow for n followers is assumed to be bidirectional and can be described by a weighted undirected graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, n\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the