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Brief paper

Stability of discrete-time positive switched linear systems with stable and marginally stable subsystems ☆

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ABSTRACT

In this paper we investigate the stability of discrete-time positive switched linear systems. The highlight of this work is that each subsystem is only assumed to be stable or marginally stable. Some sufficient conditions, imposed on the subsystem matrices and switching signals, are established to guarantee exponential stability of the considered positive switched linear systems. An extension to the case of time-varying and unbounded delay is also given. Compared with the continuous-time counterparts in the literature, our discrete-time case is much more complicated, involving the transitive connectivity between the subsystems specified by the switching signals.

1. Introduction

Positive linear systems (Farina & Rinaldi, 2011; Luenberger, 1979) are a class of linear time-invariant (LTI) systems whose state variables are confined to the positive orthant. Such systems find applications in modeling economic systems, biomedical systems, dynamic networks, chemical processes, etc. Motivated by the applications, a growing number of researchers have studied various aspects of positive systems. As one of the most fundamental system properties, stability of positive systems has been extensively investigated, see e.g., Ait Rami (2011), De Leenheer and Aeyels (2001) and Farina and Rinaldi (2011). It is worth noting that the existence of some copositive Lyapunov functions is sufficient to ensure the stability, due to the positivity of the systems. In addition, the stability of positive systems with constant delay is independent of the delay (Haddad & Chellaboina, 2004).

Recently, researchers showed increasing interest in the stability analysis of positive switched linear systems, which consist of a finite family of positive linear subsystems and a switching signal specifying the active subsystem at each time instant. Because of their rich structure, positive switched linear systems find wide applications in various areas, such as congestion control, formation flying, networks employing TCP, to name a few. Since positive linear systems have many interesting and elegant stability properties, one naturally expects that positive switched linear systems can inherit those properties. In fact, the stability conditions for positive switched linear systems with all stable subsystems under arbitrary switching can be characterized in terms of the existence of some common or multiple copositive Lyapunov functions, see, e.g., Blanchini, Colaneri, and Valcher (2015), Fornasini and Valcher (2010, 2012), Gurvits, Shorten, and Mason (2007) and Liu (2009). Moreover, such positive switched systems are always stable when the dwell time (Blanchini et al., 2015) or average dwell time (Zhao, Zhang, Shi, & Liu, 2012) of the switching signal is sufficiently large. In addition, their stability is also insensitive to a class of time-varying delays (Liu & Dang, 2011; Liu, Yu, & Wang, 2010). However, in some applications, such as, problems of consensus and congestion control (Jadbabaie, Lin, & Morse, 2003; Shorten, Leith, Foy, & Kilduff, 2003), one may encounter positive switched systems with some marginally stable subsystems. Such switched systems fail to preserve stability under arbitrary switching but may be stable under some restricted switching signals. Motivated by this, in Valcher and Zorzan (2016), the stability of a continuous-time compartmental switched system was studied. When some or all subsystems are only marginally stable, switching signals with special persistence and dwell-time properties can ensure the asymptotic stability. Further the authors of Meng, Xia, Johansson, and Hirche (2017) studied a continuous-time positive switched linear system with all marginally stable subsystems. A stability condition, called weak excitation condition, was proposed to ensure exponential stability of the system with or without time delay. It is worth pointing out that for a general switched linear system

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(without positivity constraint) with some common weak quadratic Lyapunov function, the ergodicity assumption for switching signals (Wang, Cheng, & Hu, 2009), which is analogous to the weak excitation condition, is not sufficient for the stability.

We note that in a number of applications, including Jadbabaie et al. (2003) and Olshevsky and Tsitsiklis (2011), discrete-time positive switched systems are encountered. Hence it is of interest to address the stability problem of discrete-time positive switched linear systems whose subsystems can be marginally stable. A key question one may ask here is whether a discrete version of the weak excitation condition in Meng et al. (2017) is still enough for the stability of the discrete-time case. It is worth noting that the extension of stability conditions for positive switched systems from the continuous-time case to the discrete-time case in some cases is not straightforward. For example, it is pointed out in Fornasini and Valcher (2012) that the investigation of quadratic stability for the discrete-time case is much harder than the one for the continuous-time case.

In this paper, we focus on the stability of discrete-time positive switched linear systems whose subsystems could be stable or marginally stable. Our goal is to answer the question mentioned above, namely, whether the considered discrete-time systems can be stabilized by the switching signals satisfying the weak excitation condition. It is found that the answer to this question depends on the zero/nonzero structure of the diagonal elements of the subsystem matrices. More precisely, it is shown that when all the diagonal elements of the subsystem matrices are positive, a discrete version of the weak excitation condition is sufficient to ensure exponential stability of the considered discrete-time switched systems, while if there are zeros in some diagonal positions, the sufficiency is not true. We provide a counterexample to show the insufficiency. To obtain the exponential stability in this "bad" case, we introduce an additional condition based on the transitive connectivity between the subsystems specified by switching signals satisfying the weak excitation condition. We observe that the difficulty encountered in analyzing discrete-time systems is due to the existence of zero elements in the diagonal positions of the subsystem matrices. However, this cannot occur for the discretized systems of continuous-time positive switched systems considered in Meng et al. (2017) and Valcher and Zorzan (2016) by noting that all the diagonal elements of the exponential of a Metzler matrix are always positive. This is the main reason for the distinction between the discrete-time and continuous-time cases. In addition, an extension to the case of time-varying and unbounded delay is also discussed. A modification of the weak excitation condition, which is independent of system delay, is obtained to ensure the stability of the delayed switched positive systems whose diagonal elements of the subsystem matrices are all positive.

The remainder of this paper is organized as follows. In Section 2, we present the problem formulation. The main results are provided in Section 3. A conclusion follows in Section 4.

Notation: $\mathbb{N}_0$ stands for the set of non-negative integers and $\mathcal{N} \triangleq \{1, \ldots, n\}$. A real matrix $A \in \mathbb{R}^{n_1 \times n_2}$ with all of its entries being nonnegative (resp., positive) is called nonnegative (resp., positive) and is denoted by $A \geq 0$ (resp., $A > 0$). The same definition and notation are adopted for nonnegative and positive vectors.

2. Problem formulation

In this paper, we consider the stability of discrete-time positive switched linear systems with and without time delay.

A discrete-time delay-free switched linear system is described by

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k),\ k \in \mathbb{N}_0, \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, and $\sigma : \mathbb{N}_0 \to \mathcal{P} \triangleq \{1, \ldots, m\}$ is a switching signal which determines the active subsystem at each time instant, i.e., subsystem A_i is activated at time k when $\sigma(k) = i$.

We also consider a delayed switched linear system in the form of

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}x(k-\tau(k)),\ k \in \mathbb{N}_0, \\ x(k) &= \phi(k),\ k = -\tau_0, \ldots, 0, \end{aligned} \tag{2}$$

where $\phi : \{-\tau_0, \ldots, 0\} \to \mathbb{R}^n$ is the initial function and τ_0 is decided by the delay function $\tau(\cdot) : \mathbb{N}_0 \to \mathbb{N}_0$, which satisfies the following assumption.

Assumption 1. The delay function $\tau(\cdot)$ is such that $\lim_{k\to\infty}(k - \tau(k)) = +\infty$.

Assumption 1 imposes a certain but mild constraint on the delay. It is easy to see that all bounded constant (or time-varying) delays satisfy Assumption 1. Some unbounded time-varying delays are also included, e.g., $\tau(k) = \frac{k}{2}$. Under Assumption 1,

$$\tau_0 = -\inf_{0 \leq k \leq \check{k}} \{k - \tau(k)\},$$

where $\check{k} = \sup\{k \geq 0 : k - \tau(k) < 0\}$.

System (1) (or delayed system (2)) is said to be positive if the solution $x(k)$ always stays in the positive orthant whenever the initial state x_0 is given in the positive orthant (or the initial function ϕ maps any $k \in \{-\tau_0, \ldots, 0\}$ to the positive orthant). It is well-known that system (1) is positive if and only if A_p is nonnegative for all $p \in \mathcal{P}$, while system (2) is positive if and only if A_p and B_p are nonnegative for all $p \in \mathcal{P}$.

We are concerned with the stability of discrete-time positive switched linear systems (1) and (2) which allow each subsystem being stable or marginally stable. Our standing assumptions on the subsystem matrices of systems (1) and (2) are given, respectively, as follows.

Assumption 2. For system (1), there exists a vector $v > 0$ such that $A_p v \leq v$ for all $p \in \mathcal{P}$.

Assumption 3. For system (2), there exists a vector $v > 0$ such that $(A_p + B_p)v \leq v$ for all $p \in \mathcal{P}$.

Remark 1. An extreme situation of Assumption 2 is that there exists a vector $v > 0$ such that $A_p v < v$ for all $p \in \mathcal{P}$, which is equivalent to the existence of a common copositive Lyapunov function ensuring the stability of system (1) under arbitrary switching (Blanchini et al., 2015). Generally, Assumption 2 only implies the existence of a common "weak" copositive Lyapunov function, which fails to ensure stability under arbitrary switching. A similar observation can be made from Assumption 3.

Indicated by Remark 1, to achieve the stability of systems (1) and (2) satisfying Assumptions 2 and 3, respectively, we focus on investigating the stability conditions imposed on the switching signals.

3. Results

3.1. Stability without delay

In this subsection, we establish the stability conditions for system (1). Let $v > 0$ be the vector satisfying Assumption 2. Our standing assumption on the switching signal is given as follows.