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Brief paper

Yield trajectory tracking for hyperbolic age-structured population systems☆

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ABSTRACT

For population systems modeled by age-structured hyperbolic partial differential equations (PDEs) that are bilinear in the input and evolve with a positive-valued infinite-dimensional state, global stabilization of constant yield set points was achieved in prior work. Seasonal demands in biotechnological production processes give rise to time-varying yield references. For the proposed control objective aiming at a global attractivity of desired yield trajectories, multiple non-standard features have to be considered: a non-local boundary condition, a PDE state restricted to the positive orthant of the function space and arbitrary restrictive but physically meaningful input constraints. Moreover, we provide Control Lyapunov Functionals ensuring an exponentially fast attraction of adequate reference trajectories. To achieve this goal, we make use of the relation between first-order hyperbolic PDEs and integral delay equations leading to a decoupling of the input-dependent dynamics and the infinite-dimensional internal one. Furthermore, the dynamic control structure does not necessitate exact knowledge of the model parameters or online measurements of the age-profile. With a Galerkin-based numerical simulation scheme using the key ideas of the Karhunen–Loève-decomposition, we demonstrate the controller's performance.

1. Introduction

We design an asymptotically tracking control for age-structured chemostats modeled by hyperbolic partial differential equations (PDEs) with a bilinearly acting input. Based on our prior work on the stabilization of constant yield set points, we guarantee global attractivity of output trajectories with an exponential convergence rate. In addition, we developed an efficient numerical scheme based on Galerkin-methods, which ensures accurate asymptotic properties.

Motivation. In the context of mathematical biology and demography age-structured continuous-time models are a common way of describing the evolution of a certain population with respect to the independent variables of age and time (Boucekkine, Hritonenkoand, & Yatsenko, 2013; Brauer & Castillo-Chavez, 2001). Continuous bioreactors encountered in bioengineering and pharmaceutical research are usually modeled by ordinary differential equations (ODEs) (Robledo, Grognard, & Gouzé, 2012; Smith & Waltman, 1995). Since multiple ecological concepts like resistance and resilience of ecosystems are closely related to the framework of robustness in system theory, these aspects have been studied rigorously (Ellermeyer, Pilyugin, & Redheffer, 2001; Karafyllis, Kravaris, & Kalogerakis, 2009). An analysis of the ergodicity problem is given in Inaba (1988a, b). Moreover, for age-structured models there is an extensive literature on optimal control problems (Boucekkine et al., 2013; Feichtinger, Tragler, & Veliov, 2003) as well as on the stability of certain PDE models (Robledo et al., 2012).

The model. Throughout this paper we focus on the McKendrick–vonFoerster PDE, which is introduced in Section 2. For this setting the dilution rate, which is the ratio of the volumetric flow to the constant volume in the growth chamber, is a natural control variable (Smith & Waltman, 1995; Toth & Kot, 2006). On the other hand, the output is chosen as a weighted integral of the population's age-distribution. As a result, it is possible to represent all products which are proportional to the overall population as well as possibly age-dependent synthesized products. Furthermore, the dependence of the microorganisms' growth rate on the nutrient

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concentration in the bioreactor is not captured in the model and we hence assume that the nutrient concentration is maintained constant.

Time-varying yields. Having a biomass in mind which is used for the production of antibiotics in pharmaceutical industry, the relevance of the trajectory tracking issue can be elucidated in a demonstrative way. Choosing the yield of an antibiotic as a valid output, its production rate is subjected to external influences like seasonal effects or the current demand. With the prediction of these exogenous factors on an adequate time scale it is possible to determine an optimal production rate governing desired yield trajectories. Due to the fact that a periodic reactor operation may produce a higher yield than the yield achieved by an equilibrium point (Bittanti, Fronza, & Guardabassi, 1973), a special focus is placed on periodical reference trajectories. Beyond this, if the considered bioreactor is a part of a cascaded process, the tracking of predefined trajectories makes it possible to accelerate starting processes and changes of operating points.

Results of the paper. The present paper extends our prior work (Karafyllis & Krstic, 2016), which focused on the global stabilization of desired equilibrium points of the system class under consideration. We now aim at ensuring the global attractivity of desired yield trajectories and therefore generalize the already established concepts, such that constant set point are included as a special case. For this purpose, a definition of the control objective is given in Section 3. The suggested approach exploits the relation of first-order hyperbolic PDEs to delay models in the sense of integral delay equations (IDEs, see Karafyllis & Krstic, 2014). More specifically, we decompose the PDE problem to an input-dependent finite-dimensional subsystem and an autonomous delay subsystem which is correlated to the microorganisms' reproduction. For special cases of integral kernels we are in the position to construct Control Lyapunov Functionals (CLFs).

In contrast to the prior work, we use a two-degrees-of-freedom (DOF) control structure with a separate feedforward control part evoked by the reference trajectory (Meurer & Kugi, 2009), as introduced in Section 4. In this case the feedforward controller does not solely enhance the tracking behavior of the closed loop, but plays an essential role in the overall attractivity concept. The consideration of input constraints is a crucial issue of the present control design assuming a bounded interval for the accessible dilution rate. Moreover, our output-feedback controller does not demand online measurements of the population's entire age-distribution. Even the knowledge of exact system parameters is not necessary, since the controller handles uncertainties in a robust way. In addition, it is important to guarantee that the PDE state, which represents the population density, remains positive at all times and ages. This fact, in conjunction with a control input directly acting on the whole profile (not simply on the boundary), differentiates our work to other control problems of hyperbolic PDEs (Bastin & Coron, 2011; Coron, Vazquez, Krstic, & Bastin, 2013).

Lastly, we provide simulation results of the closed-loop system in Section 6 with a Galerkin-based simulation scheme, which conserves important system properties even at low orders and enables independent age and time discretizations.

Notation.

- The set $\mathbb{R}^+$ denotes all positive-valued real numbers, $\mathbb{R}_0^+$ all non-negative real numbers.
- The inner product of L^2 is denoted $\langle f, g\rangle := \int_0^A f(a)g(a)\mathrm{d}a$ where $f, g \in L^2([0, A])$.
- $\|f(a)\|_\infty = \max_{a\in[0,A]}|f(a)|$ is the maximum- resp. L^∞-norm for $f \in \mathscr{C}^0([0, A])$.
- Given the functions $f : \mathbb{R}^+ \times X \to \mathbb{R}$, $z : \mathbb{R}^+ \to X$ with the metric space X, we define the right temporal Dini-derivative $\dot{f}^+(t, z(t)) := \overline{\lim}_{h\to 0^+} \frac{f(t+h,z(t+h))-f(t,z(t))}{h}$.
- $\mathscr{K}_\infty$ is the class of all strictly increasing, unbounded functions $\kappa \in \mathscr{C}^0(\mathbb{R}_0^+; \mathbb{R}_0^+)$ with $\kappa(0) = 0$.
- The saturation function with respect to $f \in [D_{\min}, D_{\max}]$ is defined $\mathrm{sat}(f) = \min(D_{\max}, \max(D_{\min}, f))$; other intervals are explicitly denoted as an index.
- For any $S \subseteq \mathbb{R}$ and $A > 0$, $P\mathscr{C}^1([0, A]; S)$ denotes the class of all functions $f(a) \in \mathscr{C}^0([0, A]; S)$ for which there exists a finite (or empty) set $B \subseteq (0, A)$ such that: (i) the derivative $f'(a)$ exists at every $a \in (0, A)\setminus B$ and is a continuous function on $(0, A) \setminus B$, (ii) all meaningful right and left limits of $f'(a)$ when a tends to a point in $B \cup \{0, A\}$ exist and are finite.

2. Age-structured population models

Consider the McKendrick–vonFoerster PDE (1) valid in the age-time domain $(a, t) \in (0, A) \times (0, \infty)$

$$\frac{\partial x(a,t)}{\partial t} + \frac{\partial x(a,t)}{\partial a} = -[\mu(a) + D(t)]x(a,t) \tag{1}$$

$$x(0,t) = \int_0^A k(a)x(a,t)\mathrm{d}a \tag{2}$$

$$x(a,0) = x_0(a) \tag{3}$$

which describes the evolution of the population density $x : [0, A]\times[0, \infty) \to \mathbb{R}^+$ as a part of an initial–boundary value problem (IBVP) on the same domain with an arbitrary large but finite maximum reproductive age $A > 0$. Strictly speaking, the state $(x[t])(a) = x(a, t)$ describes the density of the overall population which has reached a specific age a at a certain time t. In addition, the function $\mu(a)$ denotes the age-dependent mortality rate and $D(t)$ the dilution rate which is the control input. In particular, the non-local boundary condition (BC) (2) is valid for $t \geq 0$ and models the production of new-born individuals $x(0, t)$ determined by the birth modulus resp. the kernel $k(a)$. Furthermore, Eq. (3) is the initial condition, i.e. the initial distribution of the population density in the age-domain $[0, A]$ at $t = 0$. In addition, the output is defined by the equation

$$y(t) = \int_0^A p(a)x(a,t)\mathrm{d}a, \tag{4}$$

which takes the possibly age-dependent production rate $y(t)$ of a specific (bio)chemical species into account. For instance, we have the overall population with $p(a) = 1$.

The distributed parameter system Σ_x: (1)–(4) with input $D(t)$ and output $y(t)$ is of bilinear single-input–single-output type. Subsequently, we introduce three assumptions to guarantee the existence of a meaningful unique solution of (1)–(2) aware of the state and input constraints (see also Karafyllis & Krstic, 2016):

(A_1) The parameters functions are restricted to $k, p \in \mathscr{P}$ and $\mu \in \mathscr{C}^0([0, A]; \mathbb{R}_0^+)$, where $\mathscr{P} := \{f \in P\mathscr{C}^1([0, A]; \mathbb{R}_0^+) \big| \langle 1, f\rangle > 0\}$.

(A_2) The control $D(t)$ takes values in $[D_{\min}, D_{\max}] \subset \mathbb{R}_0^+$, where $D_{\min} < D_{\max}$.

(A_3) The initial condition (IC) (3) is compatible with (2), i.e. $x_0 \in \mathscr{X} := \{f \in P\mathscr{C}^1([0, A]; \mathbb{R}^+) \big| f(0) = \langle k, f\rangle > 0\}$.

3. Control objective and PDE decomposition

The asymptotic tracking of a reference trajectory $y_{\text{ref}}(t)$ with respect to the output $y(t)$ given by (4) defines the key objective of the contribution. For designing an asymptotic tracking control