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Brief paper

Adaptive backstepping-based tracking control of a class of uncertain switched nonlinear systems ☆

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ABSTRACT

In this paper, we propose two adaptive control schemes to solve the output tracking control problem of a class of uncertain switched nonlinear systems. The first is a robust scheme constructed based on common Lyapunov function method, aiming to achieve the insensitivity of closed-loop system to parameter switches by estimating the bound on switching parameters. It is analyzed that the system stability under arbitrary switching is established with such a scheme, but the control input required may be relatively large since the estimate of the bound on switching parameters is utilized. To avoid this, we further propose a direct adaptive control scheme by estimating the switching parameters directly, and derive a dwell-time condition for parameter switches based on an extended multiple Lyapunov functions method newly developed, such that the global boundedness of all the closed-loop signals and the convergence of tracking error to a residual around zero are ensured. The effectiveness of the proposed control schemes is illustrated through simulation studies.

1. Introduction

In the past few decades, backstepping technique has been extensively used in adaptive control design of uncertain nonlinear systems, see typically in Chen, Liu, Liu, and Lin (2009a, b), Krstic, Kanellakopoulos, and Kokotovic (1995), Wen and Zhou (2007), Wen, Zhou, Liu, and Su (2011), Xin, Wen, Su, Cai, and Wai (2015), Xin, Wen, Zhu, Su, and Liu (2016), Zhou and Wen (2007) and Zhou, Wen, and Yang (2014). Note that these schemes mainly target to nonswitched control systems which indeed can be treated as a certain mode of switched systems studied as in Liberzon (2003), Ma and Zhao (2010), Zhao, Yin et al. (2015), Vu and Liberzon (2005), Wu (2009), Zhao, Zheng, Niu, and Liu (2015) and Zhao, Shi, Zheng, and Zhang (2015). Besides, a large number of physical control systems are able to be modeled as switched systems in practice, e.g., the networked control systems (Zhao, Hill, & Liu, 2009), the near-space vehicle control systems (Bao, Li, Chang, Niu, & Yu, 2010), etc. Thus, it is of both theoretical and practical importance to investigate the tracking control problem of switched nonlinear systems.

As analyzed in Liberzon (2003) and Vu and Liberzon (2005), a switched system under arbitrary switching is stable asymptotically, provided that a common Lyapunov function exists for all subsystems. With the stability theory, several control schemes have been developed in Chiang and Fu (2014), Jiang, Shen, and Shi (2015), Lai, Liu, Zhang, and Philip Chen (2016), Ma and Zhao (2010), Wu (2009), Zhao, Shi et al. (2015) and Zhao, Zheng et al. (2015), for switched nonlinear systems. In Wu (2009), by assuming the existence of common virtual controllers for all subsystems, a backstepping control scheme was proposed to stabilize a class of switched nonlinear systems in strict-feedback form. In Ma and Zhao (2010), some certain inequality constraints were imposed to virtual controllers to remove the assumption in Wu (2009). Note that the system nonlinear functions considered in both (Ma & Zhao, 2010; Wu, 2009) are required to be available for feedback control design. Such a requirement was eliminated in Zhao, Zheng et al. (2015) with the aid of fuzzy logic systems, and moreover the developed approach in Zhao, Zheng et al. (2015), which mainly is for constructing common virtual controllers in the presence of unknown system functions, was successfully extended to stochastic switched systems in Zhao, Shi et al. (2015). The stability results obtained in Zhao, Shi et al. (2015) and Zhao, Zheng et al. (2015) are all semiglobal since the fuzzy approximation property is only valid on a compact set. In Chiang and Fu (2014), the global stabilization schemes for uncertain switched nonlinear system were further proposed based on adaptive backstepping control design. It is worthy to point out that the control schemes mentioned above are

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all based on the well-known common Lyapunov function method. However, such a method usually involves in the estimate of the bound on switching parameters, and thus may cause large control input. Besides, a common Lyapunov function for subsystems is difficult to be constructed in some cases such as those in Long and Zhao (2015b) and Tong, Sui, and Li (2015). Due to the restrictions, a multiple Lyapunov functions analysis was presented in Liberzon (2003), which promotes the development of switched control design greatly.

By using multiple Lyapunov functions method in stability analysis, a state-feedback control scheme was developed in Long and Zhao (2014) for switched nonlinear systems with an unknown exosystem. In traditional multiple Lyapunov functions method, the Lyapunov function for each individual subsystem is required to decrease exponentially, which significantly restricts the application range of this method especially when the closed-loop system suffers from external disturbances. To remove this restriction, the traditional multiple Lyapunov functions method was generalized in Long and Zhao (2015b), and the generalized result has shown to be attractive in the field as seen in Li and Yang (2016a, b) Long and Zhao (2015a), Sui and Tong (2016), Tong et al. (2015) and Zhang, Li, and Tong (2015) . However, no matter in Liberzon (2003) or Li and Yang (2016a, b) Long and Zhao (2015a, b), Sui and Tong (2016), Tong et al. (2015) and Zhang et al. (2015), the Lyapunov functions for any two subsystems are required to satisfy a restrictive condition that $V_i \leq \mu V_j$ with a positive constant μ. In nonlinear control design, guaranteeing such a condition is difficult, and hence how to establish a weaker condition urgently needs to be considered in switched control field.

In this paper, we consider the problem of output tracking control for a class of uncertain switched nonlinear systems. Two backstepping-based adaptive solutions to the problem are proposed to address the cases of arbitrary switching and dwell-time switching.

Robust adaptive control scheme: The scheme is designed based on common Lyapunov function approach. It is noticed that the switching parameters are different for each subsystem, leading to a nontrivial task to construct a common Lyapunov function for all subsystems, as pointed out in Ma and Zhao (2010), Wu (2009) and Zhao, Zheng et al. (2015). To address such an issue, we propose a new robust adaptive control strategy in this paper, in which the switching parameters are firstly transformed into their common bound via robust control design, and the transformed bound is then estimated online using adaptive approach. With the scheme, a common Lyapunov function for all subsystems is constructed successfully, and the closed-loop signal boundedness under arbitrary switching is well ensured.

Direct adaptive control scheme: In the scheme, the switching parameters are estimated directly by using adaptive approach. In this sense, the Lyapunov function is different for each subsystem, and thus the consideration of stability at switching time instants becomes rather difficult. Worse still, we cannot resolve such a difficulty as in Li and Yang (2016a, b), Liberzon (2003), Long and Zhao (2015a, b), Sui and Tong (2016), Tong et al. (2015) and Zhang et al. (2015), by employing the existing multiple Lyapunov functions method, because the method requires a restrictive condition $V_i \leq \mu V_j$ as mentioned earlier, which cannot be satisfied in our design. To overcome this difficulty, we firstly propose a less restrictive condition, i.e., $V_i \leq \mu V_j + \varDelta$ with a bounded constant $\varDelta$, and show how such a weaker condition is constructed via control design as seen in Lemma 2. Then we further propose an extended multiple Lyapunov functions method to establish the closed-loop system stability under such a weaker condition. It is shown that the global boundedness of all the closed-loop signals is ensured with the direct adaptive control scheme, and the tracking error is controlled into a residual around zero.

The paper is organized as follows. In Section 2, the considered control problem is formulated. Section 3 is devoted to constructing a robust adaptive control scheme based on common Lyapunov function method so that the closed-loop signal boundedness under arbitrary switching is ensured. Note that the scheme may cause large control input, and hence we further propose a new and novel direct adaptive control scheme in Section 4 based on an extended multiple Lyapunov functions method. The effectiveness of the proposed control schemes is illustrated in Section 5 through simulation results. Finally, we conclude the whole paper in Section 6 and point out a possible future research topic.

1.1. Notations and definitions

- $\mathbb{R}^n$, n dimensional vector space, where $n = 1, 2, \ldots$;
- $\mathbb{R}^+$, the set of positive real numbers;
- $f(t^+)$, right limit of the function $f(t)$ at t;
- $f(t^-)$, left limit of the function $f(t)$ at t;
- $\mathrm{sign}(\cdot)$, sign function;
- $|x(t)|$, absolute value of the scalar function $x(t)$;
- $\|z(t)\|$, Euclidean norm of the vector function $z(t)$, i.e., $\|z(t)\| = \sqrt{z^T(t)z(t)}$;
- $z(t) \in \mathcal{L}_\infty$, if $\sup_{t\geq 0}\|z(t)\| < \infty$.

2. Problem statement

In this paper, we consider the following class of uncertain switched nonlinear systems as in Chiang and Fu (2014), Zhu, Wen, Su, and Liu (2014) and Zhu, Wen, Su, Xu, and Wang (2013).

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta_{\sigma(t)}^T \varphi_i(x_1, x_2, \ldots, x_i), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= b_{\sigma(t)}\beta(x)u + \theta_{\sigma(t)}^T \varphi_n(x) + \psi_0(x), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ is a vector of system states, $u \in \mathbb{R}$ is control input, and $y \in \mathbb{R}$ denotes system output. $\varphi_i(x_1, \ldots, x_i) \in \mathbb{R}^p$ for $i = 1, 2, \ldots, n$, $\psi_0(x)$ and $\beta(x)$ are some known smooth nonlinear functions. $\theta_{\sigma(t)} \in \mathbb{R}^p$ and $b_{\sigma(t)} \in \mathbb{R}$ are unknown switching parameters with switching signal $\sigma : \{0\} \cup \mathbb{R}^+ \to M = \{1, 2, \ldots, m\}$.

Assumption 1. The reference trajectory $y_r(t)$ and its first n-order time derivatives $y_r^{(i)}(i = 1, 2, \ldots, n)$ are known, smooth, and bounded.

Assumption 2. Suppose that $\beta(x) \neq 0$ and $0 < b_0 \leq |b_{\sigma(t)}| \leq b_{\max} < \infty$, where b_0 and $b_{\max}$ denote some unknown positive constants. Without loss of generality, $\mathrm{sign}\big(b_{\sigma(t)}\big) > 0$ is further assumed.

Assumption 1 is rather general in the literature on backstepping control design, see Chen et al. (2009a, b), Chen, Wen, Liu, and Liu (2015), Krstic et al. (1995), Wang, Chen, Lin, and Li (2017), Wang, Chen, Lin, Zhang, and Meng (in press), Xin et al. (2015), Xin et al. (2016) and Zhou and Wen (2007), for examples. Assumption 2 is a basic condition to ensure the controllability of the switched nonlinear system (1).

Two control schemes will be presented in the following Sections 3 and 4 to address the cases of arbitrary switching and dwell-time switching, respectively, such that all the closed-loop signals are ensured to be bounded and the reference signal $y_r(t)$ is tracked by the system output $y(t)$.