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Brief paper

Rigid-body attitude stabilization with attitude and angular rate constraints[☆]

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ABSTRACT

In this paper, a solution to the problem of rest-to-rest three-axis attitude reorientation of a fully actuated rigid body under multiple attitude-constraint zones and angular velocity limits is presented. Based on the unit-quaternion parameterized attitude-constrained zones, a quadratic potential function is developed with a global minimum locating at the desired attitude and high potential closing to the constrained zones. In addition, to limit the magnitude of the angular velocity, another logarithmic potential function is also designed. Using these two potential functions and sliding mode control technique, a nonlinear attitude control law is obtained to guarantee asymptotic convergence of the closed-loop system with consideration of attitude and angular rate constraints, and external disturbances. The effectiveness of the constrained attitude control method is demonstrated through numerical simulation.

1. Introduction

Rigid-body attitude control is one of the most widely studied research fields in control literature. Extensive nonlinear control algorithms have been proposed for three-axis attitude control problem of a fully actuated rigid body, such as sliding mode control (Boskovic, Li, & Mehra, 2001; Lu & Xia, 2014; Shen, Wang, Zhu, & Poh, 2015b), backstepping control (Kristiansen, Nicklasson, & Gravdahl, 2008), adaptive control (di Gennaro, 2003; Shen, Wang, Zhu, & Poh, 2015a), hybrid control (de Angelis, Giulietti, de Ruiter, & Avanzini, 2016), and inverse optimal control (Krstic & Tsiotras, 1999; Luo, Chu, & Ling, 2005). For the case of underactuated dynamical systems, several approaches of solving the stabilization problem have also been developed, such as Avanzini, de Angelis, & Giulietti (2014), Gasagrandea, Astolfi, & Parisini (2008), Li, Yan, & Shi (2017) and Morin & Samson (1997), just to name a few. Recently, multiple application-specific constraints in rigid-body attitude maneuver have attracted a great deal of interest. For rigid spacecraft implementations, instruments equipped on the spacecraft are required to point their boresight along a target direction while keeping away from direct exposure to sunlight or other bright objects (Lee & Mesbahi, 2014). For example, the infrared telescopes may slew from one direction to another without direct exposure to the sun vector or other infrared bright regions in space (McInnes, 1994). This kind of constraint is regarded as attitude constraint. Another constraint to be taken into account is angular rate constraint caused by the saturation limitation of low-rate gyro or mission specification requirement. A practical example is X-ray Timing Explorer (XTE) spacecraft that is required to maneuver within the saturation limit of rate gyros (Wie & Lu, 1995). In view of these practical considerations, this paper studies the three-axis reorientation problem of a fully actuated rigid body subject to both of attitude and angular rate constraints.

Methods dealing with attitude constrained rigid-body reorientation problem can be generalized into two main categories: path planning methods and potential function methods. In literature, several attitude path planning strategies (de Angelis, Giulietti, & Avanzini, 2015; Frazzoli, Dahleh, Feron, & Kornfeld, 2001; Hablani, 1999; Kjellberg & Lightsey, 2013) have been developed to find the admissible rotation trajectory. However, these methods have a complex structure which gives rise to demanding computation burden (Avanzini, Radice, & Ali, 2009; Lee & Mesbahi, 2014). Potential function methods utilize the artificial potential to model the admissible attitude path. In general, the developed artificial potential is formulated with a global attractive minimum at the desired orientation and high potential closing to the exclusion zones. Then, the potential function is incorporated in the attitude

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controller design to stabilize the system while satisfying attitude constraints. Since this kind of approach is analytical without changing the overall structure of the attitude control software or hardware, it is suitable for on-board computation and provides flexible autonomous operations. In McInnes (1994), the potential function was formulated in the form of Gaussian functions, and an attitude controller was developed to converge the attitude without violating pre-defined pointing constraints. However, since Euler angles were used to represent attitude in McInnes (1994), the proposed control algorithm may suffer from singularity. In Rafal and Piotr (2005), an attitude control law was synthesized applying the potential function method to prevent the camera from exposing to the Sun light directly during the slew maneuver. In Lee and Mesbahi (2014), a convex logarithmic barrier potential was formulated in the unit-quaternion space, and the backstepping technique based controllers were proposed to ensure attitude convergence and forbidden attitude avoidance. In Shen, Yue, and Goh (2017), a velocity-free attitude controller was developed for a flexible spacecraft in the presence of attitude constraints.

Another challenge in practical rigid-body attitude control is the constraint on angular rate. To ensure that angular rate is always within a pre-defined bound determined by saturation limit of rate gyros or performance requirements, several methods have been proposed. In Wie and Lu (1995), a quaternion feedback control law was developed for the near-minimum-time eigenaxis reorientation problem of the XTE spacecraft with consideration of angular velocity and control torque constraints. Although this approach is commonly used in practical spacecraft mission, a rigorous stability proof of the closed-loop system is not given. In Verbin, Lappas, and Ben-Asher (2011), a time-efficient angular steering law was developed to handle several state constraints, where the angular rate and acceleration limits were determined by a braking curve-like angular velocity trajectory. In Hu, Li, and Zhang (2016), a robust nonlinear controller incorporating a control allocation scheme was proposed for a rigid spacecraft under angular velocity constraints and actuator saturation, where a logarithmic barrier potential function was developed. In Hu, Li, and Friswell (2015), an attitude stabilization strategy was proposed to solve the unwinding problem for a rigid spacecraft in the presence of angular velocity constraints.

In this paper, to handle attitude constraints and angular rate limitations simultaneously in attitude maneuver, an adaptive attitude controller based on two different potential functions defined in attitude orientation and angular velocity domain is presented. We prove that the proposed attitude controller is able to achieve asymptotic stabilization of the closed-loop system, while attitude and angular velocity constraints are satisfied concurrently. The main contributions of this study are summarized as the following three key-points:

(1) Comparing with aforementioned literatures (Hu et al., 2015, 2016; Lee & Mesbahi, 2014; McInnes, 1994; Rafal & Piotr, 2005; Shen et al., 2017; Verbin et al., 2011; Wie & Lu, 1995), this study presents a solution to deal with both attitude constraints and angular rate limits in attitude control.
(2) A logarithmic potential function in terms of sliding vector is first proposed, whose largest potential is placed at the maximal angular velocity respectively. Based on this potential function, angular velocity constraint is satisfied through limiting the magnitude of the sliding vector.
(3) The proposed two potential functions for attitude and angular velocity are smooth and strictly convex with global minima located at the desired attitude and angular velocity. This ensures that attitude and angular velocity could be stabilized to the global minima while avoiding multiple attitude constrained zones and limiting the magnitude of angular velocity.

The remainder of this paper is organized as follows. In Section 2, unit-quaternion is introduced for attitude representation, and rigid-body dynamics and modeling of attitude-constraint zones as well as angular rate limits are described. In Section 3, two potential functions are designed to describe the attitude constrained zones and angular velocity limits, respectively. Then, an adaptive attitude control law using sliding mode control technique is developed to guarantee asymptotic stability. The simulation results are given in Section 4, followed by conclusions in Section 5.

2. Preliminaries

In this paper, the unit-quaternion representation is used to describe the orientation of a rigid body. The set of unit quaternion $\mathcal{Q}_u$ is given by

$$\mathcal{Q}_u = \{\boldsymbol{Q} = [\boldsymbol{q}^T \; q_0]^T \in \mathcal{R}^3 \times \mathcal{R} \mid \boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1\} \tag{1}$$

where $\boldsymbol{q}$ and q_0 denote the vector part and the scalar part of a quaternion, respectively. The unit-quaternion conjugate or inverse is defined as $\boldsymbol{Q}^* = [-\boldsymbol{q}^T \; q_0]^T$. The properties of quaternion can be found in Chou (1992).

2.1. Kinematics equation

The spacecraft kinematics in terms of the unit quaternion is given by Shuster (1993)

$$\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \otimes \boldsymbol{\nu}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix}\boldsymbol{S}(\boldsymbol{q}) + q_0\boldsymbol{I}_3 \\ -\boldsymbol{q}^T\end{bmatrix}\boldsymbol{\omega} \tag{2}$$

where $\boldsymbol{\omega} \in \mathcal{R}^3$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, the notation "$\otimes$" denotes the quaternion multiplication operator, the function $\boldsymbol{\nu}$: $\mathcal{R}^3 \to \mathcal{R}^4$ is defined as the mapping $\boldsymbol{\nu}(\boldsymbol{\omega}) = [\boldsymbol{\omega}^T \; 0]^T$, and the matrix $\boldsymbol{S}(\boldsymbol{x}) \in \mathcal{R}^{3\times3}$ is a skew-symmetric matrix satisfying $\boldsymbol{S}(\boldsymbol{x})\boldsymbol{y} = \boldsymbol{x}\times\boldsymbol{y}$ for any vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{R}^3$, and "$\times$" denotes vector cross product.

Let $\boldsymbol{Q}_d \in \mathcal{Q}_u$ denote the desired attitude. The unit-quaternion error $\boldsymbol{Q}_e = [q_{e1} \; q_{e2} \; q_{e3} \; q_{e0}]^T = [\boldsymbol{q}_e^T \; q_{e0}]^T \in \mathcal{Q}_u$ is given by $\boldsymbol{Q}_e = \boldsymbol{Q}_d^* \otimes \boldsymbol{Q} = [\boldsymbol{q}_e^T \; q_{e0}]^T$. Let $\boldsymbol{\omega}_d$ denote the desired angular velocity in the desired reference frame $\mathcal{N}$. Since the rest-to-rest attitude maneuver is considered in this paper, the relative angular velocity defined as $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{R}(\boldsymbol{Q}_e)^T\boldsymbol{\omega}_d$ is simplified to $\boldsymbol{\omega}_e = \boldsymbol{\omega}$, where $\boldsymbol{R}(\boldsymbol{Q}_e)$ is $\boldsymbol{Q}_e$ related rotation matrix defined as $\boldsymbol{R}(\boldsymbol{Q}_e) = (q_{e0}^2 - \boldsymbol{q}_e^T\boldsymbol{q}_e)\boldsymbol{I}_3 + 2\boldsymbol{q}_e\boldsymbol{q}_e^T - 2q_{e0}\boldsymbol{S}(\boldsymbol{q}_e)$ (Sidi, 1997) . Then, the kinematics represented by unit-quaternion error is described as (Shuster, 1993)

$$\dot{\boldsymbol{Q}}_e = \frac{1}{2}\boldsymbol{Q}_e \otimes \boldsymbol{\nu}(\boldsymbol{\omega}_e) = \frac{1}{2}\begin{bmatrix}\boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3 \\ -\boldsymbol{q}_e^T\end{bmatrix}\boldsymbol{\omega}. \tag{3}$$

2.2. Rigid-body dynamics

The dynamics for the attitude motion of a rigid body is expressed by the following equations (Sidi, 1997):

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\tau} + \boldsymbol{d} \tag{4}$$

where $\boldsymbol{J} \in \mathcal{R}^{3\times3}$ denotes the positive definite inertia matrix of a rigid body, $\boldsymbol{\tau} \in \mathcal{R}^3$ denotes the control torque about the body axes, $\boldsymbol{d} \in \mathcal{R}^3$ denotes the external disturbances. To design the attitude controller, a sliding vector $\boldsymbol{s} = [s_1, s_2, s_3]^T \in \mathcal{R}^3$ is given by

$$\boldsymbol{s} = \boldsymbol{\omega} + k\boldsymbol{q}_e \tag{5}$$

where k is a positive constant. Consequently, the attitude dynamics in terms of the sliding vector can be written as

$$\boldsymbol{J}\dot{\boldsymbol{s}} = \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) + \boldsymbol{\tau} + \boldsymbol{d} \tag{6}$$

where the nonlinear term $\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) = -\boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{J}\boldsymbol{\omega} + \frac{k}{2}\left(\boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3\right)\boldsymbol{\omega}$.