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Brief paper

Interval estimation for continuous-time switched linear systems ☆

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ABSTRACT

This paper deals with the design of interval observers for switched linear systems (SLS), a class of hybrid systems. Under the assumption that the disturbances and the measurement noise are bounded, upper and lower bounds for the state are calculated. New conditions of cooperativity in discrete-time instants are firstly proposed. Then, some techniques for interval estimation are developed in continuous-time. It is shown that it is possible to calculate the observer gains making the estimation error dynamics cooperative and stable via some change of coordinates under arbitrary switching sequences. The performances of the developed techniques are illustrated through numerical examples.

1. Introduction

Switched systems are very flexible modeling tools, which appear in several fields such as networked control systems, electrical devices and congestion modeling (Liberzon, 2012). These systems are one of the most important classes of Hybrid Dynamical Systems (HDS). They consist of a set of continuous dynamical systems and a switching rule orchestrates among them, their detailed description can be found in various monographs like Djemai and Defoort (2015) and Liberzon (2012).

In order to study the stability of SLS, specific results have been developed. For example, in Hu, Zhai, and Michel (2002) a common Lyapunov function yields sufficient conditions for the global asymptotic stability. However, it may not always be possible to get this common function. Therefore, multiple Lyapunov functions were proposed for instance in Liberzon (2012). Approaches based on restrictions on admissible rates of commutation, like average dwell time, have also been studied to ensure the stability of SLS (see Hetel, 2007; Lin and Antsaklis, 2009) and the references therein).

As far as the stability problem is widely concerned, it is worth pointing out that the state is not always directly measured but may be estimated from the input and the output of the process. State estimation of those systems has received considerable attention over past decades. Meanwhile, some recent researches have been carried out in this domain. In the case of SLS with state jumps, necessary and sufficient conditions for the observability have been established based on geometric approach (Tanwani, Shim, & Liberzon, 2013). In Arichi, Djemai, Cherki, and Manamanni (2015) and Ríos, Mincarelli, Efimov, Perruquetti, and Davila (2015) sliding mode observers have been designed to estimate the continuous and discrete states for the switched system in observability canonical form. In Zhao, Liu, Zhang, and Li (2015) the authors have considered the problem of existence of Luenberger-type observers (Luenberger, 1966) in the case when the activated observer is time delayed compared to the activated subsystem of the switched system.

The estimation problem becomes much more involved if we consider systems subject to model and/or signals uncertainties. Therefore, state estimation approaches based on the set-membership (Meslem & Ramdani, 2011) and interval observers get more attention for uncertain systems (a survey is given in Efimov and Raïssi, 2016). According to the interval observers theory, the uncertainties are assumed to be unknown but bounded with a priori known bounds. In the literature, interval observers are applied to several classes of nonlinear systems (Meslem & Ramdani, 2011;

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Raissi, Efimov, & Zolghadri, 2012), discrete-time systems (Efimov, Perruquetti, Raïssi, & Zolghadri, 2013; Mazenc, Dinh, & Niculescu, 2014) and linear systems (Gouzé, Rapaport, & Hadj-Sadok, 2000; Lamouchi, Amairi, Raïssi, & Aoun, 2016).

For HDS, an interval observer has been applied to the estimation of linear impulsive systems (Degue, Efimov, & Richard, 2016), while the extension of those estimators to switched systems has not been fully considered in the literature. To the best of the authors knowledge, only a preliminary work has been developed in Ifqir, Oufroukh, Ichalal, and Mammar (2017) and He and Xie (2015, 2016) with the strong assumption that there exist observer gains providing the cooperativity and stability conditions simultaneously. Unfortunately, this assumption is rather restrictive and rarely verified in practice and different attempts to overcome this restriction have been performed in Ethabet, Raïssi, Amairi, and Aoun (2017), Rabehi, Efimov, and Richard (2017), and the present paper is a journal extension of the latter works.

In this work, the main contribution is to design interval estimators for SLS subject to disturbances. The measurement noise and the state disturbance are assumed to be unknown but bounded with known bounds. Discrete-time and continuous-time interval estimations have been considered. In the first approach, the cooperativity property of the observation error is ensured, under some approximations, through judicious observer gains computation, and the stability analysis is verified by the feasibility of matrix inequalities with respect to a matrix variable given in a quadratic Lyapunov function. While in the second one, the stability and cooperativity conditions are given in terms of Linear Matrix Inequalities (LMIs). It will be shown that the constructive methodologies can be applied for a large class of SLS.

The paper has the following structure. Some preliminaries are described in Section 2. The main results of designing the interval observers are developed in Section 3. Simulation results are shown in Section 4 to illustrate the efficiency of the proposed methods. Section 5 concludes the paper.

2. Preliminaries

2.1. Notations

The sets of real and natural numbers are denoted by $\mathbb{R}$ and $\mathbb{N}$ respectively. Denote the sequence of integers $\{1, \ldots, N\}$ by $\overline{1, N}$. E_p denotes a $(p \times 1)$ vector whose elements are equal to 1. I is the identity matrix of proper dimension. For a matrix $P = P^T$, the relation $P \prec 0$ ($P \succ 0$) means that the matrix $P \in \mathbb{R}^{n\times n}$ is negative (positive) definite. Denote by $\underline{x}$ and $\overline{x}$ the lower and upper bounds of a variable x such that $\underline{x} \leq x \leq \overline{x}$. $|\cdot|$ denotes the elementwise absolute value of a vector $x \in \mathbb{R}^n$ resp. a matrix $A \in \mathbb{R}^{n\times m}$. The relation $\leq$ should be interpreted elementwise for vectors as well as for matrices, i.e. $A = (a_{i,j}) \in \mathbb{R}^{p\times m}$ and $B = (b_{i,j}) \in \mathbb{R}^{p\times m}$ such that $A \geq B$ if and only if, $a_{i,j} \geq b_{i,j}$ for all $i \in \overline{1, p}, j \in \overline{1, m}$. $Diag\ (\lambda)$ denotes a diagonal matrix with the elements of the vector λ on the main diagonal.

For a matrix $A \in \mathbb{R}^{m\times n}$, define $A^+ = \max\{0, A\}$ and $A^- = A^+ - A$.

Lemma 1 (Hardy, Littlewood, & Pólya, 1952). *Let $\delta > 0$ be a scalar and $S \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix, then*

$$2x^T y \leq \frac{1}{\delta} x^T S x + \delta y^T S^{-1} y \qquad x, y \in \mathbb{R}^n. \tag{1}$$

2.2. Cooperativity

Definition 1 (Minc, 1988). A matrix $A = (a_{i,j}) \in \mathbb{R}^{n\times n}$ is said to be Metzler if all its off-diagonal elements are nonnegative i.e. $a_{i,j} \geq 0$, $\forall (i, j)$, $i \neq j$. It is said to be nonnegative if all the entries are nonnegative: $A \geq 0$.

Lemma 2 (Farina & Rinaldi, 2000; Gouzé et al., 2000). *Consider the system described by:*

$$\dot{x}(t) = Ax(t) + u(t), \quad x(0) = x_0 \tag{2}$$

The system (2) is said to be cooperative if A is a Metzler matrix and $u(t) \geq 0$. For any initial condition $x_0 \geq 0$ the solution of (2) satisfies $x(t) \geq 0, \forall t \geq 0$.

2.3. Interval relations

Property 1 (Chebotarev, Efimov, Raïssi, & Zolghadri, 2015). *Given a matrix $A \in \mathbb{R}^{n\times n}$. A is Metzler if there exists a diagonal matrix $S \in \mathbb{R}^n_+$ such that*

$$A + S > 0. \tag{3}$$

Lemma 3 (Chebotarev et al., 2015). *Let $x \in \mathbb{R}^n$ be a vector satisfying $\underline{x} \leq x \leq \overline{x}$ and $A \in \mathbb{R}^{m\times n}$ be a constant matrix, then*

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{4}$$

2.4. Basic interval observer theory

For the sake of clarity, let us recall some definitions of interval observers (Efimov & Raïssi, 2016). Consider the following system:

$$\begin{cases} \dot{x} = Ax + \phi(t) \\ y = Cx \end{cases} \tag{5}$$

where ϕ is a continuous function of time. Assume that there exist two known functions $\overline{\phi}$ and $\underline{\phi} : \mathbb{R} \to \mathbb{R}^n$ such that $\underline{\phi}(t) \leq \phi(t) \leq \overline{\phi}(t), \forall t \geq 0$.

Interval observers compute a guaranteed set that contains admissible values for the state vector of the system consistent with the output measurements. Upper and lower bounds of the set are given in the following theorem.

Theorem 1 (Gouzé et al., 2000). *Let $\underline{x}_0 \leq x_0 \leq \overline{x}_0$. If there exists a gain K such that $(A - KC)$ is Metzler and Hurwitz then*

$$\begin{cases} \dot{\overline{x}} = A\overline{x} + \overline{\phi} + K(y - C\overline{x}) \\ \dot{\underline{x}} = A\underline{x} + \underline{\phi} + K(y - C\underline{x}) \end{cases} \tag{6}$$

is an interval observer with $\underline{x}(t) \leq x(t) \leq \overline{x}(t), \forall t \geq 0$.

2.5. Stability

Consider the following continuous-time SLS:

$$\dot{x}(t) = A_{\sigma(t)}x(t), \ \sigma(t) \in \mathcal{I} = \overline{1, N}, N \in \mathbb{N} \tag{7}$$

where $x \in \mathbb{R}^n$ is the state, N is the number of linear subsystems, the finite set $\mathcal{I}$ is an index set and it stands for the collection of subsystems $A_q \in \mathbb{R}^{n\times n}$, $\forall q \in \mathcal{I}$. The switching between the subsystems is ensured via a switching signal, a piecewise constant function, $\sigma(t) : \mathbb{R}_+ \to \mathcal{I}$. The index $q = \sigma(t)$ specifies, at each instant of time, the system that currently being followed.

Lemma 4 (Liberzon, 2012). *Let $S \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix that satisfies the LMIs*

$$A_q^T S + S A_q \prec 0, \ q \in \mathcal{I} = \overline{1, N} \tag{8}$$

Then $V(x) = x^T Sx$ is a Common Quadratic Lyapunov Function (CQLF) for the system (7).

This lemma establishes conditions of the internal stability (without taking into account the effect of external inputs). For SLS with inputs and properly assigned dwell-time switching signals,