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Brief paper

A new integral sliding mode design method for nonlinear stochastic systems[☆]

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ABSTRACT

Recently, several integral sliding mode control (ISMC) methodologies have been put forward to robust stabilization of nonlinear stochastic systems depicted by T–S fuzzy models. However, these results employ very restrictive assumptions on system matrices, which impose a great limitation to real applications. This paper aims to remove these assumptions and present a new ISMC method for fuzzy stochastic systems subjected to matched/mismatched uncertainties. To this end, a novel fuzzy integral sliding manifold function is adopted such that the matched uncertainties are completely rejected while the mismatched ones will not be enlarged during the sliding mode phase. Sufficient conditions are derived to ensure the stochastic stability of the closed-loop system under sliding motion. A fuzzy sliding mode controller is further presented to maintain the states of fuzzy stochastic system onto the predefined fuzzy manifold in the presence of uncertainties. The effectiveness and benefit of the developed new method are demonstrated by the inverted pendulum system.

1. Introduction

During the past few decades, the study on Itô stochastic systems has received considerable attention due to the typical exhibition of stochastic phenomenon in many real-world situations, and a large number of works have been reported (Mao, 2007; Wang, Liu, & Liu, 2010). However, it is should be pointed out that the available literatures mainly focus on the linear stochastic models, while most practical stochastic systems are highly nonlinear. For nonlinear stochastic systems, there lacks a systematic methodology on controller synthesis due to the difficulties in finding an appropriate Lyapunov functions (Berman & Shaked, 2004). A feasible and efficient way to solve this problem is to represent a nonlinear stochastic system by T–S fuzzy models (Chiu, Lian, & Liu, 2005; Wang, Xia, & Zhou, 2016). By adopting some fuzzy rules, a complex nonlinear stochastic system can be depicted in the form of a set of linear sub-models, which can be referred to fuzzy stochastic system. Thus, the well established linear stochastic system theory can be utilized to study a highly nonlinear stochastic control system (Wu & Zheng, 2009). This explains the reason for recent surge of research attention on fuzzy stochastic systems (see, e.g. Wu, Yang, & Lam, 2014).

As a fruitful research topic of the control community, sliding mode control (SMC) has been widely applied to various complex dynamical systems (Basin & Rodriguez-Ramirez, 2014; Chiu & Liu, 2016; Edwards & Spurgeon, 1998; Huang & Mao, 2010; Kao, Xie, Wang, & Karimi, 2015; Levant & Fridman, 2010; Shi, Xia, Liu, & Rees, 2006). Besides fast response, the main appeal of SMC is that it has the ability to compensate matched uncertainties during the sliding mode phase of system. However, during the reaching phase, the systems are vulnerable to uncertainties and disturbance. As a solution to this problem, an integral sliding mode control (ISMC) strategy was established (Chiu, 2012; Hamayun, Edwards, & Alwi, 2012; Rubagotti, Estrada, Castanos, & Fridman, 2011; Utkin & Shi, 1996), where the reaching phase existed in normal sliding mode control is eliminated, and sliding motion will be achieved from the initial stage of the control action while maintaining the order of the original system. In this situation, the robustness of the system can be ensured throughout the entire system response. During the past years, the ISMC methodology has been successfully extended to stochastic systems (Basin, Rodriguez-Ramirez, Fridman, & Acosta, 2005; Niu, Ho, & Lam, 2005). Specially, several ISMC design methods have extended to accommodate fuzzy stochastic

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systems. Specially, in Ho and Niu (2007), the ISMC problem was first studied for a type of fuzzy stochastic time-delay systems, which is formulated in form of

$$\begin{aligned} dz(t) = & \sum_{i=1}^{r} \varphi_i(\nu(t)) \{A_i z(t) + A_{di} z(t-\tau) + Bu(t)\} dt \\ & + \sum_{i=1}^{r} \varphi_i(\nu(t)) C_i g_i(x(t), x(t-\tau)) d\varpi(t). \end{aligned} \tag{1}$$

However, the obtained ISMC approach employs two very restrictive assumptions, which impose a great limitation to real applications. One is all the local input matrices are required to be the same, while many practical physical plants including the well known inverted pendulum cannot fulfill this requirement. The other is the projection matrix W in the proposed sliding manifold is needed to satisfy WB is nonsingular and $WC_i = 0$ simultaneously. To remove the above two assumptions, Gao, Feng, Liu, Qiu, and Wang (2014) and Gao, Liu, Feng, and Wang (2014) put forward a dynamic ISMC methodology by resorting to the following distinguished integral sliding manifold:

$$\begin{aligned} s(t) = & W_x[z(t) - z(0)] + W_u[u(t) - u(0)] \\ & - \int_0^t \sum_{i=1}^{r} \varphi_i(\nu(\tau)) W_x (A_i z(\tau) + B_i u(\tau)) d\tau \\ & - \int_0^t \sum_{i=1}^{r} \varphi_i(\nu(\tau)) W_u (F_i z(\tau) + W_i u(\tau)) d\tau \end{aligned} \tag{2}$$

where the related matrices $\begin{bmatrix} A_i & B_i \\ F_i & W_i \end{bmatrix}$ are required to be Hurwitz for $i = 1, \ldots, r$. It is worth mentioning that, however, parts of their eigenvalues are the same as ones of matrices A_i, and the eigenvalues of A_i cannot be adjusted by given matrices F_i and W_i. In this case, if the system is not inherently stable (i.e., A_i are not Hurwitz for all $i = 1, \ldots, r$), it is hard to find suitable matrices F_i and W_i to meet this requirement. Thus, this assumption is also very restrictive for the practical application of the method.

The main purpose of the paper is to propose an alternative ISMC methodology for fuzzy stochastic systems, which can remove the very restrictive assumptions in previous results. To this end, a novel fuzzy integral sliding manifold function is utilized to better accommodate the available features of fuzzy stochastic systems, which is the key contribution of the paper. The existence condition for the fuzzy manifold is offered. Sufficient conditions to ensure the stochastic stability of the closed-loop system under sliding motion are then presented. Moreover, a fuzzy sliding mode law is synthesized to ensure reaching condition. Finally, the effectiveness and benefit of the developed method are illustrated by the inverted pendulum system.

2. Problem formulation and preliminaries

Consider a nonlinear Itô stochastic system described by the following T–S fuzzy model (Gao, Feng et al., 2014; Ho & Niu, 2007):
Model rule i: **IF** $\nu_1(t)$ is τ_{i1} and $\cdots$ $\nu_p(t)$ is τ_{ip} , **THEN**

$$\begin{aligned} dz(t) = & [(A_i + \Delta A_i)z(t) + B_i(u(t) + f_m(z(t), t))] dt \\ & + g(z(t), t) d\varpi(t), i \in \mathcal{R} = \{1, 2, \ldots, r\} \end{aligned} \tag{3}$$

with $\nu_1(t), \nu_2(t), \ldots, \nu_p(t)$ being the premise variables and functions of system state, $\tau_{i1}, \tau_{i2}, \ldots, \tau_{ip}$ being the fuzzy sets, $\varpi(t)$ representing an l-dimension Wiener process (Brownian motion) defined on probability space $(\Omega, \mathbb{F}, \mathbb{P})$, $u(t) \in \mathbb{R}^m$ being the input signal, r being the number about fuzzy rules, $z(t) \in \mathbb{R}^n$ being the system state, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ being appropriate dimensioned constant matrices. The matched uncertainty $f_m(z(t), t) \in \mathbb{R}^m$ and nonlinear function $g(z(t), t) \in \mathbb{R}^{m\times l}$ are unknown and satisfy

$$\|f_m(z(t), t)\| \le \kappa \|z(t)\|, \tag{4}$$

$$trace\left[g^T(z(t), t) g(z(t), t)\right] \le \|Ez(t)\|^2 \tag{5}$$

with $\kappa > 0$ being a known scalar, E being a known matrix.

The mismatched uncertainties ΔA_i are assumed to be norm bounded, i.e.,

$$\|\Delta A_i\| \le \sigma_i, i \in \mathcal{R} \tag{6}$$

with $\sigma_i > 0$ being known constants.

By adopting a standard inference approach, the compact presentation of fuzzy stochastic system (3) is described as

$$\begin{aligned} dz(t) = & \sum_{i=1}^{r} \varphi_i(\nu(t)) [(A_i + \Delta A_i)z(t) + B_i(u(t) \\ & + f_m(z(t), t))] dt + g(z(t), t) d\varpi(t) \end{aligned} \tag{7}$$

where $\varphi_i(\nu(t))$ are fuzzy basis functions satisfying

$$\varphi_i(\nu(t)) = \frac{\prod_{i=1}^{p} \tau_{ij}\left(\nu_j(t)\right)}{\sum_{l=1}^{r}\prod_{j=1}^{p} \tau_{lj}\left(\nu_j(t)\right)} \ge 0, \ \sum\nolimits_{i=1}^{r} \varphi_i(\nu(t)) = 1$$

with $\tau_{ij}\left(\nu_j(t)\right)$ denoting the grade of membership about variable $\nu_j(t)$ in τ_{ij}. Yet the general, the input matrix $\mathcal{B}_z = \sum_{i=1}^{r} \varphi_i(\nu(t)) B_i$ is assumed to be with full column rank.

Throughout the paper, the following lemma will be used.

Lemma 1. *For any full column rank state-dependent matrix $\mathcal{B}_z \in \mathbb{R}^{n\times m}$, if the distribution $\Delta(z) = span\left\{\mathcal{B}_{z,k}^{\perp}\right\}$, $k = 1, \ldots, n-m$, is involutive, that is*

$$\begin{aligned} \left[\mathcal{B}_{z,k}^{\perp}, \mathcal{B}_{z,l}^{\perp}\right] = & \frac{\partial \mathcal{B}_{z,l}^{\perp}}{\partial z}\mathcal{B}_{z,k}^{\perp} - \frac{\partial \mathcal{B}_{z,k}^{\perp}}{\partial z}\mathcal{B}_{z,l}^{\perp} \in \Delta(z), \\ & \forall k, l = 1, \ldots, n-m \end{aligned} \tag{8}$$

where $\mathcal{B}_{z,k}^{\perp}$ denotes the kth column about $\mathcal{B}_z^{\perp}$, $[\cdot, \cdot]$ indicates the two-vector-fields-based Lie bracket. Then there exists a nonlinear function $w(z) \in \mathbb{R}^{m\times 1}$ such that $\frac{\partial w(z)}{\partial z} = W(z) = N(z)\mathcal{B}_z^T$, with $N(z) \in \mathbb{R}^{m\times m}$ being with full rank.

Proof. By resorting to Frobenius' Theorem (Isidori, 1996), the involutivity of $\Delta(z)$ indicates that there are m independent functions $w_k(z)$ satisfying $\frac{\partial w_k(z)}{\partial z}\mathcal{B}_{z,l}^{\perp} = 0, \forall 1 \le k \le m, 1 \le l \le n-m$. Note that the matrix $W^T(z)$ is column full rank, it spans the orthogonal complement of $\Delta(z)$, e.g. $span\left\{W_k^T(z)\right\} = \left(span\left\{\mathcal{B}_{z,k}^{\perp}\right\}\right)^{\perp}$, which is equivalent to $span\left\{W_k^T(z)\right\} = span\left\{\mathcal{B}_{z,k}\right\}$ by resorting to double orthogonal complement (Luenberger, 1969). Therefore, the columns of $W^T(z)$ and $\mathcal{B}_z$ are bases of the same subspace, and satisfy $W^T(z) = \mathcal{B}_z N^T(z)$, with matrix $N^T(z)$ being the transformation matrix between them. This completes the proof.

3. Integral sliding manifold design and sliding motion analysis

In this section, a fuzzy integral sliding manifold will be designed and stochastic stability analysis will be provided for the corresponding closed-loop system.

In this paper, we propose a novel fuzzy integral sliding manifold function for fuzzy stochastic system (7) as follows:

$$\begin{aligned} s(t) = & \int_0^t W(z)dz - \int_0^t \Bigg[W(z(\tau)) \sum_{i=1}^{r} \varphi_i(\nu(\tau)) \\ & \times \sum_{j=1}^{r} \varphi_j(\nu(\tau)) \left(A_i z(\tau) + B_i K_j z(\tau)\right)\Bigg] d\tau \end{aligned} \tag{9}$$