Automatica 90 (2018) 310–316

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust stability conditions for feedback interconnections of distributed-parameter negative imaginary systems ☆

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ARTICLE INFO

Article history:
Received 3 May 2016
Received in revised form 1 August 2017
Accepted 20 August 2017
Available online 15 February 2018

Keywords:
Negative imaginary systems
Robust feedback stability
Integral quadratic constraints
Distributed-parameter systems

ABSTRACT

Sufficient and necessary conditions for the stability of positive feedback interconnections of negative imaginary systems are derived via an integral quadratic constraint (IQC) approach. The IQC framework accommodates distributed-parameter systems with irrational transfer function representations, while generalising existing results in the literature and allowing exploitation of flexibility at zero and infinite frequencies to reduce conservatism in the analysis. The main results manifest the important property that the negative imaginariness of systems gives rise to a certain form of IQCs on positive frequencies that are bounded away from zero and infinity. Two additional sets of IQCs on the DC and instantaneous gains of the systems are shown to be sufficient and necessary for closed-loop stability along a homotopy of systems.

1. Introduction

The notion of negative imaginary systems was introduced in Lanzon and Petersen (2008) and Petersen and Lanzon (2010) as a natural counterpart to positive real systems (Anderson & Vongpanitlerd, 2007; Bao & Lee, 2007; Khalil, 2002; van der Schaft, 2016). The negative imaginary property commonly arises from the dynamics of a lightly damped structure with colocated force actuators and position sensors (such as piezoelectric sensors) (Bhikkaji & Moheimani, 2009; Petersen & Lanzon, 2010). Such a system exhibits positive real dynamics from the force input to the velocity output, but negative imaginary dynamics from the force input to the position output, whose transfer function may be of relative degree 2. Furthermore, negative imaginary systems theory may also be employed to study certain systems that are not passive, for which positive real results do not hold. Another area where negative imaginary dynamics can be found is that of nano-positioning systems (Devasia, Eleftheriou, & Moheimani, 2007). Owing to the prevalence of negative imaginary properties in real world applications, such systems have been studied extensively in the literature (Das, Pota, & Petersen, 2013; Lanzon & Petersen, 2008; Petersen & Lanzon, 2010; Xiong, Petersen, & Lanzon, 2010). Feedback interconnections of negative imaginary systems are interpreted from a geometric Hamiltonian systems viewpoint in van der Schaft (2011). In Wang, Lanzon, and Petersen (2015), the problem of robust output consensus of networked negative imaginary systems is considered. Characterisations of negative imaginary systems with symmetric irrational transfer functions are considered in Ferrante and Ntogramatzidis (2013) and Ferrante, Lanzon, and Ntogramatzidis (2015). A nonlinear generalisation of negative imaginary dynamics, termed counterclockwise input–output dynamics, is given in Angeli (2006).

The robustness of feedback interconnections of open-loop stable negative imaginary systems is investigated in Lanzon and Petersen (2008) as a parallel to the positive real stability results (Anderson & Vongpanitlerd, 2007); see Fig. 1. It is shown that if the instantaneous gain of $\bar{G}$ is positive semidefinite, i.e. $\bar{G}(\infty) \geq 0$, and the product of the instantaneous gains of $\bar{G}$ and G is 0, i.e. $G(\infty)\bar{G}(\infty) = 0$, then the closed-loop system $[G, \bar{G}]$ is internally stable if, and only if, the DC gain condition $\bar{\lambda}(G(0)\bar{G}(0)) < 1$ is satisfied, where $\bar{\lambda}$ denotes the spectral radius. This result is further generalised in Xiong et al. (2010) to the case where G may have imaginary-axis poles that are not located at the origin. Physical interpretations of these results in terms of mass–spring–damper systems and RLC electrical networks are provided in Petersen (2015). In particular, it is demonstrated using the negative imaginary theory that certain mass–spring–damper systems with

☆ This work was supported in part by the Institute for Mathematics and its Applications with funds provided by the National Science Foundation, the Australian Research Council, and the Swedish Research Council through the LCCC Linnaeus centre. The material in this paper was partially presented at the 14th annual European Control Conference, July 15–17, 2015, Linz, Austria. This paper was recommended for publication in revised form by Associate Editor Yasuaki Oishi under the direction of Editor Richard Middleton.

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https://doi.org/10.1016/j.automatica.2017.09.010

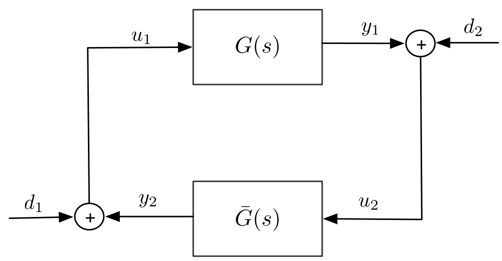

Fig. 1. Positive feedback interconnection of negative imaginary systems.

negative spring constants or RLC networks with negative inductances or capacitances are stable, whereas the standard positive real theory is inapplicable to such non-passive systems. These stability conditions are robust in the sense that they are invariant to negative-imaginary perturbations on the systems, provided that the aforementioned gain conditions are not violated. Stability conditions for negative imaginary systems with poles at the origin are studied in Mabrok, Kallapur, Petersen, and Lanzon (2014).

When the presuppositions of the stability theorems in Lanzon and Petersen (2008) and Xiong et al. (2010) do not hold, such as $\bar{G}(\infty)$ being sign-indefinite or $G(\infty)\bar{G}(\infty) \neq 0$, the DC gain condition $\bar{\lambda}(G(0)\bar{G}(0)) < 1$ is not necessary. This paper derives generic sufficient and necessary conditions for feedback stability of negative imaginary systems with respect to a specified homotopy using the theory of integral quadratic constraints (IQC) (Cantoni, Jönsson, & Kao, 2012; Cantoni, Jönsson, & Khong, 2013; Megretski, Jönsson, Kao, & Rantzer, 2010; Megretski & Rantzer, 1997). In particular, it is established that the negative imaginary properties of the systems give rise to complementary IQCs on a set of frequencies which do not include 0 and ∞ but can be arbitrarily large. This interpretation clarifies the role of negative imaginariness in robust feedback stability analysis. Furthermore, it leads to the observation that feedback stability follows if, and only if, there exist constant multipliers such that the corresponding complementary IQCs hold at frequencies of 0 and ∞, of which the condition in Lanzon and Petersen (2008) that $\bar{\lambda}(G(0)\bar{G}(0)) < 1$, $\bar{G}(\infty) \geq 0$, and $G(\infty)\bar{G}(\infty) = 0$ is a special case. The robust stability result is shown to extend to negative imaginary systems that are only marginally stable, i.e. have poles on the imaginary axis. To this end, a recently developed notion of IQCs for marginally stable systems from Khong, Lovisari, and Rantzer (2016) is employed to conclude closed-loop stability. This paper considers distributed-parameter linear time-invariant systems that admit irrational transfer functions. Such a class of systems corresponds to infinite-dimensional state–space systems in the time domain (Curtain & Zwart, 1995). Furthermore, no explicit state–space realisations are exploited in any of the proofs for the main results. This contrasts the preceding works (Lanzon & Petersen, 2008; Xiong et al., 2010), where state matrices and the negative imaginary lemma (the counterpart to the positive real lemma) are heavily employed. Preliminary results in this direction can be found in Khong, Petersen, and Rantzer (2015), where only sufficient IQC conditions were given for the class of proper real-rational transfer functions. Moreover, the results have been further strengthened in this paper via the removal of an assumption on a certain residual matrix and a reconciliation with the existing results is provided. Note that due to space limitations, the proofs of some of the results have been omitted from this paper for cases in which similar proofs are given in Khong et al. (2015). However, full details of all of the proofs can be found in the archive version of this paper (Khong, Petersen, & Rantzer, 2017). It is noteworthy that similar necessary and sufficient IQC based results for robustness analysis involving time-delays can be found in Scorletti (1997) and the idea of combining IQCs which hold on subsets of the imaginary axis can be located in Jun and Safonov (2002).

The paper evolves along the following lines. Section 2 introduces the notation of the paper and defines the classes of negative imaginary systems considered. Robust stability of feedback interconnections of *stable* negative imaginary systems is examined in Section 3. Sufficient robust stability conditions for negative imaginary systems with imaginary-axis poles are derived in Section 4. The necessity of IQC conditions for feedback stability of negative imaginary systems is established in Section 5, and a reconciliation with the existing robustness results takes place in Section 6. Two numerical examples are given in Section 7 to illustrate the theory. Finally, concluding remarks are provided in Section 8.

2. Notation and preliminaries

The notation used in this paper is defined in this section. Let $\mathbb{R}$ and $\mathbb{C}$ denote, respectively, the real and complex numbers. The real part of an $s \in \mathbb{C}$ is denoted as $\Re(s)$. $\mathbb{C}_+$ denotes the open right half plane and $\bar{\mathbb{C}}_+$ its closure. Given an $A \in \mathbb{C}^{m\times n}$ (resp. $\mathbb{R}^{m\times n}$), $A^* \in \mathbb{C}^{n\times m}$ (resp. $A^T \in \mathbb{R}^{n\times m}$) denotes its complex conjugate transpose (resp. transpose). Denote by $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$, the largest and smallest singular values of matrix A, respectively, and by $\bar{\lambda}(B)$, the spectral radius of B. I_n denotes the identity matrix of dimensions $n \times n$. Subsequently, the subscript n will often be dropped for simplicity.

Let $\mathcal{R}^{n\times m}$ denote the set of real-rational proper transfer function matrices of dimensions $n \times m$ and

$$\mathbf{H}_\infty^{n\times m} := \left\{ X : \mathbb{C} \to \mathbb{C}^{n\times m} \text{ (a.e.)} \,\middle|\, \begin{array}{l} X \text{ is analytic in } \mathbb{C}_+ \\ \sup_{s\in\mathbb{C}_+} \bar{\sigma}(X(s)) < \infty \end{array} \right\}$$

the set of stable transfer functions. The norm of the elements in $\mathbf{H}_\infty$ is denoted $\|\cdot\|_\infty$. Let $\mathbf{C}$ be the class of functions $f : \mathbb{C} \to \mathbb{C}^{n\times m}$ (a.e.) that are continuous on $j\mathbb{R} \cup \{\infty\}$, and $\mathbf{S} := \mathbf{H}_\infty \cap \mathbf{C}$. The positive feedback interconnection of two transfer functions G and $\bar{G}$, denoted by $[G, \bar{G}]$, is described by:

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} I & -\bar{G} \\ -G & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix};$$

see Fig. 1.

Definition 1. A positive feedback interconnection of G and $\bar{G}$ is said to be *internally stable* if

$$\begin{bmatrix} I & -\bar{G} \\ -G & I \end{bmatrix}^{-1} = \begin{bmatrix} I + \bar{G}(I - G\bar{G})^{-1}G & \bar{G}(I - G\bar{G})^{-1} \\ (I - G\bar{G})^{-1}G & (I - G\bar{G})^{-1} \end{bmatrix}$$

is an element in $\mathbf{H}_\infty$.

Define

$$\hat{\mathbf{N}} := \{R \in \mathbf{S}^{n\times n} : \\ j[R(j\omega) - R(j\omega)^*] \geq 0 \; \forall \omega \in (0, \infty)\} \quad \text{and}$$
$$\mathbf{N}_s := \{R \in \mathbf{S}^{n\times n} : \\ j[R(j\omega) - R(j\omega)^*] > 0 \; \forall \omega \in (0, \infty)\} \subset \hat{\mathbf{N}}.$$

$\hat{\mathbf{N}}$ denotes the set of *stable negative imaginary* transfer functions, while $\mathbf{N}_s$ denotes the set of *strictly negative imaginary* transfer functions. The set of stable (strictly) negative imaginary real-rational proper transfer functions defined in Lanzon and Petersen (2008) is a subclass of $\hat{\mathbf{N}}$ ($\mathbf{N}_s$). In particular, an $R \in \mathcal{R} \cap \hat{\mathbf{N}}$ satisfies $R(0) = R(0)^T \in \mathbb{R}^{n\times n}$ and $R(\infty) = R(\infty)^T \in \mathbb{R}^{n\times n}$ (Lanzon & Petersen, 2008, Lem. 2). Therefore, it follows that $j[R(j\omega) - R(j\omega)^*] = 0$ when $\omega = 0$ or $\omega = \infty$. The set of *negative imaginary* transfer functions is defined below.

Definition 2. A transfer function $R : \mathbb{C} \to \mathbb{C}^{n\times n}$ (a.e.) is said to be *negative imaginary* if