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Optimal communication scheduling and remote estimation over an additive noise channel[☆]

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ABSTRACT

This paper considers a sequential sensor scheduling and remote estimation problem with one sensor and one estimator. The sensor makes sequential observations about the state of an underlying memoryless stochastic process, and makes a decision as to whether or not to send this measurement to the estimator. The sensor and the estimator have the common objective of minimizing expected distortion in the estimation of the state of the process, over a finite time horizon. The sensor is either charged a cost for each transmission or constrained on transmission times. As opposed to the prior work where communication between the sensor and the estimator was assumed to be perfect (noiseless), in this work an additive noise channel with fixed power constraint is considered; hence, the sensor has to encode its message before transmission. Under some technical assumptions, we obtain the optimal encoding and estimation policies within the piecewise affine class in conjunction with the optimal transmission schedule. The impact of the presence of a noisy channel is analyzed numerically based on dynamic programming. This analysis yields some rather surprising results such as a phase-transition phenomenon in the number of used transmission opportunities, which was not encountered in the noiseless communication setting.

1. Introduction

The communication scheduling and remote state estimation problem arises in the applications of wireless sensor networks, such as environmental monitoring and networked control systems. As an example of environmental monitoring, researchers at the National Aeronautics and Space Administration (NASA) Earth Science group are interested in monitoring the evolution of the soil moisture, which is used in weather forecast, ecosystem process simulation, etc. (Shuman et al., 2010). In order to achieve that goal, the sensor networks are built over an area of interest. The sensors collect data on the soil moisture and send them to the decision unit at NASA via wireless communication. The decision unit at NASA forms estimates on the evolution of the soil moisture based on the messages received from the sensors. Similarly, in networked control systems, where the objective is to control some remote plants, sensor networks are built to measure the states of the remote plants. Sensors transmit their measurements to the controller via a wireless communication network, and the controller estimates the state of the remote plant and generates a control signal based on that estimate (Hespanha, Naghshtabrizi, & Xu, 2007). In both scenarios, the quality of the remote state estimation strongly affects the quality of decision making at the remote site, that is, weather prediction or control signal generation. The networked sensors are usually constrained by limits on power (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002). They are not able to communicate with the estimator at every time step and thus, the estimator has to produce its best estimate based on the partial information received from the sensors. Therefore, the communication between the sensors and the estimator should be scheduled judiciously, and the estimator should be designed properly, so that the state estimation error is minimized under the communication constraints.

Research on the general sensor scheduling problem dates back to the 1970s. In one of the earliest works (Athans, 1972), the problem formulation is such that only one out of several sensors can be selected at each instant of time to observe the output of a linear stochastic system. Using the measurements over a finite time interval, the goal is to form prediction on some future state of the

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system. Furthermore, each sensor is associated with a certain measurement cost. The author proposed an off-line deterministic sensor scheduling strategy that minimizes the sum of measurement cost over the time interval and prediction error. Gupta, Chung, B. Hassibi, and Murray (2006) studied the sensor scheduling problem over infinite time horizon. Similar to the problem in Athans (1972), only one sensor can be selected at each instant of time. However, there is no measurement cost associated with each sensor. The authors proposed an off-line stochastic sensor scheduling strategy such that the expected steady state estimation error is minimized. Yang and Shi (2011) studied the off-line sensor scheduling problem where there is only one sensor observing the state of a linear stochastic system. The sensor can communicate with the remote estimator only a limited number of times. The objective was to minimize the cumulative estimation error over a finite time horizon. It was shown that the optimal sensor scheduling strategy is to distribute the limited communication opportunities uniformly over the time horizon. The authors of the papers discussed above considered off-line sensor scheduling problems. "Off-line sensor scheduling" means the sensor is scheduled to take observation or conduct communication based on some a priori information about the system (e.g. statistics of random variables, system matrices). The on-line information (e.g. sensor's observation, battery's energy level) is not taken into account when making schedules. Some other selected work on off-line sensor scheduling problems can be found in Mo, Garone, Casavola, and Sinopoli (2011), Ren, Cheng, Chen, Shi, and Sun (2013) and Shi and Zhang (2012).

With the advances in hardware devices, sensors are endowed with stronger computational capabilities. Consequently, the sensors are able to make schedules based on all the information they have (a priori information as well as on-line information), which motivates the formulation of on-line sensor scheduling problems. Åström and Bernhardsson (2002) considered a state estimation problem with a first-order stochastic system. They compared the estimation error over infinite time horizon obtained by periodic sampling and threshold event-triggered sampling. The periodic sampling is one of the off-line sensor scheduling strategies while the threshold event-triggered sampling is one of the on-line sensor scheduling strategies. They showed that the threshold event-triggered sampling, which is also called "threshold-based communication strategy", leads to better performance in state estimation compared with periodic sampling. The global optimality of threshold-based communication strategy in this context is proved later by Nar and Başar (in press). Imer and Başar (2010) considered the on-line sensor scheduling and remote state estimation problem over a finite time horizon. In the formulation, the sensor is restricted to communicate only a limited number of times. By considering the communication strategies within the class of threshold-based strategies, the paper has shown that there exists a unique threshold-based communication strategy achieving the best performance on remote state estimation. Furthermore, the optimal threshold can be computed by solving a dynamic programming equation. Bommannavar and Başar (2008) later extended the result of Imer and Başar (2010) to multi-dimensional systems. The continuous-time version of the problem in Imer and Başar (2010) has been studied by Rabi, Moustakides, and Baras (2006). Xu and Hespanha (2004) considered the networked control problem involving state estimation and communication scheduling, which can be viewed as a sensor scheduling and remote estimation problem. They fixed the estimator to be Kalman-like and designed an event-triggered sensor that minimizes the time average of the sum of the communication cost and estimation error over infinite time horizon. They showed that the optimal communication strategy is deterministic and stationary, and is a function of the estimation error. Wu, Jia, Johansson, and Shi (2013) considered the sensor scheduling and estimation problem subject to constraints on the average communication rate over infinite time horizon. The authors assumed that the sensor has noisy observations on the system state. By restricting the sensor scheduling strategies to the threshold event-triggered class, they derived the exact minimum mean square error (MMSE) estimator. However, the exact MMSE estimator is nonlinear and thus computationally intractable. Under a Gaussian assumption on the a priori distribution, the authors derived an approximate MMSE estimator, which is Kalman-like. Based on the approximated MMSE estimator, the authors derived conditions on the thresholds so that the average sensor communication rate will not exceed its upper bound. You and Xie (2013) extended the work in Wu et al. (2013) by deriving conditions on the thresholds so that the estimator is stable. Han, Mo, Wu, Weerakkody, Sinopoli, and Shi (2015) showed that if the sensor is fixed to apply some stochastic event-triggered strategy, then the exact MMSE estimator is Kalman-like. Other selected work on remote estimation with event-based sensor operations can be found in Shi, Elliott, and Chen (2016) and Weerakkody, Mo, Sinopoli, Han, and Shi (2013). The work in Han et al. (2015), Wu et al. (2013) and You and Xie (2013) can also be viewed as Kalman-filtering with scheduled observations, which is related to Kalman-filtering with intermittent observations studied in Sinopoli, Schenato, Franceschetti, Poolla, Jordan, and Sastry (2004) and You, Fu, and Xie (2011).

The approaches of Wu et al. (2013) and Xu and Hespanha (2004) involved fixing the communication strategies or estimation strategies to be of a certain type and then deriving the corresponding optimal estimation strategies and communication strategies, respectively. The approach of Imer and Başar (2010), on the other hand, is to derive the jointly optimal communication strategies and estimation strategies. Similarly, Lipsa and Martins (2011) considered the sensor scheduling and remote estimation problem where the sensor is not constrained by communication times but is charged a communication cost. They proposed a threshold event-triggered sensor and a Kalman-like estimator and proved that the proposed sensor and estimator are jointly optimal, minimizing the sum of communication cost and estimation error over a finite time horizon. Nayyar, Başar, Teneketzis, and Veeravalli (2013) considered a similar problem where the sensor is equipped with an energy harvesting sensor. In the work of Nayyar et al. (2013), the problem formulation is such that the sensor is constrained by the energy level of the battery and is also charged a communication cost. It is shown in Nayyar et al. (2013) that an energy dependent threshold event-triggered sensor and a Kalman-like estimator are jointly optimal. Hence, the result of Nayyar et al. (2013) can be viewed as generalization of the results of Imer and Başar (2010) and Lipsa and Martins (2011). In both Lipsa and Martins (2011) and Nayyar et al. (2013), majorization theory was used to prove the optimality of the respective results, which is closely related to the approach in Hajek, Mitzel, and Yang (2008).

It is worth drawing attention to the two different types of constraints that arise in the works mentioned above – hard and soft constraints – as featured in the problem setups of Imer and Başar (2010) and Lipsa and Martins (2011). In the problem of Imer and Başar (2010), the sensor can only communicate for a prespecified number of times. Such a communication constraint is called *hard constraint*. In the work of Lipsa and Martins (2011), however, the sensor is charged a communication cost. This kind of communication constraint is called *soft constraint*. In the problem with hard constraint, the communication strategy must take the remaining communication opportunities as a variable and schedule no communication if there is no remaining opportunity. Such communication strategies guarantee that the number of transmissions made over the time horizon of interest will not exceed the given constraint. In the problem with soft constraint, however, the sensor is not constrained by the number of transmissions,