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Brief paper

Global stabilization of feedforward nonlinear time-delay systems by bounded controls ☆

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ABSTRACT

The problem of global stabilization for a family of feedforward nonlinear time-delay systems by bounded controls is considered. Based on a special canonical form of the considered nonlinear system, two types of new nonlinear control laws are proposed to achieve global stabilization. The new special canonical form used in this paper contains not only time delay in the input but also time delays in the state, which leads to natural cancellation in the recursive design. Moreover, some free parameters are introduced into these controllers. These advantages can help to simplify the proof for the global stability of the closed-loop system and improve the transient performance of the closed-loop system significantly. A practical example is given to illustrate the effectiveness of the proposed approaches.

1. Introduction

The existence of time delay can cause performance deterioration and even instability of the control system (Hale, 1977). Therefore, control of time-delay systems has been an active research topic for several decades, and many effective approaches have been established to handle various problems (see Du, Lam, and Shu, 2010; Koo and Choi, 2015, 2016; Krstic, 2010; Meng, Lam, Du, and Gao, 2010 and the references therein). On the other hand, practical control systems are subject to input saturation. Ignoring the saturation nonlinearity in controller design can degrade the system performance of the resulting closed-loop system when the saturation occurs and may even lead to instability. Thus, much research has been devoted to deal with saturation nonlinearity because of its significant influence (see Marchand & Hably, 2005; Sussmann, Sontag, & Yang, 1994; Teel, 1992; Wang, Xue, & Lu, 2015; Zhou & Duan, 2009 and the references therein). It is thus natural to consider the problem of stabilization of control systems by bounded and delayed controls, which, as we can expect, is even more difficult than the problems of stabilizing control systems by either bounded or delayed controls, and only few results are available in the literature (see Mazenc, Mondié, & Niculescu, 2003; Yakoubi & Chitour, 2007; Zhou, Duan, & Lin, 2010 and the references cited there).

Feedforward nonlinear systems, which have an upper triangular structure, are an important class of nonlinear systems (Francisco, Mazenc, & Mondié, 2007; Ye & Wang, 2007). For example, both the planar vertical takeoff and landing (PVTOL) aircraft model and the inertia wheel pendulum (IWP) model can be transformed into chains of integrators with nonlinear perturbations, which are special feedforward nonlinear systems. During the past two decades, many important stabilization results have been proposed for feedforward nonlinear systems (see Choi & Lim, 2010; Jo, Choi, & Lim, 2014; Ye, 2003; Zhang, Feng, & Sun, 2012; Zhang, Liu, Baron, & Boukas, 2011; Zhang, Liu, Feng, & Zhang, 2013 and the references therein). Motivated by Teel's forwarding design (Teel, 1992), Mazenc et al. firstly solved the global stabilization problem for the multiple integrators system by using bounded and delayed controls in Mazenc et al. (2003). Later on, these results were extended to a family of feedforward nonlinear systems with delay and saturation in the input in Mazenc, Mondié, and Francisco (2004). Also inspired by Teel's forwarding design (Teel, 1992), an adaptive stabilizer was proposed to solve the stabilization problem for feedforward nonlinear systems with time delays in Ye (2011), where the stabilizer consisted of a nested saturation feedback, and a set of switching logics were designed to tune online the saturation levels in a switching manner. Based on the transformed nonlinear system given by Mazenc et al. (2004), a new nonlinear control law consisting of cascade saturation functions was proposed in Ye, Jiang, Gui, and Yang (2012). Recently, in Ye

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(2014) a saturated and delayed controller was proposed to stabilize globally a class of feedforward nonlinear systems whose nominal dynamics is the cascade of multiple oscillators and multiple integrators. In Zhou and Yang (2016), we revisited the problem of global stabilization of multiple integrators system by using delayed and bounded controls, and three globally stabilizing nonlinear control laws were proposed based on some special canonical forms. Motivated by the results in Zhou and Yang (2016), the problem of global stabilization of the multiple oscillator system by using delayed and bounded controls was considered in Yang, Zhou, and Lam (2017), in which a nonlinear control law consisting of nested saturation functions was proposed.

In this paper, we consider the problem of bounded feedback global stabilization for a family of feedforward nonlinear time-delay systems. Motivated by our recent results in Zhou and Yang (2016), two classes of nonlinear control laws consisting of nested/cascade saturation functions are respectively proposed to solve the problem based on a new special canonical form, which contains both the current and the delayed state vectors. The main contribution of this paper and the significance of the obtained results can be stated as follows. Firstly, as pointed out in Zhou and Yang (2016), for the nominal dynamics of the considered feedforward nonlinear systems, the proposed special canonical form contains time delays in its state, which, because of the existence of time delay in the input, allows us to cancel all the other state components at every step of the recursive design so that only a scalar system decoupled from the other state components is required to be handled in every step. This is different from the design in Mazenc et al. (2004), which needs to consider a scalar time-delay system coupled with the former state components in every step of the recursive design. Moreover, the proposed controllers also contain some free parameters that can be well designed to improve the control performance. Secondly, the design approach proposed in this paper can deal with feedforward nonlinear systems whose nonlinearities contain not only the current states but also the delayed states, which are more general than the systems considered in Mazenc et al. (2004) and Ye et al. (2012). Finally, although the invertible state transformation used in this paper is similar to that in Zhou and Yang (2016), there still exist some difficulties because of the presence of the unknown nonlinearities. For example, due to the specific characteristic of the invertible transformation, the constraints imposed on the nonlinearities in the transformed system depend on the delayed state vectors (see Eq.(12)), which make the analysis more difficult than that in Zhou and Yang (2016). Moreover, compared with Zhou and Yang (2016), the recursive design in this paper is more challenging since the dynamics of the state y_i depends on the other states $y_j, j \geq i$ at the step i because of the presence of the nonlinearities, which leads to a quite complicated analysis. As a result, the resulting closed-loop system is still a nonlinear time-delay system and its stability should be verified carefully.

Notation: The notation used in this paper is fairly standard. For two integers p and q with $p \leq q$, the symbol $\mathbf{I}[p,q]$ refers to the set $\{p, p+1, \ldots, q\}$. Let $X_i = (x_i, x_{i+1}, \ldots, x_n, x_{n+1})^{\mathrm{T}}$, for any $i \in \mathbf{I}[1, n+1]$, where $x_{n+1} = u$. For a positive constant ε, $\sigma_\varepsilon(x) \triangleq \varepsilon \operatorname{sign}(x) \min\{|x/\varepsilon|, 1\}$ denotes the standard saturation function. The notation $|\cdot|$ refers to both the usual Euclidean norm for vectors and the induced 2-norm for matrices. For any two integers p and q and any functions g_i, $i \in \mathbf{I}[p,q]$, we denote $\Sigma_{i=p}^{q} g_i = 0$ if $q < p$. At last, for any constants a and b with $b \geq a$, we let $y_{[a,b]} = y(s)$, $s \in [a,b]$ and $|y|_{[a,b]} \triangleq \sup_{s\in[a,b]} |y(s)|$.

2. Problem formulation and preliminaries

In this paper, we consider the following feedforward nonlinear system

$$\begin{cases} \dot{x}_1(t) = a_2 x_2(t-h_2) + \mathcal{L}_1\left((X_3)_{[t-h,t]}\right) + f_1, \\ \quad\vdots \\ \dot{x}_{n-1}(t) = a_n x_n(t-h_n) + \mathcal{L}_{n-1}\left((X_{n+1})_{[t-h,t]}\right) + f_{n-1}, \\ \dot{x}_n(t) = a_{n+1} x_{n+1}(t-h_{n+1}) + f_n, \end{cases} \tag{1}$$

in which $f_i = f_i((X_{i+1})_{[t-r,t]})$, $i \in \mathbf{I}[1,n]$, and $\mathcal{L}_k(\cdot)$, $k \in \mathbf{I}[1, n-1]$ are linear operators defined by

$$\mathcal{L}_k\left((X_{k+2})_{[t-h,t]}\right) = \sum_{j=k+2}^{n+1} \sum_{i=1}^{m_{kj}} a_{kji} x_j\left(t - h_{kji}\right),$$

where $n \geq 2$, $m_{kj} \geq 1$ are integers, a_i, a_{kji} are known constants satisfying $a_i \neq 0$, $\{h_i, h_{kji}\}$ are known non-negative numbers, $h = \max\{h_{kij}\}$, $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector, $x_{n+1} = u \in \mathbf{R}$ is the control input, and r is a non-negative constant that can be unknown. The functions $f_i(\cdot)$, $i \in \mathbf{I}[1,n]$ are continuous and satisfy the following assumption.

Assumption 1. There exist positive scalars ϕ_i, $i \in \mathbf{I}[1,n]$ such that

$$\left|f_i\left((X_{i+1})_{[t-r,t]}\right)\right| \leq \phi_i |X_{i+1}|^2_{[t-r,t]}, \tag{2}$$

whenever $|X_{i+1}|_{[t-r,t]} \leq 1$.

Our main work is to solve the following problem:

Problem 1. Find a state feedback control u satisfying $|u| \leq 1$ such that the closed-loop system is globally asymptotically stable and locally exponentially stable at the origin.

Remark 1. The upper bounds "1" in Assumption 1 and Problem 1 can be replaced by any given positive constant ρ. For example, if we study Problem 1 with $|u| \leq \rho$ for system (1) satisfying Assumption 1, then by the change of variable $v = u/\rho$, the system still satisfies Assumption 1 where the scalars ϕ_i are updated accordingly.

In the absence of state delay, and when $a_i = 1$ and $a_{kji} = 0$, the nonlinear system (1) becomes

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) + f_i\left(\underline{x}_{i+1}(t)\right), \ i \in \mathbf{I}[1, n-1], \\ \dot{x}_n(t) = u(t - h_{n+1}), \end{cases}$$

and, in this case, condition (2) reduces to $|f_i(\underline{x}_{i+1}(t))| \leq M|\underline{x}_{i+1}(t)|^2$, whenever $|\underline{x}_{i+1}(t)| \leq 1$, where $\underline{x}_i = (x_i, x_{i+1}, \ldots, x_n)^{\mathrm{T}}$ and M is a given constant. For this system the corresponding Problem 1 has been solved in Mazenc et al. (2004), where a nonlinear control law consisting of nested saturation functions was proposed. The controller established in Mazenc et al. (2004) satisfies $|u| \leq \varepsilon^*$ with $\varepsilon^* = 1/(20(kh_{n+1})^n)$, where $k = \max\{16n^3[4n\sqrt{n}(1+n^2)^{n-1}+1], 4(20)^{n+1}n(n+2)\}$. It follows that the saturation level decreases sharply as n increases (h_{n+1} is fixed), which indicates that the actuator capacity may not be fully utilized when n is relatively large. Based on a canonical form introduced in Mazenc et al. (2004), a new nonlinear control law consisting of cascade saturation functions was proposed in Ye et al. (2012) recently. In order to guarantee the stability of the closed-loop system, the saturation level of the controller established there is required to be sufficiently low.

In this paper, motivated by our recent results in Zhou and Yang (2016) and Yang et al. (2017), based on a novel canonical form for the feedforward nonlinear system (1), two new nonlinear control laws will be proposed to solve Problem 1. Compared with the canonical form introduced in Mazenc et al. (2004), the canonical