Automatica 88 (2018) 31–37

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Rendezvous with connectivity preservation of mobile agents subject to uniform time-delays[☆]

Maobin Lu

Department of Mechanical and Biomedical Engineering, City University of Hong Kong, Kowloon, Hong Kong SAR, China

ARTICLE INFO

Article history:
Received 28 May 2016
Received in revised form 16 April 2017
Accepted 20 September 2017

Keywords:
Connectivity preservation
Rendezvous
Multi-agent systems
Distributed control
Time-delay

ABSTRACT

This paper investigates the problem of rendezvous with connectivity preservation for a group of mobile agents subject to both unknown input delay and unknown communication delay. We propose a new potential function and design a potential function based distributed control law. To tackle unknown time-delays, we introduce a control gain in the control law and develop an energy functional to characterize the energy of the whole system. By analyzing the energy change, the control gain is designed. It is shown that as long as the communication network is connected at the initial time, the proposed control law can make all agents reach the same location and maintain connectivity of the communication network for all time. Meanwhile, it can accommodate arbitrarily bounded uniform constant input delay and communication delay.

1. Introduction

For the past decade, the problem of rendezvous with connectivity preservation has been extensively studied (Dong & Huang, 2013, 2014, 2017; Fan & Hu, 2015; Fan, Liu, Feng, Song, & Wang, 2013; Ji & Egerstedt, 2005, 2007; Su, 2015; Su, Wang, & Chen, 2010). Its control objective is to design some distributed control law such that under limited sensing and communication ranges, the mobile agents can reach the same location, and at the same time, maintain connectivity of the communication network for all time. The connectivity preserving feature distinguishes the rendezvous problem from many other cooperative control problems, such as consensus, formation and synchronization (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber & Murray, 2004), where connectivity of the communication network for all time is given as a basic assumption.

The problem of rendezvous with connectivity preservation is first introduced in Ji and Egerstedt (2005, 2007) for single integrator multi-agent systems. In particular, it proposes a potential function approach and develops a potential function based distributed control law to solve the problem. Then, along this line, the problem of rendezvous with connectivity preservation is studied under various conditions. In particular, Su et al. (2010) consider leaderless and leader-following rendezvous problem with connectivity preservation for double integrator multi-agent systems. The similar problem is studied in Dong and Huang (2013, 2014), where external disturbances are tackled by the output regulation approach. Recently, a distributed internal model approach is proposed in Su (2015) to handle the problem of rendezvous with connectivity preservation subject to both external disturbances and plant uncertainties. In addition, a constraint function approach is adopted in Fan and Hu (2015) and Fan et al. (2013) to solve the problem of rendezvous with connectivity preservation. It is worth mentioning that the connectivity preservation problem is also studied in Chen, Fan, and Zhang (2015), Kantaros and Zavlanos (2017), Poonawala and Spong (2017), Su, Wang, and Chen (2009) and Zavlanos and Pappas (2007).

In regard to cooperative control of multi-agent systems, a basic yet challenging problem is to take time-delays into account. In fact, both input delay and communication delay pose great threats to the system stability. The influence of time-delays has been considered in some cooperative control problems, see, e.g., Moreau (2004) and Olfati-Saber and Murray (2004). However, to the best of our knowledge, the time-delay has never been considered in the problem of rendezvous with connectivity preservation. It is known that the problem of rendezvous with connectivity preservation can be solved by the potential function based approach and the key to this approach is to guarantee that an energy function of the system is non-increasing under the designed distributed control law (Ji & Egerstedt, 2005). However, the existence of time-delays makes it extremely difficult to achieve this objective.

In this paper, we consider the problem of rendezvous with connectivity preservation for single integrator multi-agent systems subject to both unknown input delay and unknown communication delay. In contrast to existing results for similar problems,

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Bert Tanner under the direction of Editor Christos G. Cassandras.

E-mail address: maobinlu@cityu.edu.hk.

https://doi.org/10.1016/j.automatica.2017.11.003
0005-1098/

say Dong and Huang (2013, 2014), Ji and Egerstedt (2007), Su (2015) and Su et al. (2009), the current problem is much more challenging. On one hand, the system under consideration is a time-delay system. Thus, instead of the energy function, an energy functional has to be constructed to characterize the energy of the whole system. As a consequence, it makes the problem much more complex. To guarantee that the energy functional is non-increasing, one parameter is introduced in the distributed controller, which depends on the maximal energy of the whole system. On the other hand, as in Dong and Huang (2013, 2014) and Ji and Egerstedt (2007), the effect of hysteresis is introduced to allow the creation of new communication links in the time-varying network. Therefore, the change of the energy functional at each switching time instant has to be analyzed to obtain the maximal energy, which has never been encountered in any similar problem. To tackle these challenges, a sequence of technical lemmas are established to lay the foundation of our approach. It is shown that the proposed distributed control law can make all agents reach the same location, and at the same time, maintain connectivity of the communication network as long as it is connected at the initial time. Meanwhile, it can accommodate arbitrarily bounded uniform constant input delay and communication delay.

The remainder of this paper is organized as follows. Section 2 gives the problem formulation for rendezvous with connectivity preservation. Section 3 introduces some technical lemmas and presents our main result. Section 4 illustrates our design by one example. Finally, some conclusions are made to end this paper in Section 5.

Notation. For $x_i \in \mathbb{R}^n$, $i = 1, \ldots, m$, $\mathrm{col}(x_1, \ldots, x_m) = [x_1^T, \ldots, x_m^T]^T$. $\mathbf{1}_N$ denotes the column vector of dimension N with all its elements being 1. For a symmetric real matrix T, $\lambda(T)$ denotes the eigenvalue of T.

2. Problem formulation and preliminaries

Consider a group of mobile agents with the dynamics of each agent being described by the single integrator as follows:

$$\dot{x}_i(t) = u_i(t - \tau_a),\ i = 1, \ldots, N \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and the control input of the ith agent, respectively, and τ_a is the input delay.

Associated with the multi-agent system (1), we can define a time-varying graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$, where the node set $\mathcal{V} = \{1, \ldots, N\}$ with node i associated with the ith agent, and the edge set $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. The edge set $\mathcal{E}(t)$ is defined by the following rules.

Given any $r > 0$ and $0 < r_0 < r$, the edge set $\mathcal{E}(t)$ is defined such that for any $t \le 0$, $\mathcal{E}(t) = \{(i,j) |\ \|x_i(t) - x_j(t)\| < (r - r_0), i \neq j, i, j = 1, \ldots, N\}$; and for any $t > 0$,

1) if $\|x_i(t) - x_j(t)\| \ge r$, then $(i,j) \notin \mathcal{E}(t)$;
2) if $(i,j) \notin \mathcal{E}(t^-)$ and $\|x_i(t) - x_j(t)\| < (r - r_0)$, then $(i,j) \in \mathcal{E}(t)$;
3) if $(i,j) \in \mathcal{E}(t^-)$ and $\|x_i(t) - x_j(t)\| < r$, then $(i,j) \in \mathcal{E}(t)$.

Then, the neighbor set of agent i at time t can be defined as $\mathcal{N}_i(t) = \{j | (j,i) \in \mathcal{E}(t)\}$.

Remark 2.1. The definition of the graph $\mathcal{G}(t)$ is similar to that in Dong and Huang (2013) and Ji and Egerstedt (2007). The physical meaning of r is the sensing and communication range of the mobile agents and the switching threshold r_0 is to introduce the effect of hysteresis.

Consider the distributed control laws of the form

$$u_i(t) = f_i\left(x_i(t - \tau_c) - x_j(t - \tau_c)\right), j \in \mathcal{N}_i(t), i = 1, \ldots, N \tag{2}$$

where $f_i(\cdot)$ are some nonlinear functions vanishing at the origin and τ_c is the communication delay in the network. Let $\tau = \tau_a + \tau_c$. It is assumed that $\tau \le \bar{\tau}$ for some known positive real number $\bar{\tau}$.

Now, the problem of rendezvous with connectivity preservation is defined as follows.

Problem 2.1. Given the multi-agent system (1), $r > 0$ and $r_0 \in (0, r)$, and arbitrarily bounded input delay τ_a and communication delay τ_c, find the distributed control laws of the form (2) such that, for any initial condition that makes $\mathcal{G}(\theta), \theta \in [-2\tau, 0]$, connected, the closed-loop system has the following properties:

1) $\mathcal{G}(t)$ is connected for all $t \ge 0$;
2) $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0,\ i, j = 1, \ldots, N$.

Remark 2.2. It has been shown in Ji and Egerstedt (2007) that when the input delay $\tau_a = 0$ and the communication delay $\tau_c = 0$, Problem 2.1 can be solved by the distributed control laws of the form (2). However, as is shown later, the existence of time-delays makes the problem much more challenging.

Remark 2.3. It is well known that the problem of rendezvous with connectivity preservation is studied under the condition that the graph at the initial time is connected. Thus, in what follows, without additional emphasis, the initial graph $\mathcal{G}(\theta), \theta \in [-2\tau, 0]$, is always assumed to be connected.

3. Main result

To achieve connectivity preservation, we adopt the potential function approach to design the control law. In particular, we propose a potential function as follows:

$$\psi(s) = \frac{s^2}{r^2 - s^2 + \frac{r^2}{Q}},\ 0 \le s \le r \tag{3}$$

where Q is some positive real number. The potential function (3) is inspired by the one in Su et al. (2010) and that in Su (2015). It is noted that the function $\psi(s)$ in (3) is nonnegative, differentiable and bounded for $s \in [0, r]$, and it satisfies four properties: 1) $\frac{d\psi(s)}{ds} > 0$, for $s \in (0, r)$; 2) $\lim_{s\to0^+}\left(\frac{d\psi(s)}{ds}\cdot\frac{1}{s}\right)$ is nonnegative and bounded; 3) For any $a_0 > 0$ and $0 < r_0 < r$, there exists Q such that $a_0\psi(r - r_0) < \psi(r) = Q$; 4) $\lim_{s\to0^+}\left(\frac{d(\frac{d\psi(s)}{ds}\cdot\frac{1}{s})}{ds}\cdot\frac{1}{s}\right)$ is bounded. In fact, in existing literature, all kinds of potential functions have been proposed (Ji & Egerstedt, 2007; Su et al., 2010). However, those potential functions do not satisfy property 4), which is instrumental to the construction of a bounded energy functional defined later.

The distributed control laws of the form (2) are defined as follows:

$$u_i(t) = -k\sum_{j\in\mathcal{N}_i(t)}\nabla_{x_i}\psi(\|x_i(t - \tau_c) - x_j(t - \tau_c)\|) \tag{4}$$

where $i = 1, \ldots, N$, and k is some positive real number to be determined.

For $i = 1, \ldots, N, j \in \mathcal{N}_i(t)$, denote $x_{ij}(t) = x_i(t) - x_j(t)$, and $\rho_{ij}(t) = \nabla_{x_i}\psi(\|x_{ij}(t)\|)$. For $i, j = 1, \ldots, N$, let

$$w_{ij}(t) = \begin{cases} \dfrac{2(r^2 + \frac{r^2}{Q})}{\left(r^2 - \|x_{ij}(t)\|^2 + \frac{r^2}{Q}\right)^2}, & \text{if } (j,i) \in \mathcal{E}(t) \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Then, it is obtained that

$$\rho_{ij}(t) = \nabla_{x_i}\psi(\|x_{ij}(t)\|) = w_{ij}(t)x_{ij}(t). \tag{6}$$