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Identifiability of linear dynamic networks[☆]

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ABSTRACT

Dynamic networks are structured interconnections of dynamical systems (modules) driven by external excitation and disturbance signals. In order to identify their dynamical properties and/or their topology consistently from measured data, we need to make sure that the network model set is identifiable. We introduce the notion of *network identifiability*, as a property of a parametrized model set, that ensures that different network models can be distinguished from each other when performing identification on the basis of measured data. Different from the classical notion of (parameter) identifiability, we focus on the distinction between network models in terms of their transfer functions. For a given structured model set with a pre-chosen topology, identifiability typically requires conditions on the presence and location of excitation signals, and on presence, location and correlation of disturbance signals. Because in a dynamic network, disturbances cannot always be considered to be of full-rank, the reduced-rank situation is also covered, meaning that the number of driving white noise processes can be strictly less than the number of disturbance variables. This includes the situation of having noise-free nodes.

1. Introduction

Dynamic networks are structured interconnections of dynamic systems and they appear in many different areas of science and engineering. Because of the spatial connections of systems, as well as a trend to enlarge the scope of control and optimization, interesting problems of distributed control and optimization have appeared in several domains of applications, among which robotic networks, smart grids, transportation systems, multi agent systems etcetera. An example of a (linear) dynamic network is sketched in Fig. 1, where excitation signals r and disturbance signals v, together with the linear dynamic modules G induce the behavior of the node signals w.

When structured systems like the one in Fig. 1 become of interest for analyzing performance and stability, it is appropriate to also consider the development of (data-driven) models. In system identification literature, where the majority of the work is focused on open-loop or feedback controlled (multivariable) systems, there is an increasing interest in data-driven modeling problems related to dynamic networks. Particular questions that can be addressed are, e.g.:

(a) Identification of a single selected module G_{ji}, on the basis of measured signals w and r;
(b) Identification of the full network dynamics;
(c) Identification of the topology of the network, i.e. the Boolean interconnection structure between the several nodes w_i.

The problem (a) of identifying a single module in a dynamic network has been addressed in Van den Hof, Dankers, Heuberger, and Bombois (2013), where a framework has been introduced for prediction error identification in dynamic networks, and classical closed-loop identification techniques have been generalized to the situation of structured networks. Using this framework, predictor input selection (Dankers, Van den Hof, Heuberger, & Bombois, 2016) has been addressed to decide on which node signals need to be measured for identification of a particular network module. Errors-in-variables problems have been addressed in Dankers, Van den Hof, Bombois, and Heuberger (2015) to deal with the situation when node signals are measured subject to additional sensor noise.

The problem (b) of identifying the full network can be recast into a multivariable identification problem, that can then be

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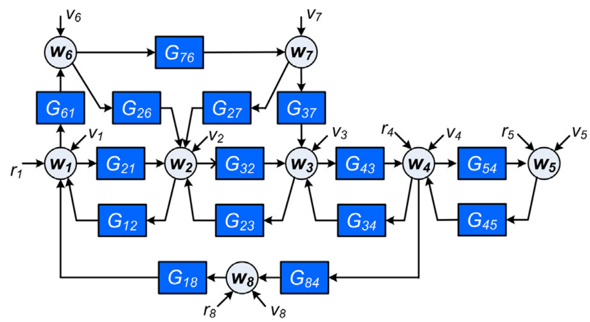

Fig. 1. Dynamic network where node variables w_i are the outputs of the summation points indicated by circles.

addressed with classical identification methods (Söderström & Stoica, 1989). Either structured model sets can then be used, based on an a priori known interconnection structure of the network, or a fully parametrized model set, accounting for each and every possible link between node signals.

The problem (c) of topology detection has been addressed in e.g. Materassi and Salapaka (2012) where Wiener filters have been used to reconstruct the network topology. In Chiuso and Pillonetto (2012) a Bayesian viewpoint has been taken and regularization techniques have been applied to obtain sparse estimates. Topology detection in a large scale network has been addressed in Sanandaji, Vincent, and Wakin (2011, 2012) using compressive sensing methods, and in a biological network in Yuan (2012) and Yuan, Stan, Warnick, and Gonçalves (2011) using also sparse estimation techniques. Causal inference has been addressed in Quinn, Kiyavash, and Coleman (2011).

Not only in problem (b) but also in problem (c), the starting point is most often to model all possible links between node signals, in other words to parametrize all possible modules G_{ji} in the network. However when identifying such a full network model, care has to be taken that different network models can indeed be distinguished on the basis of the data that is available for identification. In Adebayo et al. (2012) and Gonçalves and Warnick (2008) specific local conditions have been formulated for injectivity of the mapping from the network transfer function (transfer from external signals r to node signals w) to network models. This is done outside an identification context and without considering (non-measured) disturbance inputs. Uniqueness properties of a model set for purely stochastic networks (without external excitations r) have been studied in Hayden, Yuan, and Gonçalves (2013) and Materassi and Salapaka (2012) where the assumption has been made, like in many of the works in this domain, that each node is driven by an independent white noise source.

In this paper we are going to address the question: under which conditions on the experimental setup and choice of model set, different network models in the set can be distinguished from each other on the basis of measured data? The typical conditions will then include presence and location of external excitations, presence of and modeled correlations between disturbance signals, and modeled network topology.

This question will be addressed by introducing the concept of network identifiability as a property of a parametrized set of network models. We will study this question for the situations that

- Disturbance terms v_i are allowed to be correlated over time but also over node signals, i.e. v_i and v_j, $i \neq j$ can be correlated.
- The vector disturbance process $v := [v_1^T \; v_2^T \cdots]^T$ can be of reduced-rank, i.e. has a driving white noise process that has a dimension that is strictly less than the dimension of v. This includes the situation that some disturbance terms can be 0.
- Direct feedthrough terms are allowed in the network modules.

The presence of possible correlations between disturbances, limits the opportunities to break down the modeling of the network into several multi-input single-output MISO) problems, as e.g. done in Van den Hof et al. (2013). For capturing these correlations among disturbances all relevant signals will need to be modeled jointly in a multi-input multi-output (MIMO) approach.

If the size of a dynamic network increases, the assumption of having a full rank noise process becomes more and more unrealistic. Different node signals in the network are likely to experience noise disturbances that are highly correlated with and possibly dependent on other node signals in its direct neighborhood. One could think e.g. of a network of temperature measurements in a spatial area, where unmeasured external effects (e.g. wind) affect all measured nodes in a strongly related way. In the identification literature little attention is paid to this situation. In a slightly different setting, the classical closed-loop system (Fig. 3) also has this property, by considering the input to the process G to be disturbance-free, rendering the two-dimensional vector noise process of reduced-rank. Closed-loop identification methods typically work around this issue by either replacing the external excitation signal r by a stochastic noise process, as e.g. in the joint-IO method (Caines & Chan, 1975), or by only focussing on predicting the output signal and thus identifying the plant model (and not the controller), as e.g. in the direct method (Ljung, 1999). In econometrics dynamic factor models have been developed to deal with the situation of high dimensional data and rank-reduced noise (Deistler, Anderson, Filler, Ch. Zinner, & Chen, 2010; Deistler, Scherrer, & Anderson, 2015).

The notion of identifiability is a classical notion in system identification, but the concept has been used in different settings. The classical definition as present in Ljung (1976) and Söderström, Ljung, and Gustavsson (1976) is a consistency-oriented concept concerned with estimates converging to the true underlying system (system identifiability) or to the true underlying parameters (parameter identifiability). In the current literature, identifiability has become a property of a parametrized model set, referring to a unique one-to-one relationship between parameters and predictor model, see e.g. Ljung (1999). As a result a clear distinction has been made between aspects of data informativity and identifiability. For an interesting account of these concepts see also the more recent work (Bazanella, Gevers, & Miskovic, 2010). In the current literature the structure/topology of the considered systems has been fixed and restricted to the common open-loop or closed-loop cases. In our network situation we have to deal with additional structural properties in our models. These properties concern e.g. the choices where external excitation and disturbance signals are present, and how they are modeled, whether or not disturbances can be correlated, and whether modules in the network are known and fixed, or parametrized in the model set. In this paper we will particularly address the structural properties of networks, and we will introduce the concept of network identifiability, as the ability to distinguish networks models in identification. Rather than focussing on the uniqueness of parameters, we will focus on uniqueness of network models.

We are going to employ the dynamic network framework as described in Van den Hof et al. (2013), and we will introduce and analyze the concept of *network identifiability* of a parametrized model set. We will build upon the earlier introduction of the problem and preliminary results presented in Weerts, Dankers, and Van den Hof (2015) and Weerts, Van den Hof, and Dankers (2016b), but we will reformulate the starting points and definitions, as well as extend the results to more general situations in terms of correlated noise, reduced-rank noise, and absence of delays in network modules.