Automatica 88 (2018) 48–56

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

An incremental harmonic balance-based approach for harmonic analysis of closed-loop systems with Prandtl–Ishlinskii operator☆

Lei Fang[a], Jiandong Wang[b,*], Xiaobo Tan[c]

[a] College of Engineering, Peking University, Beijing, China
[b] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China
[c] Smart Microsystems Lab, Department of Electrical and Computer Engineering, Michigan State University, East Lansing, MI, USA

ARTICLE INFO

Article history:
Received 8 March 2016
Received in revised form 17 March 2017
Accepted 24 May 2017

Keywords:
Nonlinear system analysis
Harmonic response
Hysteresis
Incremental harmonic balance

ABSTRACT

Analyzing hysteretic systems presents a significant challenge due to the memory effect of hysteresis. In this paper we present an incremental harmonic balance (IHB)-based approach to compute the steady-state response of a closed-loop system with hysteresis under a sinusoidal excitation, where the hysteresis element is modeled by the Prandtl–Ishlinskii (PI) operator. While the describing function method (DFM) can be used to obtain an approximate solution for the closed-loop system based on first-order harmonics, the proposed IHB-based approach iteratively calculates the harmonic components of the hysteretic system up to an arbitrary order. The main challenge is the harmonic calculation of the periodic output of the PI operator for a multi-harmonic input. In order to address this problem, an alternative definition of the play operator is utilized as the hysteron for the PI operator. By using the alternative definition, a set of switching time instants, when the play operator enters or exits the boundary region, are determined by a bisection method. The calculation of the incremental harmonic components is finally reformulated as a linear matrix equality that can be solved efficiently. As an illustration, numerical results for a system involving a proportional–integral feedback controller are presented to demonstrate the advantage of the IHB-based approach over the DFM in approximating the harmonic response of the hysteretic system.

1. Introduction

Modeling and control of hysteretic systems has gained significant attention over the past several decades (Janaideh, Rakotondrabe, & Tan, 2016; Tan & Iyer, 2009). One reason for the rapid development is the wide application of smart materials, which exhibit considerable hysteretic behaviors (Bertotti & Mayergoyz, 2005; Smith, 2005). Another important reason is the hysteretic stiction from which many control valves in industrial processes suffer; as a result, modeling, detection, quantification, and compensation of control valve stiction have been active research topics recently (Choudhury, Shah, & Thornhill, 2008; Jelali & Huang, 2010). On the modeling side, one effective model for these systems takes a Hammerstein structure consisting of a hysteresis operator followed by a linear subsystem (Fang & Wang, 2015; Hsu & Ngo, 1997; Iyer & Tan, 2009). Several hysteresis operators are often adopted, including the Preisach operator (Tan & Baras, 2004), the Prandtl–Ishlinskii (PI) operator (Janaideh, Rakheja, & Su, 2009; Kuhnen, 2003), and the Preisach–Krasnosel'skii–Pokrovskii (PKP) operator (Riccardi, Naso, Janocha, & Turchiano, 2012; Webb, Lagoudas, & Kurdila, 1998), each of which is based on a weighted superposition of elementary hysteretic operators. On the control side, a popular control scheme is to construct an inverse hysteresis operator to compensate the hysteresis effect and to design a feedback controller to deal with the inversion error and remaining dynamics (Iyer & Tan, 2009; Kuhnen, 2003). In order to deal with sticky control valves, the knocker method (Hägglund, 2002), the constant reinforcement method (Ivan & Lakshminarayanan, 2009), the two-movement method (Cuadros, Munaro, & Munareto, 2012), and the controller tuning method (Mohammad & Huang, 2012) have been formulated.

Compared with the extensive work on modeling and control of systems with hysteresis, analysis for such systems is relatively limited (Cavallo, Natale, abd Pirozzi, & Visone, 2005; Edardar, Tan, & Khalil, 2014; Esbrook, Tan, & Khalil, 2014; Macki, Nistri, & Zecca, 1992). A major challenge is the memory effect, a key feature of

☆ This work was supported in part by the National Natural Science Foundation of China (No. 61061130559), Research Fund for the Taishan Scholar Project of Shandong Province of China, China Scholarship Council (No. 201406010203) and US National Science Foundation (CMMI 1301243). The material in this paper was partially presented at the 2016 American Control Conference, July 6–8, 2016, Boston, MA, USA. This paper was recommended for publication in revised form by Associate Editor Zongli Lin under the direction of Editor Daniel Liberzon.
* Corresponding author.
E-mail addresses: fanglei@pku.edu.cn (L. Fang), jiandong@sdust.edu.cn (J. Wang), xbtan@egr.msu.edu (X. Tan).

https://doi.org/10.1016/j.automatica.2017.11.005
0005-1098/

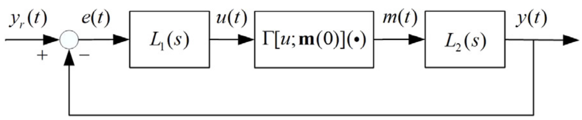

Fig. 1. Configuration of a nonlinear closed-loop system with hysteresis.

hysteresis, which results in complicated dynamic behaviors and significantly hinders the extension of analysis tools for systems with static nonlinearities (Brokate & Sprekels, 1996; Mayergoyz, 2003). An existing method for analyzing hysteretic systems is the describing function method (DFM) (Gelb & Velde, 1968), which utilizes the fundamental harmonic approximation of the periodic signals. However, the DFM is only effective when the hysteresis is weak; for systems with pronounced hysteresis, the accuracy of DFM drops quickly due to the influence of the higher-order harmonics (Mickens, 1984).

In this work, we present a novel approach to computing the frequency response of a closed-loop system with hysteresis. A key element of our approach is the extension of an incremental harmonic balance (IHB) method (Chen, Liu, & Meng, 2012; Shen, Yang, & Liu, 2006). The IHB method is an iterative algorithm, where the input to the nonlinearity of concern is assumed to consist of a series of harmonics, and the corresponding change in the output of the nonlinearity, due to an increment in the input harmonics, is computed via a first-order Taylor approximation. The IHB method has been used in analyzing systems with static nonlinearities, such as cubic polynomials (Chen et al., 2012) and dead zones (Shen et al., 2006); however, the memory effect of hysteresis makes the extension of the IHB method to hysteresis systems challenging. To the best of our knowledge, this paper is the first attempt to extend the IHB method to the harmonic analysis of hysteretic systems.

We consider the PI operator as the hysteresis model, which is a weighted superposition of multiple play operators. The PI operator and its extension have been widely utilized in control of hysteretic systems (Chen, Hisayama, & Su, 2009; Chen, Ren, & Zhong, 2016; Huang, Zhang, & Zhang, 2016; Janaideh & Kreji, 2013; Liu, Su, & Li, 2014; Riccardi, Naso, Turchiano, & Janocha, 2013; Shan & Leang, 2012; Wang & Su, 2006). The main challenge in extending the original IHB method to systems with a PI operator is the calculation of the periodic output of a PI operator for a multi-harmonic input. In order to address this problem, we exploit the results in Esbrook and Tan (2012), namely, the output of the play operator is described as a function of its input and a series of pulse waves determined by a set of switching time instants when the play operator enters or exits the interior region. With the switching time instants determined with a bisection method, the calculation of the incremental harmonic components is finally reformulated as a linear matrix equality that can be solved efficiently. Numerical results demonstrate that the performance of the proposed IHB-based approach is almost independent of the hysteresis severity, while the DFM deteriorates quickly as the hysteresis severity increases.

A preliminary study of this paper was presented as a conference paper (Fang, Wang, & Tan, 2016), where the hysteresis element is modeled by one single play operator. This paper extends the preliminary study to the PI hysteresis operator, so that the technical complexity is much higher. In addition, the proposed IHB-based approach is more complete, e.g., new steps are provided to determine the harmonic order.

The rest of this paper is organized as follows. Section 2 describes the problem to be solved. The details of the proposed IHB-based approach are presented in Section 3, while the DFM is briefly revisited in Appendix A. Numerical results are given in Section 4 to illustrate the effectiveness of the proposed approach. Concluding remarks are provided in Section 5.

2. Problem formulation

Consider a closed-loop system shown in Fig. 1, where a hysteresis operator $\Gamma[u;\mathbf{m}(0)](\cdot)$ is sandwiched by two linear components $L_1(s)$ and $L_2(s)$. Generally, $L_1(s)$ can represent a controller, while $L_2(s)$ may stand for the controlled linear dynamics. The signals $y_r(t)$, $e(t)$, $u(t)$, $m(t)$, and $y(t)$ denote the reference, control error, hysteresis input, hysteresis output, and system output, respectively. Here t is an nonnegative integer standing for the sampling index. In general, linear controllers are preferred owing to their simplicities in terms of design, parameter tuning and maintenance. Thus, more than 95% of industrial control loops use proportional–integral–differential controllers (Aström & Hägglund, 2006). When a hysteresis arises, e.g., when control valves become sticky as the time in service grows, a natural question is how much the performance of linear controllers is negatively affected by the hysteresis. If the effect is minor, then linear controllers are still preferred. To answer the question, a basic step is to perform some theoretical analysis including the harmonic analysis on the closed-loop system as represented in Fig. 1 with a linear controller $L_1(s)$.

The hysteresis Γ is assumed to be a classical PI operator consisting of a weighted superposition of basic hysterons called play operators. For a play operator P_r with an initial condition $m_r(0)$, when its input $u(t)$ is continuous and monotone, the output $m_r(t) = P_r[u;m_r(0)](t)$ is

$$P_r[u;m_r(0)](t) = \max\{\min\{u(t)+r, m_r(t-1)\}, u(t)-r\},$$

where $r > 0$ stands for the play radius. For a general continuous input, the input signal is broken into monotone segments, and the output is then calculated by setting the last output of one monotone segment as the initial condition for the next. Then, a finite-dimensional PI operator can be represented as

$$\Gamma[u;\mathbf{m}(0)](t) = \sum_{i=0}^{N} \theta_i P_{r_i}[u;m_i(0)](t) = \boldsymbol{\theta}^T \mathbf{P_r}[u;\mathbf{m}(0)](t), \tag{1}$$

where the play radii of the $N+1$ play operators satisfy $0 = r_0 < r_1 < \cdots < r_N < \infty$, and θ_i is the weighting of the play operator P_{r_i}. Moreover, $\boldsymbol{\theta} \triangleq [\theta_0, \theta_1, \cdots, \theta_N]^T$, $\mathbf{r} \triangleq [r_0, r_1, \cdots, r_N]^T$, $\mathbf{m}(t) \triangleq [m_0(t), m_1(t), \cdots, m_N(t)]^T$, and $\mathbf{P_r} \triangleq [P_{r_0}, P_{r_1}, \cdots, P_{r_N}]^T$. Since the play output $m_i(t)$ represents the state of the play operator P_{r_i} at the time instant t, the vector $\mathbf{m}(0)$ stands for the initial state of the PI operator.

Assume that the reference signal $y_r(t)$ is a sinusoid

$$y_r(t) = A_{y_r}\sin(\omega t). \tag{2}$$

Assume also that the input $u(t)$ of the PI operator Γ has an amplitude A_u larger than the largest play radius r_N, which ensures a contraction property for the play operators and enables the independence of the $2\pi/\omega$-periodic steady-state output of the PI operator from its initial state (Tan & Khalil, 2009). The objective of this paper is to compute the steady-state responses of $u(t)$, $m(t)$ and $y(t)$ in Fig. 1 under the sinusoidal reference $y_r(t)$ in (2).

3. The proposed IHB-based approach

This section proposes the IHB-based approach for the harmonic analysis of the system in Fig. 1.