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Brief paper

Asymptotic stabilization of continuous-time linear systems with quantized state feedback ☆

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ABSTRACT

This paper considers an asymptotic stabilization problem of continuous-time linear systems subject to state quantization. Based on spherical polar coordinates, a quantizer is proposed with a desired relation between the quantized data and the corresponding quantization error for this problem. Owing to the quantization error the resulting closed-loop system is described by a discontinuous right-hand side differential equation and the notion of Krasovskii solution is adopted for the equation. By the proposed quantizer, a time-invariant coding scheme is first proposed and further a state feedback controller is designed to achieve convergence of the closed-loop trajectories toward a compact surface surrounding the origin, on which sliding motion occurs. Then, a time-varying coding scheme is proposed to avoid chattering phenomena induced by the sliding motion on the surface and achieve asymptotic stability of systems.

1. Introduction

In this paper we formulate and solve an asymptotic stabilization problem with a limited communication channel. Our task involves designing quantizer and controller for continuous-time linear systems with state quantization and achieving asymptotic stability of systems.

The stabilization problem of quantized feedback systems motivated by numerous applications is a very active and expanding research area, where communication between the plant and the controller is limited owing to capacity or security constraints. Concerning the effect of quantization in control systems, such a topic on discrete-time quantized systems has been extensively addressed by researchers over the last years; see, e.g., Delchamps (1990), Elia and Mitter (2001), Fu and Xie (2005), Liberzon (2003), Martins, Dahleh, and Elia (2006), Matveev and Savkin (2006), Picasso and Colaneri (2008), Sharon and Liberzon (2012), Tatikonda and Mitter (2004), Wang and Yan (2014), Wang and Li (2015) and Wong and Brockett (1999) and the references therein. Also, quantization has been applied to finite abstraction problems; see Tazaki and Imura (2008). By state-quantization operation, Tazaki and Imura (2008) design an approximately bisimilar finite state system for discrete-time linear system, which is applicable to optimal control problems.

Different from discrete-time quantized systems, when a continuous-time plant is controlled with quantized information, since the resulting closed-loop system is described by a discontinuous right-hand side differential equation owing to quantization error, the existence of classical solutions is not guaranteed to the closed-loop system; see Cortés (2008a) or Filippov (1988). Fortunately, Krasovskii solutions are arbitrarily close to the solutions to such a differential equation; see Goebel, Sanfelice, and Teel (2012) and Hájek (1979), so the notion of Krasovskii solution is adopted to tackle the problem under consideration for the closed-loop system; see, e.g., Cortés (2008a, b), Ceragioli and De Persis (2007); Ceragioli, De Persis, and Frasca (2011) and Ferrante, Gouaisbaut, and Tarbouriech (2015), just to cite a few. Cortés (2008b) presents analysis and design results for distributed consensus algorithms in continuous-time multi-agent networks. Ceragioli and De Persis (2007) consider the problem of stabilizing continuous-time nonlinear systems with quantized control input. In Ceragioli et al. (2011) continuous-time average consensus dynamics is considered, in which the agents' states are uniformly quantized. Ferrante et al. (2015) deal with the stabilization of continuous-time linear systems subject to uniform input quantization. These papers consider the consensus or the stabilization of resulting closed-loop systems with a discontinuous right-hand side differential equation. In contrast to these works, utilizing a proposed quantizer we are

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concerned with asymptotic stabilization of continuous-time linear systems with state quantization instead of only stabilization.

The objective of this paper is to propose appropriate coding scheme to achieve asymptotic stability of continuous-time linear systems subject to state quantization. Based on spherical polar coordinates, a quantizer is proposed for this problem with a desired relation between the quantized data and the corresponding quantization error. By the proposed quantizer, two kinds of coding schemes are presented: one is time-invariant and the other time-varying. Under the time-invariant coding scheme, a state feedback controller is designed to achieve convergence of the closed-loop trajectories toward a compact surface surrounding the origin, on which sliding motion occurs, and the evolution behavior of the system is analyzed after the system state reaches the surface. Then, to avoid chattering phenomena induced by the sliding motion on the surface, a time-varying coding scheme is proposed and further asymptotic stability of the system is achieved. Under the time-varying coding scheme, the quantizer is dynamic and uses zooming-in/out technique, however, different from the dynamic quantizer under Cartesian coordinates, apart from the desired relation between the quantized data and the corresponding quantization error, another advantage of the proposed quantizer is that it can be combined with quadratic Lyapunov function harmoniously and Lyapunov level set can match the ellipsoidal quantization region, which facilitates the analysis of the evolution behavior of the system. The proposed quantizer is a continuous version of the quantizer in our previous work (Wang & Yan, 2014). Compared to Wang and Yan (2014), this paper considers a continuous-time system and utilizes Lyapunov function for the design of the quantizer. Our work is related to the work of Ferrante et al. (2015), the major difference is the quantizer with the different quantization nonlinearity condition. In Ferrante et al. (2015), uniform quantizer under Cartesian coordinates is used, by which the quantization nonlinearity conditions are given, while the quantizer in this paper is proposed based on spherical polar coordinates. The proposed quantizer brings benefits to the systems as follows: on one hand, a new quantization nonlinearity condition is developed. The developed nonlinearity condition shows that the magnitude of the quantized data is proportional to an upper bound of the magnitude of the corresponding quantization error, which is a desired property of the quantizers; on the other hand, since the new quantization nonlinearity condition implies that the quantizer resolution will become fine as the quantized data tends to the origin and coarse as it is far from the origin, the coding scheme based on the proposed quantizer guarantees that the closed-loop trajectories converge toward the origin and achieves asymptotic stability of the system.

Notation: E and O denote respectively the identity matrix and the null matrix of appropriate dimensions. The set $\mathfrak{B}(x, \delta)$ denotes the δ radius closed Euclidean ball centered at x. The Krasovskii operator $\mathcal{K}$ is defined by $\mathcal{K}(f(x)) := \bigcap_{\delta>0} f(\mathfrak{B}(x, \delta))$. For a matrix A, A^T denotes its transpose. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. $*$ stands for symmetric blocks in matrices. For two sets S_1 and S_2, $S_1 \setminus S_2$ denotes the set S_1 deprived of S_2. $\|\cdot\|$ denotes the Euclidean norm for a vector and the corresponding matrix induced norm for a matrix, $\langle \cdot, \cdot \rangle$ denotes the standard Euclidean inner product of two vectors, $\delta_{\min}(\cdot)$ the minimum singular value of a matrix, $\nabla(\cdot)$ the gradient of a function and $\lceil \cdot \rceil$ the ceiling function.

2. Problem statement

Consider the following continuous linear system

$$\begin{cases} \dot{x} = Ax + Bu \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x \in \mathbb{R}^d$ and $u \in \mathbb{R}^m$ are the state and the input of the system. Matrices A, B are real constant matrices with appropriate dimensions. The input of the system is the result of a quantized state feedback control law: $u = Kq(x(t))$ where K is a real constant matrix of appropriate dimension, $q(\cdot)$ is the quantizer function defined in Section 3. A vector x with appropriate dimension is quantized as $q(x)$, the estimate of x, with the same dimension. System (1) subject to state quantization is described by

$$\dot{x} = Ax + BKq(x). \tag{2}$$

By defining quantization error $\Xi(x) = q(x) - x$, a type of quantization nonlinearity, the system (2) is written as

$$\begin{cases} \dot{x} = (A + BK)x + BK\Xi(x) \\ x(0) = x_0. \end{cases} \tag{3}$$

Since the righthand side of (3) is a discontinuous function of the state owing to quantization error, the existence of solutions in a classical sense is not guaranteed; see Cortés (2008a) or Filippov (1988). So in this paper we adopt Krasovskii solutions to system (3), which is the solutions to the following differential inclusion:

$$\dot{x} \in \mathcal{K}((A + BK)x + BK\Xi(x)) \tag{4}$$

where $\mathcal{K}$ is the Krasovskii operator. By Filippov (1988) every solution to

$$\dot{x} \in (A + BK)x + BK\mathcal{K}(\Xi(x)) \tag{5}$$

is a Krasovskii solution to (3). The reasons to consider Krasovskii solutions are as follows: on one hand, the existence of Krasovskii solutions is guaranteed under local boundedness of the right-hand side of (3), which is satisfied in our case, on the other hand, such solutions contain sliding modes which induce chattering phenomena.

In this paper, we aim to develop a quantizer with a new quantization nonlinearity condition to achieve asymptotic stability of the system. The problem we intend to solve can be summarized as follows:

Problem 2.1. Determine a quantizer and a coding scheme along with a state feedback gain K such that the system (1) is asymptotically stable.

Compared with Ceragioli et al. (2011) and Ferrante et al. (2015), the key problem is to design a quantizer with a new quantization nonlinearity condition and a coding scheme such that the state of the system (1) converges toward the origin instead of a set containing the origin.

3. Quantizer based on spherical polar coordinates

The quantizer in this paper will be based on spherical polar coordinates. Let the vector $x = [x_1 \ x_2 \ \cdots \ x_{d-1} \ x_d]^{\mathrm{T}} \in \mathbb{R}^d$. Then we call the column $[x_1 \ x_2 \ \cdots \ x_{d-1} \ x_d]^{\mathrm{T}}$ as the Cartesian rectangular coordinate of x. The vector can also be represented using spherical polar coordinate

$$\begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} \in \mathbb{B}^d := \left\{ \begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} : \begin{matrix} 0 \le \theta_1, \theta_2, \ldots, \theta_{d-2} \le \pi, \\ 0 \le \theta_{d-1} \le 2\pi, 0 \le r < \infty \end{matrix} \right\}$$

via the coordinate transformation pair

$$\begin{aligned} x_1 &= r\cos\theta_1 \\ x_2 &= r\sin\theta_1\cos\theta_2 \\ &\vdots \\ x_{d-1} &= r\sin\theta_1\sin\theta_2\cdots\sin\theta_{d-2}\cos\theta_{d-1} \\ x_d &= r\sin\theta_1\sin\theta_2\cdots\sin\theta_{d-2}\sin\theta_{d-1} \end{aligned}$$