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A power consensus algorithm for DC microgrids☆

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ABSTRACT

A novel power consensus algorithm for DC microgrids is proposed and analyzed. DC microgrids are networks composed of DC sources, loads, and interconnecting lines. They are represented by differential–algebraic equations connected over an undirected weighted graph that models the electrical circuit. The proposed algorithm features a second graph, which represents the communication network over which the source nodes exchange information about the instantaneous powers, and which is used to adjust the injected current accordingly. This gives rise to a nonlinear consensus-like system of differential–algebraic equations that is analyzed via Lyapunov functions inspired by the physics of the system. We establish convergence to the set of equilibria, where weighted power consensus is achieved, as well as preservation of the weighted geometric mean of the source voltages. The results apply to networks with constant impedance, constant current and constant power loads.

1. Introduction

The proliferation of renewable energy sources and storage devices that are intrinsically operating using the DC regime is stimulating interest in the design and operation of DC microgrids, which have the additional desirable feature of preventing the use of inefficient power conversions at different stages. These DC microgrids might have to be deployed in areas where an AC microgrid is already in place, creating what is called a hybrid microgrid (Loh, Li, Chai, & Blaabjerg, 2013), for which rigorous analytical studies are still in their infancy. Furthermore, the envisioned future in which power generation is far away from the major consumption sites raises the problem of how to transmit power with low losses, a problem for which High Voltage Direct Current (HVDC) networks perform comparatively better than AC networks. Finally, also mobile grids on ships, aircrafts, and trains are based on a DC architecture.

With DC and hybrid microgrids, as well as HVDC networks, on the rise, we need to develop a deeper system-theoretic understanding of this interesting class of dynamical networks. In this paper we propose and analyze a control algorithm for a DC microgrid that enforces power sharing among the different power sources.

1.1. Literature review

The literature on DC microgrids is rapidly growing. We summarize below the contributions that share a systems and control-theoretic point of view on these networks. The work (Nasirian, Moayedi, Davoudi, & Lewis, 2015) relies on a cooperative control paradigm for DC microgrids to replace the conventional secondary control by a voltage and a current regulator. In Zhao and Dörfler (2015) a voltage droop controller for DC microgrids inspired by frequency droop in AC power networks is analyzed, and a secondary consensus control strategy is added to prevent voltage drift and achieve optimal current injection. The paper (Belk, Inam, Perreault & Turitsyn, 2016) models the DC microgrid via the Brayton–Moser equations and uses this formalism to show that with the addition of a decentralized integral controller voltage regulation to a desired reference value is achieved. Other schemes achieving desirable power sharing properties are proposed but no formal analysis is provided. In Tucci, Meng, Guerrero, and Ferrari-Trecate (2016), a secondary consensus-based control scheme for current sharing and voltage balancing in DC microgrids is designed in a Plug-and-Play fashion to allow for the addition or removal of generation units. A distributed control method to enforce power sharing among a cluster of DC microgrids is proposed in Moayedi and Davoudi (2016). Other work has focused on the challenges in the stability analysis of DC microgrids using consensus-like algorithms due to the interaction between the communication

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network and the physical one (Meng, Dragicevic, Roldán-Pérez, Vasquez, & Guerrero, 2016). Finally, feasibility of the nonlinear algebraic equations in DC power circuits is studied by Barabanov, Ortega, Grino, and Polyak (2016), Lavei, Rantzer, and Low (2011), and Simpson-Porco, Dörfler, and Bullo (2015).

A closely related research area is that of multi-terminal HVDC transmission systems. The paper (Sarlette, Dai, Phulpin, & Ernst, 2012) focuses on cooperative frequency control for these networks. In Andreasson et al. (2014) distributed controllers that keep the voltages close to a nominal value and guarantee a fair power sharing are considered, whereas passivity-based decentralized PI control for the global asymptotic stabilization of multi-terminal high-voltage is studied in Zonetti, Ortega, and Benchaib (2015). The paper (Zonetti, Ortega, & Schiffer, 2016) studies feasibility and power sharing under decentralized droop control. We refer to Zonetti (2016, Chapter 4) for an annotated bibliography of HVDC transmission systems.

1.2. Main contribution

The objective of the paper is to propose a novel control algorithm that exhibits three main features: (i) it makes sources provide power in prescribed ratios for a wide range of load magnitudes; (ii) it simultaneously guarantees that all voltages stay within a compact set around an operating point, and that the geometric voltage of the source voltages is maintained constant for all time; (iii) it tackles so-called "ZIP" (constant impedance, constant current and constant power) loads, which are known to substantially affect the stability of the system.

Power sharing is essential in microgrid operations because changing load conditions may lead to an imbalance situation in which few sources, if not a single one, provide the majority of the power demand. This might result in cases in which the overloaded sources exceed their capacity limit driving the microgrid to instability.

While droop controllers are usually employed to achieve power distribution in DC microgrids, they cannot achieve such a task exactly because they can only strike a tradeoff between power sharing and voltage control. As such, there are no generally acknowledged criteria for the selection of droop gains that provide guaranteed power sharing properties in the presence of unknown or uncertain variable load conditions. Our controller overcomes such limitations, providing a substantial improvement with respect to existing controllers.

The proposed controller is enabled by communicating the instantaneous source power measurements among neighboring source nodes, averaging these measurements and setting the voltage at the source terminals accordingly. An additional feature of the algorithm is that a weighted geometric average of the source voltages is preserved. In absence of a communication environment, our distributed consensus-based algorithm can also be implemented by power talk communication via the DC microgrid (Angjelichinoski et al., 2015).

The system dynamics present interesting features. By averaging the power measurements that the sources communicate amongst each other, the system dynamics becomes an intriguing combination of the physical network (the weighted Laplacian of the electrical circuit appearing in the power measurements) and the communication network (over which the information about the power measurements is exchanged). ZIP loads introduce algebraic equations in the system's dynamics, adding additional complexity and nonlinearities.

To analyze this system of nonlinear differential–algebraic equations without going through a linearization of the dynamics, Lyapunov-based arguments become very convenient. The Lyapunov functions in this case are constructed starting from the power dissipated in the network that is further shaped to take into account the specifics of the dynamics. In fact, it is shown that the closed-loop system can be written as a weighted gradient of the Lyapunov function (Lemma 2), a form that is crucial to carry out the stability analysis. The presence of the loads, which shift the equilibrium of interest, is taken into account by the so-called Bregman function (De Persis & Monshizadeh, 2018). The level sets of the Lyapunov functions are used to estimate the excursion of the state response of these systems and therefore, combined with the preservation of the geometric average of the source voltages, can be used to obtain an estimate of the voltage at steady state.

Reactive power sharing algorithms have been first suggested by Schiffer, Seel, Raisch, and Sezi (2016) for network-reduced AC microgrids whose voltage dynamics show similar features as in DC grids. In this paper we show that a related idea can be adopted also for network preserved DC microgrids. The novelties of this contribution with respect to Schiffer et al. (2016) are the different dynamics of the system under study, the explicit consideration of algebraic equations in the model and the use of Lyapunov arguments to prove the main results.

1.3. Paper organization

The model of the DC microgrid is introduced in Section 2. The power consensus algorithm is introduced in Section 3. The analysis of the closed-loop system is carried out in Section 4 for the general case of ZIP loads, and then specialized to the case of ZI loads, since the latter permits to obtain stronger results under weaker conditions. The simulations of the algorithm are provided in Section 5. Conclusions are drawn in Section 6.

1.4. Notation

Given a vector v, the symbol $[v]$ represents the diagonal matrix whose diagonal entries are the components of v. The notation $\mathrm{col}(v_1, v_2, \ldots, v_n)$, with v_i scalars, represents the vector $[v_1 \quad v_2 \quad \ldots \quad v_n]^T$. If v_i are matrices having the same number of columns, then $\mathrm{col}(v_1, v_2, \ldots, v_n)$ denotes the matrix $[v_1^T \quad v_2^T \quad \ldots \quad v_n^T]^T$. The symbol $\mathbb{1}_n$ represents the n-dimensional vector of all 1's, whereas $\mathbb{0}_{m\times n}$ is the $m \times n$ matrix of all zeros. When the size of the matrix is clear from the context the index is omitted. The $n \times n$ identity matrix is represented as $\mathbb{I}_n$. Given a vector $v \in \mathbb{R}^n$, the symbol $\mathbf{ln}(v)$ denotes the element-wise logarithm, i.e., the vector $[\ln(v_1) \ldots \ln(v_n)]^T$. Given a set S, the symbol $|S|$ indicates the cardinality of the set.

2. DC resistive microgrid

The DC microgrid is modeled as an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} := \{1, 2, \ldots, n\}$ the set of nodes (or buses) and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ the set of edges. The edges represent the interconnecting lines of the microgrid, which we assume here to be resistive. Associated to each edge is a weight modeling the conductance (or reciprocal resistance) $1/r_k > 0$, with $k \in \mathcal{E}$. The set of nodes is partitioned into the two subsets of n_s DC sources $\mathcal{V}_s$ and n_l loads $\mathcal{V}_l$, with $n_s + n_l = n$.

Let $I \in \mathbb{R}^n, V \in \mathbb{R}^n_{>0}$ denote the vectors of currents and potentials respectively at the nodes of $\mathcal{G}$. The current-potential relation in a resistive network is given by the identity $I = B\Gamma B^T V$, with $B \in \mathbb{R}^{n\times|\mathcal{E}|}$ being the incidence matrix of $\mathcal{G}$ and $\Gamma = \mathrm{diag}\{r_1^{-1}, \ldots, r_{|\mathcal{E}|}^{-1}\}$ the diagonal matrix of conductances. Considering the partition of the nodes in sources and loads, we let $I = \mathrm{col}(I_s, I_l)$ and $V = \mathrm{col}(V_s, V_l)$ without loss of generality, where $I_s = \mathrm{col}(I_1, \ldots, I_{n_s})$, $I_l = \mathrm{col}(I_{n_s+1}, \ldots, I_n)$, $V_s = \mathrm{col}(V_1, \ldots, V_{n_s})$, $V_l = \mathrm{col}(V_{n_s+1}, \ldots, V_n)$, and we correspondingly partition the