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Brief paper

On optimal system operation in robust economic MPC☆

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ABSTRACT

In this paper, different approaches for economic MPC under disturbances are investigated with respect to their optimal operating behavior. We derive dissipativity-based conditions under which certain optimal operating regimes can be guaranteed for the considered setups. Depending on the information about the disturbance, the system, and the algorithmic structure of the considered underlying robust economic MPC scheme, different statements can be derived. These include, inter alia, statements on stochastic and robust optimal operation. Moreover, we are able to provide converse statements showing – under a certain controllability assumption – necessity of the dissipativity statements for optimal operation at steady-state.

1. Introduction

Economic Model Predictive Control (MPC) has been an active field of research within the last decade. While in stabilizing MPC, the stage cost function is chosen in order to stabilize an a priori determined steady-state, in economic MPC arbitrary stage cost functions can be handled. This allows to consider more general control objectives, e.g., profit maximization or minimization of energy consumption. In the nominal, undisturbed case, optimal behavior of economic MPC algorithms has thoroughly been investigated. Due to the consideration of general stage cost functions in economic MPC, the optimal operating behavior might not be steady-state operation and also the closed-loop system does not necessarily converge to the optimal steady-state. Thus, one key question is: How can the optimal operating behavior be classified? In Angeli, Amrit, and Rawlings (2012), a first definition of optimal operation at steady-state is introduced for nominal economic MPC and sufficient conditions for a system to satisfy this property are presented (based on dissipativity). The converse statement is investigated in Müller, Angeli, and Allgöwer (2015), where it is shown that a certain controllability assumption is needed in order to prove necessity of dissipativity. A generalization of the above results as well as an extension to periodic systems is provided in Müller and Grüne (2016) and Zanon, Grüne, and Diehl (2016).

For most practical applications, the system is affected by disturbances. Whenever disturbances are acting on a system, it turns out that just transferring robust stabilizing MPC schemes to the economic case can result in a very poor performance, since the schemes do not account for the influence of the disturbance on the performance (see, e.g., Bayer, Müller, & Allgöwer, 2014). In order to overcome this drawback, several schemes have been presented explicitly accounting for the disturbance within the setup of the MPC algorithm (see, e.g., Bayer, Lorenzen, Müller, & Allgöwer, 2016; Bayer et al., 2014; Bayer, Müller, & Allgöwer, 2016; Broomhead, Manzie, Shekhar, & Hield, 2015; Hovgaard, Larsen, & Jorgensen, 2011; Huang, Biegler, & Harinath, 2012; Lucia, Andersson, Brandt, Diehl, & Engell, 2014; Marquez, Patiño, & Espinosa, 2014).

In this paper, we are interested in analyzing the optimal operating behavior in the context of economic MPC under disturbances. In the robust setting, only in Bayer et al. (2014) some first attempt to examine the optimal operating behavior has been made, but only for one particular setup and for a special structure of the optimization problem. Here, we present a more comprehensive treatment of the subject. The major difficulty when analyzing optimal operation at steady-state under disturbances lies in the different robust economic MPC approaches available in literature. In particular, these approaches differ in the way which information about the disturbance is assumed to be known and how it is taken into account. This necessitates different notions of optimal operation at steady-state and, at the same time, different notions of dissipativity for investigating these properties. We highlight the fundamental contrast to the nominal case, where optimal operation at steady-state is a property that is independent of the underlying MPC

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approach. The analysis is split in three parts, which differ depending on which information about the disturbance is considered and how the disturbance is accounted for. These three parts also structure the paper: First, we investigate optimality depending only on the dynamics, the cost function, and the constraints. The disturbance is only considered through bounds on the worst case disturbance. The resulting optimality notion can be interpreted as steady-state operation being approximately optimal up to an error term depending on the largest disturbance. This seems to be the most general but also the weakest statement for optimality. Thus, in a second step, we investigate optimality for robust economic MPC schemes based on nominal dynamics underlying the optimal control problem. Third, we extend the analysis to systems with additional stochastic information on the disturbance. This additional information can be employed to sharpen the statement on the optimal operating behavior.

Notation: We denote by $\mathbb{I}_{\geq 0}$ the set of all non-negative integers and by $\mathbb{I}_{[a,b]}$ the set of all integers in the interval $[a, b] \subset \mathbb{R}$. For sets $X, Y \subseteq \mathbb{R}^n$, the Minkowski set addition is defined by $X \oplus Y := \{x+y \in \mathbb{R}^n : x \in X, y \in Y\}$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class $\mathcal{K}$ function if it is strictly increasing and $\alpha(0) = 0$. It is a class $\mathcal{K}_\infty$ function if furthermore $\alpha(s) \to \infty$ for $s \to \infty$. We denote k-step ahead predictions of state or input a, which are predicted at time t, by $a(k|t)$. When a prediction is marked by $*$, e.g., $a^*(k|t)$, this indicates that this state or input is the optimal k-step ahead prediction at time t with respect to the considered MPC problem. Given a set $\mathbb{Y} \subseteq \mathbb{X} \times \mathbb{U}$, we denote its projection on $\mathbb{X}$ by $\mathbb{Y}_\mathbb{X}$. The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $|x|$.

2. Problem setup

We are interested in controlling systems of the form

$$x(t+1) = f\big(x(t), u(t), w(t)\big), \quad x(0) = x_0, \tag{1}$$

with $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^q \to \mathbb{R}^n$ continuous, where $x(t) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ is the input to the system, and $w(t) \in \mathbb{W} \subset \mathbb{R}^q$ is an external disturbance acting on the system. We assume that the disturbance set $\mathbb{W}$ is convex, compact, and contains the origin in its interior. Moreover, we assume constraints on the states and inputs of the form

$$\big(x(t), u(t)\big) \in \mathbb{Z}, \quad \forall t \in \mathbb{I}_{\geq 0}, \tag{2}$$

where $\mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U}$ is a compact and convex set containing the origin in its interior.

Our goal is to characterize the optimal system behavior a priori, given the dynamics, the constraints $\mathbb{Z}$, and the general continuous stage cost $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$. This can be done in different ways, depending on which information on the disturbances w is considered and how this information is taken into account. When considering economic stage cost functions, that is, stage cost functions where the optimal behavior is not directly determined by design, it is a relevant question to derive the optimal operating scheme. This is not only of theoretical interest but can also be used in order to investigate whether the closed-loop system "does the right thing", i.e., whether the closed-loop system achieves the best attainable performance. Thus, investigating optimal operating schemes and closed-loop performance guarantees can be seen as two sides of the same coin: while the former determines the best achievable performance, the latter provides the guaranteed achieved closed-loop performance. While the closed-loop performance bound is a property of the underlying MPC algorithm, optimal operation (at steady-state) is (in the nominal case) a property of the problem setup only. We will see later that for the robust case, this characteristic can be recovered, yet, to some extent the considered algorithm plays part in the analysis.

In order to ensure computational tractability of the repeatedly solved optimal control problem, in MPC for systems with disturbances one typically uses a feedback described by a given (fixed) continuous parametrization $u(t) = \pi(x(t), v(t))$, with $v(t) \in \mathbb{R}^m$ (see, e.g., Fontes & Magni, 2003; Limón et al., 2009). Using this parametrization results in the system

$$\begin{aligned} x(t+1) &= f\big(x(t), \pi(x(t), v(t)), w(t)\big) \\ &=: f_\pi\big(x(t), v(t), w(t)\big). \end{aligned} \tag{3}$$

Before starting the discussion, we want to introduce some notation. With regard to the constraints and the input parametrization, we introduce

$$\mathbb{Z}_\pi := \Big\{(x, v) \in \mathbb{R}^n \times \mathbb{R}^m : \big(x, \pi(x, v)\big) \in \mathbb{Z}\Big\}. \tag{4}$$

Here, $\mathbb{Z}_\pi$ need not be a subset of $\mathbb{Z}$. By nominal system, we mean the system

$$z(t+1) = f_\pi(z(t), v(t), 0), \quad z(0) = z_0, \tag{5}$$

where f_π is the same dynamics as in (3), and by z we denote the nominal state. Note that while $u(t) = \pi(x(t), v(t))$ is the input to the real system, $\pi(z(t), v(t))$ is the input to the nominal system and $v(t)$ is a free input variable. Given the nominal system, we can introduce the error $e(t) := x(t) - z(t)$. We want to determine an invariant set for the resulting error system

$$\begin{aligned} e(t+1) &= f_\pi\big(x(t), v(t), w(t)\big) - f_\pi\big(z(t), v(t), 0\big), \\ e(0) &= x_0 - z_0. \end{aligned} \tag{6}$$

Definition 1 (*Bayer, Müller et al., 2016*). A set $\Omega_\infty \subseteq \mathbb{R}^n$ is robust control invariant (RCI) for the error system (6) if there exists a feedback control law $u(t) = \pi(x(t), v(t))$ such that for all $x(t), z(t) \in \mathbb{R}^n$ and $v(t) \in \mathbb{R}^m$ with $e(t) := x(t) - z(t) \in \Omega_\infty$ and $\big(x(t), \pi(x(t), v(t))\big) \in \mathbb{Z}$, and for all $w(t) \in \mathbb{W}$, it holds that $e(t+1) \in \Omega_\infty$.

For any state $x(t)$ of the real system satisfying $x(t) \in \{z(t)\} \oplus \Omega_\infty$, the subsequent state $x(t+1)$ lies for all possible disturbances $w(t) \in \mathbb{W}$ within the set $\{z(t+1)\} \oplus \Omega_\infty$. Thus, the set Ω_∞ centered at the nominal trajectory provides an outer approximation of all possible state trajectories of the real system. Moreover, by controlling the nominal state, one can also control the real state which is by Definition 1 guaranteed to lie in the RCI set centered at the nominal state. In general, computing RCI sets for arbitrary nonlinear systems is a difficult task, but there exist different methods to compute RCI sets for (special classes of) nonlinear systems, see, e.g., Bayer, Bürger, and Allgöwer (2013), Limón, Alamo, and Camacho (2002) and Yu, Maier, Chen, and Allgöwer (2013). In case of linear dynamics in (3), i.e., $x(t+1) = Ax(t) + Bu(t) + w(t)$, and with linear feedback of the form $u(t) = Kx(t) + v(t)$, computing an RCI set boils down to computing a robust positively invariant (RPI) set, see e.g. Blanchini (1999) and Kolmanovsky and Gilbert (1998).

In order to further simplify the notation, we introduce the cost function

$$\ell_\pi\big(x(t), v(t)\big) := \ell\big(x(t), \pi(x(t), v(t))\big).$$

Remark 1. Considering the input parametrization π, we note that this is not needed in Section 3. However, to keep a consistent line of presentation, the parametrization is used throughout the whole paper.

2.1. Optimal operation in nominal economic MPC

Definition 2 (*Angeli et al., 2012*). System (5) is said to be *optimally operated at steady-state* with respect to the cost function ℓ and the