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Brief paper

Robust finite-time connectivity preserving coordination of second-order multi-agent systems[☆]

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ABSTRACT

In this paper, we consider robust finite-time connectivity preserving coordination problems for second-order multi-agent systems with limited sensing range. Based on integral sliding mode control and artificial potential field, a distributed controller is developed to achieve robust finite-time consensus and meanwhile maintain the connectivity of the communication network. The method is further extended to address a finite-time formation tracking control problem. Numerical examples are given to show the effectiveness of the proposed methods.

1. Introduction

Multi-agent systems have attracted much attention in the past decade, as it has a wide range of potential applications in both civilian and military areas, such as sensor networks and multi-vehicle formation (Dong & Hu, 2017; Dong, Sun, & Hu, 2016; Ren, Beard, & Atkins, 2007). In practical applications, the agents usually have limited sensing and communication capabilities and the connection graph for the multi-agent system is dependent on the sensing range of the agent. Thus, the connectivity of the initial network topology cannot be guaranteed for all future time, which motivates the connectivity preservation problem (Dong & Huang, 2013; Feng, Sun, & Hu, in press; Ji & Egerstedt, 2007; Kan, Klotz, Cheng, & Dixon, 2012; Su, Wang, & Chen, 2010). Connectivity preservation has a wide application in multi-robot flocking/rendezvous problems. For example, in Ji and Egerstedt (2007), a distributed gradient method was developed to maintain the initial network topology for a first-order multi-robot system.

Most of the existing works did not specify the convergence rate of the rendezvous algorithm. When considering the convergence rate and robustness to external disturbances, finite-time control laws usually have better performance (Bhat & Bernstein, 2000). Finite-time consensus problems were investigated in Cao, Ren, Casbeer, and Schumacher (2016), Cortés (2006), Guan, Sun, Wang, and Li (2012), Hui (2011), Liu, Chen, Du, and Yang (2016), Li, Du, and Lin (2011), Khoo, Xie, and Man (2009), Ou, Du, and Li (2014), Wang and Hong (2008), Wang and Xiao (2010), Xiao, Wang, Chen, and Gao (2009), Yu and Long (2015), Zhang, Yang, and Zhao (2013) and Zhao, Duan, Wen, and Chen (2016). Cortés (2006) presented nonsmooth tools to analyze finite-time stability of continuous-time systems where the differential equations have a discontinuous right-hand side, and then extended the results to networked finite-time consensus. For second order systems, Zhao et al. (2016) proposed a binary consensus protocol which only requires sign information between neighbors. Considering intrinsic nonlinear dynamics, some nonlinear consensus protocols using odd functions were developed in Guan et al. (2012) and Liu et al. (2016), and homogeneity theory is used to prove the stability. The finite-time rendezvous problem was first proposed in Hui (2011). Cao, Ren, Casbeer, and Schumacher (2016) investigated integrator-type dynamics with Lipschitz nonlinearities and Dong (2016) considered disturbance rejection. It is noted that all these works consider first-order leaderless multi-agent systems.

In this paper, we consider the finite-time connectivity preservation problem for second-order leader-following multi-agent systems with bounded nonlinearities and disturbances. We propose an integral sliding mode based framework to achieve robust finite-time consensus and formation tracking, and meanwhile maintain the connectivity of the communication network. The main contributions of this paper can be summarized as follows:

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(1) Disturbance rejection, finite-time coordination and connectivity preservation are simultaneously achieved. The disturbance rejection adopts the integral sliding mode scheme (Utkin & Shi, 1996) in order to guarantee connectivity preservation, which also deals with the nonzero acceleration of the leader. (2) We study the finite-time connectivity preserving consensus tracking problem for second-order systems, which are more complicated than first-order leaderless systems addressed in the literature Hui (2011) and Cao et al. (2016). The method is developed according to odd function based consensus protocols for second order multi-agent systems, where we design potential functions to achieve connectivity preservation. One of the difficulties is to estimate the settling time. In the existing literature such as Guan et al. (2012) and Liu et al. (2016), homogeneity theory is used to prove the stability, which leads to the difficulty in the settling time estimation. In this paper, by properly selecting the Lyapunov function and potential functions, the upper bound of the settling time can be written as an expression of the initial conditions. (3) Inspired by the derived consensus tracking controller, we propose a finite-time connectivity preserving formation control approach based on a new potential function. Compared with the existing studies on the finite-time formation tracking problem (e.g., Liu, Zhao, & Chen, 2016; Ou et al., 2014; Xiao et al., 2009), the proposed method is robust to bounded disturbances and can preserve the network pattern.

2. Notation and preliminaries

Notations: In this paper, we use $\mathbb{R}$ and $\mathbb{R}^n$ to denote the set of real numbers and n-dimensional real column vectors, respectively. $A\otimes B$ denotes the Kronecker product of matrices A, B. Let $\|\cdot\|$ be the 2-norm and $\|\cdot\|_1$ be the 1-norm. For a vector $e=[e_1,\dots,e_n]^T\in\mathbb{R}^n$, $\mathrm{sgn}(e)=[\mathrm{sgn}(e_1),\dots,\mathrm{sgn}(e_n)]^T$. Let $\mathrm{sig}^\alpha(e)=\|e\|^\alpha(e/\|e\|)$, if $e\neq\mathbf{0}$, and $\mathrm{sig}^\alpha(e)=\mathbf{0}$ if $e=\mathbf{0}$, which is continuous if $\alpha>0$. $\mathbf{1}$ and $\mathbf{0}$ are column vectors with appropriate dimensions.

Graph theory: Denote $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$ an undirected graph, where $\mathcal{V}=\{1,\dots,N\}$ and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ indicate the set of vertices and edges, respectively. An edge is an ordered pair $(i,j)\in\mathcal{E}$ if agent j can be directly supplied with information from agent i. $\mathcal{N}_i=\{j\in\mathcal{V}\,|(j,i)\in\mathcal{E}\}$ denotes the neighborhood set of vertex i. Graph $\mathcal{G}$ is connected if there is an undirected path between every pair of distinct agents. A matrix $A=\left[a_{ij}\right]\in\mathbb{R}^{N\times N}$ denotes the adjacency matrix of $\mathcal{G}$, where $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$ else $a_{ij}=0$. In this paper, we suppose $a_{ii}=0$. A matrix $L\triangleq D-A\in\mathbb{R}^{N\times N}$ is called the Laplacian matrix of $\mathcal{G}$, where $D=[d_{ii}]\in\mathbb{R}^{N\times N}$ is a diagonal matrix with $d_{ii}=\sum_{j=1}^{N}a_{ij}$.

Lemma 1. *(1) (Wang & Xiao, 2010) Let $\delta_1,\dots,\delta_N\geq 0$ and $0<p\leq 1$, then $(\sum_{i=1}^{N}\delta_i)^p\leq\sum_{i=1}^{N}\delta_i^p\leq N^{1-p}(\sum_{i=1}^{N}\delta_i)^p$. (2) Let $\delta_1,\dots,\delta_N\geq 0$ and $0<p<q$, then*[1] *$(\sum_{i=1}^{N}\delta_i^q)^{\frac{1}{q}}\leq(\sum_{i=1}^{N}\delta_i^p)^{\frac{1}{p}}$. (3) (Zuo, Yang, Tie, & Meng, 2014) Let $\delta_1,\dots,\delta_N\geq 0$ and $p>1$, then $\sum_{i=1}^{N}\delta_i^p\geq N^{1-p}(\sum_{i=1}^{N}\delta_i)^p$. (4) (Young's Inequality) Let $\delta_1,\delta_2,c>0$ and $p,q>1$. If $\frac{1}{p}+\frac{1}{q}=1$, then $\delta_1\delta_2\leq c^p\frac{\delta_1^p}{p}+c^{-q}\frac{\delta_2^q}{q}$.*

[1] This conclusion can be easily obtained by noting that $(\sum_{i=1}^{N}\delta_i^q)^{\frac{p}{q}}\leq\sum_{i=1}^{N}(\delta_i^q)^{\frac{p}{q}}$.

3. Robust finite-time connectivity preserving consensus tracking

3.1. Problem formulation

Consider a second-order multi-agent system with N followers. The dynamics of the follower i ($i\in\{1,\dots,N\}$) are described by

$$\begin{aligned}\dot{x}_i(t)&=v_i(t),\\ \dot{v}_i(t)&=u_i(t)+f_i(x_i,v_i,t)+d_i(t),\end{aligned}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ represents the position state of agent i, $v_i(t)\in\mathbb{R}^n$ represents the velocity state of agent i, $u_i(t)\in\mathbb{R}^n$ represents the control input, $f_i(x_i,v_i,t)\in\mathbb{R}^n$ is an unknown nonlinear function and $d_i(t)\in\mathbb{R}^n$ is the disturbance. Let $x(t)=[x_1^T(t),\dots,x_N^T(t)]^T\in\mathbb{R}^{Nn}$ be the position vector and $v(t)=[v_1^T(t),\dots,v_N^T(t)]^T\in\mathbb{R}^{Nn}$ be the velocity vector.

The leader for system (1) has the following dynamics:

$$\dot{x}_0(t)=v_0(t),\ \ \dot{v}_0(t)=f_0(x_0,v_0,t),\tag{2}$$

where $x_0(t)\in\mathbb{R}^n$ represents the position state, $v_0(t)\in\mathbb{R}^n$ represents the velocity state, and $f_0(x_0,v_0,t)\in\mathbb{R}^n$ is the acceleration state.

In reality, the agent usually has a limited communication capability and can only communicate with agents within its information range. If the relative distance of two neighboring agents are larger than this range, the communication link may be lost. Suppose that the initial connections are established according to the distances, i.e., $\mathcal{E}(0)=\{(i,j):\left\|x_i(0)-x_j(0)\right\|<R, i\in\{0,1,\dots,N\}, j\in\{1,\dots,N\}\}$. The initial information exchange among the $N+1$ agents is represented by graph $\mathcal{G}(0)=\{\mathcal{V},\mathcal{E}(0)\}$ with $\mathcal{V}=\{0,1,\dots,N\}$. Let $\mathcal{G}_F(0)=\{\mathcal{V}_F,\mathcal{E}_F(0)\}$ be the subgraph of the followers where $\mathcal{V}_F=\{1,\dots,N\}$ and $\mathcal{E}_F(0)\subset\mathcal{E}(0)$.

The adjacency matrix $A=[a_{ij}]\in\mathbb{R}^{N\times N}$ is defined as $a_{ij}>0$ if $\left\|x_i(t)-x_j(t)\right\|<R$ and $\left\|x_i(0)-x_j(0)\right\|<R$, and $a_{ij}=0$ elsewise. The access of agents to the leader's trajectory signal is represented by a diagonal matrix $B=[b_i]\in\mathbb{R}^{N\times N}$ where $b_i=1$ if $\|x_i(t)-x_0(t)\|<R$ and $\|x_i(0)-x_0(0)\|<R$, and $b_i=0$ elsewise. Let $H=L+B$ be the information exchange matrix with L being the Laplacian of $\mathcal{G}_F$.

The following assumptions will be used in the subsequent stability analysis.

Assumption 1. $\mathcal{G}_F(0)$ is connected and at least one agent has access to the leader's information.

Assumption 2 (*Zhao et al., 2016*)**.** $\|f_i(x_i,v_i,t)+d_i-f_0(x_0,v_0,t)\|_\infty\leq c$, $i\in\{1,\dots,N\}$, where c is a known constant.

Definition 1 (*Robust Finite-time Connectivity Preserving Consensus Tracking*)**.** Consider a multi-agent system composed of N followers with dynamics (1) and a leader with dynamics (2). Each agent can sense only up to a distance R from it. Suppose that Assumptions 1–2 hold. The robust finite-time connectivity preserving consensus tracking problem is solved if the system has the following properties: (1) the connectivity of the initial graph $\mathcal{G}(0)$ is preserved for all $t\geq 0$; (2) $x_i(t)\to x_0(t)$ and $v_i(t)\to v_0(t)$ in a finite time T, $i\in\{1,\dots,N\}$.

3.2. Control design

Consider a distributed integral sliding mode control law with the following form:

$$u_i=u_{nomi}+u_{disconi},\tag{3}$$