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Brief paper

State-feedback control of Markov jump linear systems with hidden-Markov mode observation☆

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ABSTRACT

In this paper, we study state-feedback control of Markov jump linear systems with partial information about the mode signal responsible for switching between dynamic modes. We assume that the controller can only access random samples of the mode signal according to a hidden-Markov observation process. Our formulation provides a novel framework to analyze and design feedback control laws for various Markov jump linear systems previously studied in the literature, such as the cases of (*i*) clustered observations, (*ii*) detector-based observations, and (*iii*) periodic observations. We present a procedure to transform the closed-loop system with hidden-Markov observations into a standard Markov jump linear system while preserving stability, H_2 norm, and H_∞ norm. Furthermore, based on this transformation, we propose a set of Linear Matrix Inequalities (LMI) to design feedback control laws for stabilization, H_2 suboptimal control, and H_∞ suboptimal control of discrete-time Markov jump linear systems under hidden-Markov observations of the mode signals. We conclude by illustrating our results with some numerical examples.

1. Introduction

Markov jump linear systems (Costa, Fragoso, & Marques, 2005) are an important class of switched systems in which the *mode signal*, responsible for controlling the switch among dynamic modes, is modeled by a time-homogeneous Markov chain. This class of systems has been widely used in multiple areas, such as robotics (Siqueira & Terra, 2004), economics (Blair & Sworder, 1975), and networked control (Hespanha, Naghshtabrizi, & Xu, 2007). Solutions to standard controller synthesis problems for Markov jump linear systems, such as stabilization, quadratic optimal control, H_2 optimal control, and H_∞ optimal control (see, e.g., Costa et al., 2005), can be found in the literature. These works are based on the assumption that the controller has full knowledge about the mode signal at any time instant. However, this assumption is not realistic in many practical scenarios.

To overcome this issue, several papers have investigated the effect of limited and/or uncertain knowledge about the mode signal. For example, do Val, Geromel, and Gonçalves (2002) studied H_2 suboptimal control of discrete-time Markov jump linear systems when the state space of the mode signal is partitioned into subsets, called *clusters*, and the controller only knows in which cluster the mode signal is at a given time. Similar studies in the context of H_∞ suboptimal control can be found in Fioravanti, Gonçalves, and Geromel (2014) and Gonçalves, Fioravanti, and Geromel (2012). Vargas, Costa, and do Val (2013) investigated quadratic optimal control problems in the extreme case of having a single mode cluster (i.e., when the mode signal cannot be observed). Many of the above works can be analyzed in a framework recently proposed by Costa, Fragoso, and Todorov (2015) in the context of H_2 suboptimal control, as long as the mode signal can be observed at any time instant. In a complementary line of work, we find some papers assuming that the mode signal can only be observed at particular sampling times, instead of at any time instant. In this direction, Cetinkaya and Hayakawa (2015) designed almost-surely stabilizing state-feedback gains when the sampling times follow a renewal process. Similarly, Cetinkaya and Hayakawa (2014) derived stabilizing state-feedback gains using Lyapunov-like functions under periodic observations.

In this paper, we propose a novel framework to analyze and design state-feedback control laws for discrete-time Markov jump

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linear systems when the observations of the mode signal by the controller are both clustered and randomized over time. Specifically, we assume that the random samples of the mode signal are obtained when, and only when, the mode signal takes particular values. The proposed observation process, called *hidden-Markov* – due to its similitude with hidden-Markov chains (Ephraim & Merhav, 2002) – recovers many relevant cases previously studied in the literature, such as those in Cetinkaya and Hayakawa (2014, 2015), Costa et al. (2015), do Val et al. (2002) and Gonçalves et al. (2012). It is important to remark that, since the observation process is hidden-Markovian, existing methods for analysis and control of Markov jump linear systems, such as those in Costa et al. (2005), do Val et al. (2002) and Gonçalves et al. (2012), do not apply to this case.

One of the main purposes of this paper is to show that, despite the generality of hidden-Markov observation processes, the resulting closed-loop system can be equivalently transformed into a (standard) Markov jump linear system while preserving important closed-loop properties, including mean-square stability, H_2 norm, and H_∞ norm. Furthermore, based on this transformation, we propose a set of Linear Matrix Inequalities (LMI) to design feedback control laws for stabilization, H_2 suboptimal control, and H_∞ suboptimal control of discrete-time Markov jump linear systems under hidden-Markov observations of the mode signal.

The paper is organized as follows. In Section 2, we formulate the state-feedback control problem for Markov jump linear systems with hidden-Markov observations of the mode signal. We show in Section 3 that the resulting closed-loop system can be transformed into a standard Markov jump linear system by embedding the (possibly non-Markovian) stochastic processes responsible for the random observation process into a standard Markov chain. Based on this transformation, in Section 4, we present an LMI formulation to design state-feedback gains for stabilization, H_2 suboptimal control, and H_∞ suboptimal control. We conclude by illustrating the obtained results by numerical simulations in Section 5.

The notation used in this paper is standard. Let $\mathbb{Z}$ and $\mathbb{N}$ denote the set of integers and nonnegative integers, respectively. The number of the elements of a finite set X is denoted by $|X|$. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the spaces of real n-vectors and $n \times m$ matrices, respectively. By $\|\cdot\|$, we denote the Euclidean norm on $\mathbb{R}^n$. $\Pr(\cdot)$ is used to denote the probability of an event. The probability of an event conditional on another event $\mathcal{A}$ is denoted by $\Pr(\cdot \mid \mathcal{A})$. Expectations are denoted by $E[\cdot]$. The identity matrix with dimension d is denoted by I_d. Let A be a real matrix. Define $\mathrm{He}(A) = A + A^\top$. When A is positive definite, we write $A > 0$. The symbol $\star$ is used to denote the symmetric blocks of partitioned symmetric matrices. Finally, indicator functions are denoted by $\mathbb{1}(\cdot)$.

2. Problem formulation

In this section, we formulate the problems under study. Let $r = \{r(k)\}_{k=0}^\infty$ be a time-homogeneous Markov chain taking values in a finite set Θ with transition probabilities $\Pr(r(k+1) = \theta' | r(k) = \theta) = p_{\theta\theta'}$ for $\theta, \theta' \in \Theta$. Let n, m, q, and ℓ be positive integers and, for each $\theta \in \Theta$, let $A_\theta \in \mathbb{R}^{n\times n}$, $B_\theta \in \mathbb{R}^{n\times m}$, $C_\theta \in \mathbb{R}^{\ell\times n}$, $D_\theta \in \mathbb{R}^{\ell\times m}$, and $E_\theta \in \mathbb{R}^{n\times q}$. Consider the Markov jump linear system Σ given as

$$\begin{aligned} x(k+1) &= A_{r(k)}x(k) + B_{r(k)}u(k) + E_{r(k)}w(k),\\ z(k) &= C_{r(k)}x(k) + D_{r(k)}u(k). \end{aligned} \tag{1}$$

We call x and r the *state* and the *mode* of Σ, respectively. The signal w represents an exogenous disturbance, u is the control input, and z is the controlled output. The initial conditions are denoted by $x(0) = x_0$ and $r(0) = r_0$. We will assume that x_0 and r_0 are either deterministic constants or random variables, depending on the particular control problem considered.

2.1. Hidden-Markov mode observation

In this paper, we consider the situation where the controller cannot measure the mode signal at every time instant. To study this case, we model the times at which the controller can observe the mode by a stochastic process $t = \{t_i\}_{i=0}^\infty$ taking values in $\mathbb{N} \cup \{\infty\}$. We call t the *observation process* and each t_i an *observation time*. For each i, we assume either $t_i < t_{i+1}$ or $t_i = t_{i+1} = \infty$. It is understood that, if $t_i < t_{i+1} = \infty$, then no observation will be performed after time t_i. In this paper, we focus on the following class of observation processes:

Definition 1. We say that an observation process t is *hidden-Markovian* if there exists a subset $\Theta_o \subset \Theta$ such that $t_0 = \min\{k \geq 0 : r(k) \in \Theta_o\}$ and, for every $i \geq 0$, $t_{i+1} = \min\{k > t_i : r(k) \in \Theta_o\}$, where the minimum of the empty set is understood to be ∞.

We can interpret Θ_o as the set of modes that are observable from the controller. In the extreme case of $\Theta_o = \Theta$, the mode is observed at every time instant. Although Definition 1 requires the observation process to be correlated with the dynamics of the plant, in practice, it is possible to use an observation process independent of the plant, as illustrated in the following example:

Example 2. Consider a Markov jump linear system Σ_P given by

$$\begin{aligned} x(k+1) &= A_{P,r_P(k)}x(k)+B_{P,r_P(k)}u(k)+E_{P,r_P(k)}w(k),\\ z(k) &= C_{P,r_P(k)}x(k)+D_{P,r_P(k)}u(k), \end{aligned} \tag{2}$$

for a Markov chain r_P having a finite state space Θ_P and appropriately defined coefficient matrices $A_{P,\theta_P}, \dots$, and E_{P,θ_P} for $\theta_P \in \Theta_P$. Let us consider the situation where the observation process t is defined independently of the given system. Specifically, assume the existence of another Markov chain r_K, defined over a finite set Θ_K and independent of r_P, and a subset $\Theta_{K,o} \subset \Theta_K$ such that $t_0 = \min\{k \geq 0 : r_K(k) \in \Theta_{K,o}\}$ and, for every $i \geq 0$, $t_{i+1} = \min\{k > t_i : r_K(k) \in \Theta_{K,o}\}$. We can show that this observation process, which we call an *independent hidden-Markov observation process*, can be regarded as specifying a hidden-Markov observation process in the sense of Definition 1, as we see below. For all $\theta_P \in \Theta_P$ and $\theta_K \in \Theta_K$, define $A_{(\theta_P,\theta_K)} = A_{P,\theta_P}$, $B_{(\theta_P,\theta_K)} = B_{P,\theta_P}$, …, and $E_{(\theta_P,\theta_K)} = E_{P,\theta_P}$. Let us also introduce the extended Markov chain $r = (r_P, r_K)$ taking values in $\Theta = \Theta_P \times \Theta_K$. We can then see that Σ_P is equivalent to the Markov jump linear system Σ (given in (1)). Also, we can confirm that the above observation process can be realized as the observation process in the sense of Definition 1 if we set $\Theta_o = \Theta_P \times \Theta_{K,o} \subset \Theta$. As we see in Section 5, the flexibility of being able to design observation processes that are independent of the plant to be controlled allows us to recover interesting cases in the literature.

Apart from the random uncertainties in the observation times described above, we also consider the situation in which the controller may only observe partial information about the mode signal. Specifically, we assume that the set Θ_o of observable modes is divided into nonempty subsets $\mathcal{C}_1, \dots, \mathcal{C}_N$ called clusters (do Val et al., 2002), and that the controller can only observe to which cluster the mode signal belongs at each observation time. In the particular situation where Θ_o equals the entire space Θ, the proposed observation process reduces to the case studied in do Val et al. (2002) and Gonçalves et al. (2012). Throughout the paper, we let $\pi : \Theta_o \to \{1, \dots, N\}$ be defined by

$$\pi(\theta) = k, \ \text{if}\ \theta \in \mathcal{C}_k, \tag{3}$$

which denotes the mapping of a mode into the integer index of the cluster the mode belongs to.

The combination of a hidden-Markov observation process and clustered observations enables us to realize the detector-based observations studied by Costa et al. (2015):