Automatica 89 (2018) 111–116

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

The partial pinning control strategy for large complex networks ☆

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ARTICLE INFO

Article history:
Received 10 June 2016
Received in revised form 29 May 2017
Accepted 21 September 2017

ABSTRACT

In large directed complex networks, it may result unfeasible to successfully pinning control the whole network. Indeed, when the pinner node can be connected only to a limited number of nodes, it may be impossible to guarantee pinning controllability of all the network nodes. In this paper, we introduce the partial pinning control problem, which consists in determining the optimal selection of the nodes to be pinned so as to maximize the fraction of nodes of the whole network that can be asymptotically controlled to the pinner's trajectory. A suboptimal solution to this problem is provided for a class of nonlinear node dynamics, together with the bounds on the minimum coupling and control gains required to "partially control" the network. The theoretical analysis is translated into an integer linear program (ILP), which is solved on a testbed network of 688 nodes.

1. Introduction

An increasing number of complex control systems in applications can be modeled as networks of nonlinear dynamical agents (the nodes), communicating with the others via a communication protocol defined on the network edges. Researchers in different areas of applied science and engineering have been addressing the problem of selecting the network topology and the communication protocols among agents in order for the complex network to perform a desired function. Examples include rendezvous and flocking problems in robotics (Cortes, Martinez, & Bullo, 2006; Han & Ge, 2015; Tanner, Jadbabaie, & Pappas, 2007), synchronization of sensor networks (An et al., 2011), consensus and multi-agent coordination problems in control theory (DeLellis, di Bernardo, Gorochowski, & Russo, 2010; Li, Wen, Duan, & Ren, 2015), and the emergence of coordinated motion in biological settings (Ghosh, Rangarajan, & Sinha, 2010; Paley, Leonard, Sepulchre, Grunbaum, & Parrish, 2007).

Substantial attention has been devoted on synchronization (Belykh, Belykh, & Hasler, 2006; Gao, Meng, Chen, & Lam, 2010; Liu & Chen, 2015; Pecora & Caroll, 1990) and consensus of complex networks (DeLellis, diBernardo, Garofalo, & Liuzza, 2010; Li, Chen, Su, & Li, 2016). The idea is to find strategies to regulate the behavior of large ensembles of interacting agents that ensure all systems in the network evolve towards the same asymptotic trajectory (Vishnampet, 1993). At first, diffusively coupled identical nonlinear systems were considered: given the node dynamics, the problem becomes that of determining the range of the values of the coupling gains for which the network synchronizes. This *synchronizability problem* has been solved mainly by using the so-called Master Stability Function approach (Pecora & Caroll, 1998), contraction theory (Lohmiller & Slotine, 1998; Russo & di Bernardo, 2009), and passivity tools (Gao, Chen, & Chai, 2007).

Although analytical conditions for synchronizing all nodes towards an asymptotic solution were obtained, a major problem still remains from a control viewpoint. Indeed, such common solution, if it exists, cannot be arbitrarily imposed. A possible strategy to achieve this goal would be to directly add some feedback control input on each of the systems in the network so to steer the dynamics of each agent towards the desired trajectory. In practice, when more than a handful of agents are considered, this approach is not viable. A feasible alternative is represented by *Pinning Control* (Huang & Manton, 2009; Li, Sun, Small, & Fu, 2015; Wang & Chen, 2002), where the control action is exerted through an additional node, the *pinner*, which is directly connected only to a subset of the network nodes, the *pinned nodes*. In this scenario, the problem consists not only in designing the strength and form of the control action to be exerted by the pinner, but also in determining how many, and which pinned nodes need to be selected to achieve the control objective (Porfiri & di Bernardo, 2008; Sorrentino, di Bernardo, Garofalo, & Chen, 2007). In the recent literature, an optimal location of the pinned nodes is sought, so as to guarantee that all the network nodes asymptotically follow the reference trajectory imposed by the pinner. To control directed networks,

☆ The material in this paper was partially presented at the 55th IEEE Conference on Decision and Control, December 12–14, 2016, Las Vegas, NV, USA. This paper was recommended for publication in revised form by Associate Editors Antonis Papachristodoulou and Bert Tanner under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2017.11.025
0005-1098/

under suitable assumptions on the individual dynamics, if the network graph admits a spanning tree, it suffices to pin the root of this tree (Chen, Liu, & Lu, 2007). Otherwise, it is necessary to pin at least one node in each root strongly connected component (RSCC) of the network (Lu, Li, & Rong, 2010), that is, each SCC the nodes of which have incoming edges only from nodes of the same SCC.

In this paper, we take a different point of view with respect to Refs. Chen et al. (2007) and Lu et al. (2010). Inspired by the work on controllability of large networks (DeLellis, Garofalo, & Lo Iudice, 2016; Gao, Liu, D'Souza, & Barabási, 2014; Lo Iudice, Garofalo, & Sorrentino, 2015), we consider the case in which, for technological or economic reasons, the pinner signal can only reach a limited number of nodes belonging to a given set. Moreover, to allow coping with the limitations arising when dealing with non-ideal actuators, see e.g. Ref. Ocampo-Martinez, Puig, Cembrano, and Quevedo (2013), we assume that constraints exist on the value of the coupling and control gains which could lead to pinning nodes in non root SCCs to control the whole network. As these restrictions may not allow complete pinning controllability, a question naturally arises: which nodes must be pinned to drag the greatest number of nodes to the pinner's trajectory? We call this the *partial pinning control* problem and, after providing an analytic solution, we translate it into an integer linear program (ILP). Moreover, an optimization problem is formulated and solved to select the pinning and coupling gains. The effectiveness of the approach is then illustrated on a testbed example.

2. Mathematical preliminaries and notation

Let us consider a directed graph (digraph) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ and $\mathcal{E}$ are the set of vertexes and edges of $\mathcal{G}$, respectively. A is the adjacency matrix of $\mathcal{G}$, and its ijth element a_{ij} is greater than zero if there exists an edge from j to i, while it is zero otherwise. Moreover, the ijth element ℓ_{ij} of the Laplacian matrix L of the digraph is equal to $-a_{ij}$ if $j \neq i$ while it is $\sum_{j=1}^{N} a_{ij}$ if $j = i$. Any digraph $\mathcal{G}$ can be decomposed in its σ SCCs $\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i)$, where $\mathcal{V}_i$ is the set of nodes of $\mathcal{G}_i$ and $\mathcal{E}_i = \{(l, m) \in \mathcal{E} : l, m \in \mathcal{V}_i\}$ the set of edges, and we label the SCCs so that only the first $\rho \geq 1$ are also RSCCs. The Directed Acyclic Graph (DAG) condensation $\mathcal{G}^D = (\mathcal{V}^D, \mathcal{E}^D)$ of $\mathcal{G}$ is a graph whose nodes represent the SCCs of $\mathcal{G}$ while $(i, j) \in \mathcal{E}^D$ if, in $\mathcal{G}$, there exists at least an edge connecting a node of $\mathcal{V}_j$ to one of $\mathcal{V}_i$. Every node i of $\mathcal{G}$ has a set of nodes in its downstream, as we say that node j_l is in the downstream of node j_1 (j_1 is in the upstream of j_l) if there exists a sequence $\{\ell_{j_{i+1}j_i}\}_{i=1}^{l-1}$ of nonzero entries of the Laplacian L, that is, if there exists a directed path from node j_1 to node j_l. We denote by $\Gamma(\mathcal{G}_i)$ ($\Gamma^T(\mathcal{G}_i)$) the set of nodes of $\mathcal{G}$ that are only in the downstream (upstream) of the nodes in $\mathcal{V}_i$, including the nodes in $\mathcal{V}_i$ itself. Leveraging the decomposition in layers of a DAG (Liu, Slotine, & Barabási, 2012), we can now define the levels of a graph $\mathcal{G}$.

Definition 1. Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a set of nodes $\Gamma \subseteq \mathcal{V}$, the mth level of Γ is

$$\mathcal{R}_m := \{\mathcal{V}_l \subseteq \Gamma : \exists (i, j) \in \mathcal{V}_l \times \mathcal{R}_{m-1}, \nexists (i, j) \in \mathcal{V}_l \times \mathcal{B}_m\},$$

with $\mathcal{B}_m := \{\mathcal{V}_l \subseteq (\Gamma - \cup_{k=0}^{m-1} \mathcal{R}_k)\}$, $\mathcal{R}_0 := \{\mathcal{V}_l \subseteq \Gamma : \nexists (i, j) \in \mathcal{V}_l \times (\Gamma - \mathcal{V}_l)\}$; b is the smallest integer such that $\mathcal{R}_{b+1} = \emptyset$.

In what follows, when referring to an SCC, we will use a double subscript to identify the specific SCC the nodes of which are in a given level of Γ. For instance, $\mathcal{V}_{ij}$ will identify the set of nodes of the ith SCC belonging to $\mathcal{R}_j$, see Fig. 1. Finally, given a set χ, we denote its cardinality by $|\chi|$. $\mathrm{diag}\{d_1, \ldots, d_m\}$ denotes the $m \times m$ diagonal matrix with diagonal elements $d_1, \ldots, d_m$, while 1_m is an m-dimensional vector of ones, and I_m is the $m \times m$ identity matrix. $\mathcal{D}_m^+$ is the set of positive definite diagonal matrices in $\mathbb{R}^{m \times m}$. Given a square matrix $M \in \mathbb{R}^{m \times m}$, we denote its symmetric part $M_{\mathrm{sym}} := 0.5(M + M^T)$ and its eigenvalues as $\lambda_1(M), \ldots, \lambda_m(M)$.

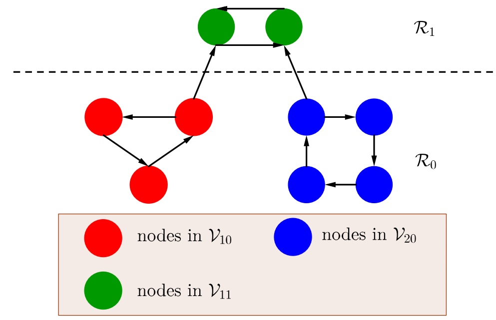

Fig. 1. Representation of the sets $\mathcal{R}_0, \mathcal{R}_1, \ldots, \mathcal{R}_b$ on a sample DAG condensation $\mathcal{G}^D$ of a network graph $\mathcal{G}$.

3. Partial pinning control

3.1. Problem formulation

We consider a linearly coupled network described by a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where the $N = |\mathcal{V}|$ nodes are nonlinear dynamical systems with state $x_i \in \mathbb{R}^n$, while the edges describe the interconnections among the nodes. An extra node, the pinner, with state $s \in \mathbb{R}^n$ and identical dynamics but different initial conditions, is added to the network. A subset of $\mathcal{V}$, say $\mathcal{C}$, is the set of *pinnable nodes* that can be directly controlled by the pinner. We call *pinned* the nodes in $\mathcal{P} \subseteq \mathcal{C}$ that actually receive an input from the pinner. The network dynamics are

$$\dot{x}_i = f(x_i, t) + c \sum_{j=1}^{N} a_{ij} H(x_j - x_i) - \kappa \delta_i H(x_i - s), \qquad (1)$$

for $i \in \mathcal{V}$, where $\dot{s} = f(s, t)$ are the pinner's dynamics, $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is the nonlinear vector field describing the node dynamics, $c, \kappa \in \mathbb{R}$ are the coupling and control gains, respectively, $H \in \mathbb{R}^{n \times n}$ is the inner coupling matrix describing the information exchanged among neighboring nodes, and $\delta_i = 1$ if $i \in \mathcal{P}$, and 0 otherwise.

Definition 2. Network (1) is q-partially pinning controlled to the pinner's trajectory when

$$\lim_{t \to +\infty} \|x_i(t) - s(t)\| = 0, \quad i \in \mathcal{Q}, \qquad (2)$$

where $\mathcal{Q} \subseteq \mathcal{V}$ and $q = |\mathcal{Q}|$; when $q = N$, network (3) is fully pinning controlled, while when $\mathcal{V}_i \subseteq \mathcal{Q}$, then we say that the SCC $\mathcal{G}_i$ is pinning controlled.

Problem 1. Partial pinning control.

$$\begin{aligned} & q^* = \max_{\mathcal{P} \subseteq \mathcal{C}} |\mathcal{Q}| \\ & |\mathcal{P}| = p \\ & c \leq c_M, \; \kappa \leq \kappa_M. \end{aligned} \qquad (3)$$

Before illustrating the problem solution, we need to give the following definition:

Definition 3 (*DeLellis, di Bernardo, & Russo, 2011; Lu & Chen, 2006*). Given two $n \times n$ matrices $V > 0$ and W, a vector field $g : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is QUAD(V, W) if $(x - s(t))V(g(x, t) - g(s(t), t)) \leq (x - s(t))^T W (x - s(t))$, for all $x \in \mathbb{R}^n$, $t \in \mathbb{R}^+$, where $s(t)$ is the pinner trajectory.

3.2. Problem solution

Here, we give the conditions ensuring pinning controllability of any given SCC to then translate Problem 1 into the maximization