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Adaptive control-based Barrier Lyapunov Functions for a class of stochastic nonlinear systems with full state constraints ☆

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ABSTRACT

An adaptive control scheme is developed in the paper for nonlinear stochastic systems with unknown parameters. All the states in the systems are required to be constrained in a bounded compact set, i.e., the full state constraints are considered in the systems. It is for the first time to control nonlinear stochastic systems with the full state constraints. In contrast to deterministic systems, the stochastic systems with the full state constraints are more difficult to be stabilized and the design procedures are more complicated. By constructing Barrier Lyapunov Functions (BLF) in symmetric and asymmetric forms, it can be ensured that all the states of the stochastic systems are not to transgress their constraint bounds. Thus, the proposed scheme not only solves the stability problem of stochastic systems, but also overcomes the effect of the full state constraints on the control performance. Finally, it is proved that all the signals in the closed-loop system are semi-global uniformly ultimately bounded (SGUUB) in probability, the system output is driven to follow the reference signals, and all the states are ensured to remain in the predefined compact sets. The validity of the proposed scheme is verified by a simulation example.

1. Introduction

In recent years, the stability of nonlinear stochastic systems has attracted much attention in the control theory community. The stochastic disturbance is often containing in the practical systems and its presence is an instability source of systems. At present, most of results focused on stability of nonlinear deterministic systems (Ahn, Shi, & Basin, 2015; Boulkroune, Bouzeriba, & Bouden, 2016; Boulkroune & M'saad, 2012; Boulkroune, Tadjine, M'Saad, & Farza, 2010, 2014; Chen, Feng, & Su, 2016; Chen & Ge, 2013; Chen, Ge, & Ren, 2011; Chen, Hua, & Ge, 2014; Fu, Ma, & Chai, 2015; Ge & Tee, 2007; Ge & Wang, 2004; Ge, Yang, Dai, Jiao, & Lee, 2009; Hamdy, 2013; Hamdy & El-Ghazaly, 2014; He & Ge, 2016; Krstic, Kokotovic, & Kanellakopoulos, 1995; Su, Stepanenko, Svoboda, & Leung, 2000; Tong, Zhang, & Li, 2016; Yoo, 2013; Yoo & Park, 2009; Yoo, Park, & Choi, 2009; Zhou, Li, Wu, Wang, & Ahn, 2017). The research and development of stochastic systems is very tardy in comparison with deterministic systems.

The result on global output-feedback stabilization in probability was first developed in Deng and Krstic (1999) for stochastic nonlinear continuous-time systems by using the backstepping design and designing inverse optimal controllers. In Deng and Krstić (2000), the control problem of stochastic nonlinear systems in output-feedback with noise and its covariance in time varying was addressed, and the designed controller guaranteed that the solutions converge (in probability) to a residual set proportional to the unknown bound on the covariance. In Deng, Krstic, and Williams (2001) solved a stochastic disturbance attenuation problem and the main task is to make the system solution bounded by a monotone function of the supremum of the covariance of the noise. A class of nonlinear stochastic systems with unmodeled dynamics and uncertain nonlinear functions were controlled in Wu, Xie, and Zhang (2007). By using the concept of input-to-state practical stability and improving small-gain theorem in stochastic case, an adaptive controller via output feedback was well constructed. It is subsequent to solve the problem of the adaptive tracking for nonlinear stochastic systems with stationary Markovian switching

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(Wu, Yang, & Shi, 2010). In Liu, Yin, Gao, Alsaadi, and Hayat (2015), an adaptive stabilization control was studied for nonlinear high-order systems with stochastic inverse and uncertain parameterization nonlinear. An adaptive partial state feedback control was framed to render globally stability in probability of the closed-loop system. Based on the neural networks (NNs), the control problem via output feedback of uncertain stochastic nonlinear systems in strict-feedback and with time-varying delays was firstly developed in Chen, Jiao, Li, and Li (2010) where the NNs are employed to approximate the unknown functions. Afterwards, the adaptive fuzzy or neural output feedback control designs were proposed for nonlinear stochastic strict feedback (Tong, Li, Li, & Liu, 2011; Zhou, Shi, Xu, & Li, 2013), pure-feedback (Chen, Liu, & Wen, 2014), nonstrict feedback (Wang, Liu, Liu, Chen, & Lin, 2015) systems. The adaptive neural tracking control was developed in Niu, Ahn, Li, and Liu (2017); Zhao, Shi, Zheng, and Zhang (2015) for a class of nonlinear switched stochastic systems form with actuator dead-zone and arbitrary switchings.

However, it can be known from the above results that all the states of the considered stochastic systems are unconstrained and this will result in lack of practicability. To this end, two pioneering developments were proposed in Tee, Ge, and Tay (2009a) and Tee, Ren, and Ge (2011) for nonlinear systems with constant and time-varying output constraints. From then on, much attention has been attracted for nonlinear constraint control and many significant works are received in recent years. In Ren, Ge, Tee, and Lee (2010), an adaptive neural control was studied for nonlinear systems in output feedback with only output constraint based on a BLF. For practical output constraint systems such as electrostatic microactuators with bidirectional drive, a flexible crane system and wheeled mobile robotic system, the authors in He, Zhang, and Ge (2014) and Tee, Ge, and Tay (2009b) proposed the adaptive constraint control schemes by using the BLF design. The constraint control of a robotic manipulator with the joint space constraints was specified in Tang, Ge, Tee, and He (2016) based on the integral BLFs where the neural networks are utilized to compensate for the unknown robot dynamics and the external force. The control problem of partial state constraints for strict-feedback systems was addressed in Tee and Ge (2011) and a novel BLF method is used in order to prevent violation of the state constraints. Subsequently, the constraint control methods focused on the full state constraints. In Kim and Yoo (2014), Liu and Tong (2016) and Liu and Tong (2017), the framework of the full state constraints on nonlinear pure-feedback systems or strict-feedback unknown control direction systems was constructed based on the BLFs and the mean value theorem. An adaptive control via an observer design was got for uncertain nonlinear systems with full-state constraints (Liu, Li, & Tong, 2014). The adaptive tracking problem for an uncertain n-link robot with full-state constraints was solved in He, Chen, and Yin (2016) based on the BLF with the neural approximation. For wheeled mobile robotic system with full state constraints, the authors in Ding, Li, Liu, Gao, Chen, and Deng (2017) proposed an adaptive constraint control strategy by using the BLF design. It should be mentioned that in these constraint controls, an important problem on the stochastic disturbance is omitted. In Yin, Yu, Shahnazi, and Haghani (2017), the adaptive control problem of output constraint control switched systems with stochastic disturbances was worked out by providing a mapping to transform the constrained system into an unconstrained one. However, there are two major shortcomings: (1) all the design procedures are obtained based on unconstrained systems, and then, lack of novel design tool is used in the stochastic constraint systems; (2) only the output is constrained in the systems and the full state constraints are not considered.

This paper will be to design an adaptive controller for uncertain nonlinear stochastic parametric systems and all the states are constrained in a bounded compact set, i.e., the systems are required to subject to the full state constraints. The full state constraints are for the first time to take into account in the stochastic systems. Due to the coexistence of the stochastic disturbances and the full state constraints, it will become more difficult in the controller design in contrast to the deterministic systems. The symmetric and asymmetric BLFs will be constructed to solve their constraint control problems and all the states of the stochastic systems are guaranteed that their constraint bounds are not transgressed. All the signals in the closed-loop system are proven to be SGUUB in probability, the system output can follow the reference signal to a small compact set, and all the states are ensured to remain in the predefined compact sets. Finally, the validity of the proposed scheme can be verified by using a simulation example.

2. Problem descriptions and basic knowledge

Consider a class of SISO stochastic nonlinear systems

$$\begin{cases} dx_i = \left(\mu_i^T \varphi_i\left(\bar{x}_i\right) + \phi_i\left(\bar{x}_i\right) x_{i+1}\right) dt \\ \quad + g_i^T\left(\bar{x}_i\right) d\omega, i = 1, \ldots, n-1 \\ dx_n = \left(\mu_n^T \varphi_n\left(\bar{x}_n\right) + \phi_n\left(\bar{x}_n\right) u\right) dt + g_n^T\left(\bar{x}_n\right) d\omega \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$, $y \in R$ and $u \in R$ are the system state vector, the system output, and the control input, respectively; $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $i = 1, 2, \ldots, n$; $\varphi_i\left(\bar{x}_i\right)$ and $\phi_i\left(\bar{x}_i\right)$ are known nonlinear function vectors; μ_i is an unknown constant vector; $g_i\left(\bar{x}_i\right)$ is a known smooth function vector; and ω is a standard Wiener process. In this study, all the states are constrained in the compact sets, i.e., $|x_i| < k_{c_i}$ where k_{c_i} is a known positive constant.

Remark 1. In He et al. (2016), Liu et al. (2014), Kim and Yoo (2014), Liu and Tong (2016) and Tee et al. (2009b), the adaptive control of the full state constraints was addressed for uncertain nonlinear systems. However, all the results omitted an important element the stochastic disturbance. In fact, when the stochastic disturbance appears in the systems, the design procedure in the above literatures will not hold and needs to be reconstructed. In this paper, the stochastic disturbance will be handled and the stochastic type of BLFs will be used.

The control objective of the paper is to develop an adaptive control scheme such that (1) all the signals in the closed-loop system are guaranteed to be SGUUB in probability; (2) y is driven to follow the reference signal $y_d(t)$ to a bounded compact set; and (3) all the states cannot violate their constrained bounds where $y_d(t)$ is a known and bounded function.

The control objective can be implemented in the following assumptions.

Assumption 1 (*Tee et al., 2011*). For any constant $k_{c_i} > 0$, there exist positive constants $\bar{y}_d$, $\underline{y}_d$, A_1, Y_i, $i = 1, 2, \ldots, n$, and $\max\left\{\underline{y}_d, \bar{y}_d\right\} \le A_1 < k_{c_1}$ such that the desired trajectory $y_d(t)$ and its ith order derivatives $y_d^{(i)}(t)$ satisfy $-\underline{y}_d \le y_d(t) \le \bar{y}_d$ and $\left|y_d^{(i)}(t)\right| \le Y_i$ where $\underline{y}_d, \bar{y}_d, Y_1, \cdots, Y_n$ are positive constants for all $t \ge 0$ and $i = 1, \ldots, n$.

Consider the stochastic nonlinear system below:

$$dx = f(x)\, dt + g(x)\, d\omega \tag{2}$$

where x is the system state, $f(x) \in R$ and $g(x) \in R^{n\times r}$ are the locally Lipschitz functions, and ω denotes an r-dimensional standard Wiener process.