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Brief paper

Consensus control for a network of high order continuous-time agents with communication delays[☆]

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ABSTRACT

This paper is concerned with the consensus control problem for multi-agent systems with agents characterized by high-order linear continuous-time systems subject to communication delays between neighbouring nodes in the network. A new consensus protocol is proposed. It requires communication between neighbouring agents only at certain sampling points, rather than at all times. It is also unique in the sense that it is nonlinear in the continuous-time domain but linear when the agents are viewed in the sampled-data domain. Under the proposed consensus protocol, marginally stable multi-agent systems can reach consensus for any large delay. Unstable multi-agent systems achieve consensus when the time delay is within a certain range. Moreover, in the single-input case, we give an optimal control gain which yields the fastest consensus speed. The proposed technique is expected to pave a new way for new theoretical studies on network properties required for consensus control.

1. Introduction

Consensus is a process that a group of agents with different initial states reach an agreement by local communication between agents. As a distributed cooperative control of multi-agent systems, consensus control is closely related to problems such as flocking (Tanner, Jadbabaie, & Pappas, 2007), formation control (Fax & Murray, 2004), and network congestion control (Paganini, Doyle, & Low, 2001). Consensus algorithms also find wide applications in many disciplines, including smart grid (Mou, Xing, Lin, & Fu, 2015), sensor networks (Kar & Moura, 2010) and distributed parameter estimation (Kar, Moura, & Ramanan, 2012).

Consensus control problems have attracted a lot of attention, see, e.g., Ma and Zhang (2010) and You and Xie (2011a, b). Reference Ma & Zhang (2010) considers the consensus control problem for the following multi-agent system

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \dots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ represent the state and the control input of the ith agent, respectively; $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. The consensus protocol is given by

$$u_i(t) = K\sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)),\ i = 1, \dots, N, \tag{2}$$

where $\{a_{ij},\ i,\ j = 1, \dots, N\}$ are elements of the adjacency matrix and $K \in \mathbb{R}^{m\times n}$ is a gain matrix. It is shown in Ma and Zhang (2010) that there exists a gain K such that the multi-agent system (1) reaches consensus under the protocol (2) if and only if (A, B) is stabilizable and the network topology has a spanning tree. In this case, such a K can be constructed by a standard Riccati equation. It is also pointed out in Ma and Zhang (2010) that the above results fail to have counterparts in discrete-time linear multi-agent systems. A necessary and sufficient consensusability condition for discrete-time multi-agent systems with a single input is presented in You and Xie (2011b). Besides a controllability requirement, this condition contains an inequality involving unstable eigenvalues of A and the ratio λ_2/λ_N (where λ_2 and λ_N are the smallest and the largest non-zero eigenvalues of the Laplacian matrix for the

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network topology, respectively). The control gain solving consensus is given by modified Riccati inequalities.

Aforementioned works all deal with consensus problems without delay. When delays happen in the information transmission between neighbours, a commonly used consensus protocol is

$$u_i(t) = K \sum_{j=1}^{N} a_{ij}(x_j(t-\tau) - x_i(t-\tau)). \tag{3}$$

Most works in the literature study consensus control problems with time delay in the following framework: for a fixed K, seek an upper bound $\bar{\tau}$ for the delay such that consensus can always be achieved under protocol (3) for any $\tau \in [0, \bar{\tau})$, see Cepeda-Gomez (2015), Munz, Papachristodoulou, and Allgower (2010), Olfati-Saber and Murray (2004) and Xu, Zhang, and Xie (2013). For example, Olfati-Saber and Murray (2004) considers integrator dynamics and obtains an exact delay bound ('exact' means that the bound is necessary and sufficient) for the protocol (3) with $K = 1$ by analysing the roots of certain characteristic equation. Cepeda-Gomez (2015) investigates high-order multi-agent systems and characterizes the exact delay bound for general gains by using the cluster treatment of characteristic roots paradigm. Departing from these works, Li and Fu (2016), Wang, Zhang, and Fu (2015), and Zhou and Lin (2014) discuss consensus control problems with time delay in another framework. They design K to be a function of delay τ, denoted by $K(\tau)$, such that protocol (3) with $K = K(\tau)$ renders system consensus when the delay is equal to τ. It is not a concern whether this control gain works for other values of delay. Zhou and Lin (2014) focuses on system (1) where all the eigenvalues of A lie on the imaginary axis. It is shown that consensus can be achieved for arbitrarily large delay. Allowing A to have eigenvalues on the open right-half plane, Wang et al. (2015) give a delay bound below which consensus can be achieved. However, this bound is presented using the maximal value of a function, which cannot be solved analytically.

This paper is concerned with the consensus problem for the multi-agent system (1) with communication delays. Different from (3), a new consensus protocol is proposed. It requires relative state and input signals between neighbouring agents only at certain sampling points, rather than all the time. This means that only a limited amount of communication is needed between neighbouring agents. Our consensus control gain is delay dependent like Wang et al. (2015) and Zhou and Lin (2014). The motivation for designing such a gain is that we hope to deal with larger delay than using a delay-independent gain as in Xu et al. (2013). The method of constructing consensus control gains is using modified Riccati inequalities and is from reference You & Xie (2011b). Our approach is as follows. First, the consensus problem for discrete-time multi-agent systems with multi-step communication delay is studied. It is transformed to a delay-free consensus problem by the reduction technique (Artstein, 1982). Then, this result is applied to the problem under consideration via the sampled-data models. The contribution of this paper includes two aspects. First, for marginally stable agents (here, "marginally stable" means that all the eigenvalues of the system are located on the closed left-half plane), consensus is guaranteed for any large delay. For unstable agents, consensus is achieved when the delay is below a bound which depends on the network topology and the agent dynamics. This bound is shown to be larger than that in Wang et al. (2015) and Xu et al. (2013) in some cases. Secondly, the influence of consensus control gains on the consensus speed is investigated and an optimal gain yielding the fastest consensus speed is provided in the single-input case.

The rest of the paper is organized as follows. The problem formulation is given in Section 2. The consensus control problem for discrete-time multi-agent systems with multi-step delay is discussed in Section 3. The problem under consideration is solved in Section 4. Performance analysis of the proposed consensus protocol is given in Section 5. Numerical examples are provided in Section 6. Conclusions are presented in Section 7. A useful proposition is given in the Appendix.

Notations: $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the sets of n-order column vectors and $n \times m$-order matrices with real elements, respectively. For a complex number c, $\mathrm{Re}(c)$, $\mathrm{Im}(c)$, $|c|$, and $\bar{c}$ stand for its real part, imaginary part, modular, and conjugate, respectively. For a matrix $X \in \mathbb{R}^{n\times m}$, X' is its transpose. For a matrix $X \in \mathbb{R}^{n\times n}$, $\rho(X)$, $\mathrm{tr}(X)$, and $\lambda_j(X)$, $j = 1, \ldots, n$, denote its spectral radius, trace and eigenvalues, respectively. For a symmetric matrix X, $X > 0$ means that it is positive definite. For a positive integer N, $\bar{N}$ represents the set $\{1, \ldots, N\}$; e^X represents the exponential of a matrix.

2. Problem formulation

Let the directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ denote the communication topology between multi-agents with the set of vertices $\mathcal{V} = \{1, 2, \ldots, N\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The ith vertex represents the ith agent and the edge $(i, j) \in \mathcal{E}$ denotes that the agent j receives information from the agent i. Self-edges are not allowed. The set of neighbours of the ith agent is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is called the weighted adjacency matrix of $\mathcal{G}$ with nonnegative elements and $a_{ij} > 0$ if and only if $j \in \mathcal{N}_i$. The in-degree of the ith vertex and the in-degree matrix are denoted by $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$, respectively. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Note that $a_{ij} = a_{ji}$, $\forall i, j \in \mathcal{V}$, if and only if $\mathcal{G}$ is an undirected graph (You & Xie, 2011b). Obviously, for an undirected graph, $\mathcal{L}$ is a symmetric, positive semi-definite matrix and all its eigenvalues λ_i, $i \in \bar{N}$, are non-negative. For a connected graph having a spanning tree, we have $0 = \lambda_1 < \lambda_2 \le \ldots \le \lambda_N$.

The dynamics of each agent is given by (1). Suppose the communication delay from agent j to agent i is $s_{ij}\tau$ where s_{ij} is a positive integer and τ is positive and constant. The maximal value of s_{ij} is $\bar{s}$, i.e., $\max_{i,j\in\bar{N}}\{s_{ij}\} = \bar{s}$. In this context, the available information for the controller $u_i(t)$ is $\{x_j(s), u_j(s) : s \le t - s_{ij}\tau,\ j \in \mathcal{N}_i\}$ and $\{x_i(s), u_i(s) : s \le t - s_{ii}\tau\}$. The aim is to design the controller $u_i(t)$ for each agent i using the above available information such that the multi-agent system (1) achieves consensus.

Definition 1. The agents in the network achieve consensus if $\lim_{t\to\infty} x_j(t) - x_i(t) = 0$, $\forall i, j \in \bar{N}$, for any initial value $x_i(0)$.

The following assumptions are made in this paper.

Assumption 1. The network topology $\mathcal{G}$ is an undirected connected graph.

Assumption 2. All the eigenvalues of A lie in the closed right-half plane.

Assumption 3. (A, B) is controllable and B has full column rank.

Remark 1. If some eigenvalues of A lie in the open left-half plane, it is a standard practice to decompose the system (1) into two subsystems, one asymptotically stable which requires no consensus control action, and one with eigenvalues in the closed right-half plane, which is considered under Assumption 2. Thus, Assumption 2 does not lose generality.