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Brief paper

# Universal strategies to explicit adaptive control of nonlinear time-delay systems with different structures[☆]

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ABSTRACT

The adaptive control problem of high-order time-delay nonlinear systems with lower-triangular or upper-triangular structure is studied in this paper. Remarkably, by using the function scaling gain strategy and homogeneous domination method, two kinds of explicit universal adaptive controllers are designed such that all the states of the closed-loop system are globally bounded and the solutions of the original system converge to zero. The robust adaptive controller is also designed by employing the $\sigma$-modification method. Examples are provided to demonstrate the validness of the theory.


## 1. Introduction

Recently, wide attention has been paid to adaptive control of nonlinear systems with uncertainties and time delays, see Koo, Choi, and Lim (2011), Liu and Tong (2017), Liu and Wu (2014) and Wu (2009). One reason for this is that nonlinear systems usually suffer from various uncertainties such as unknown parameters, time-varying disturbances and so on. Ignoring these factors will lead to poor system behavior. Another reason is that time-delay phenomenon exists in many systems (Efimov, Polyakov, Perruquetti, and Richard, 2016; Liu and Chopra, 2014; Zhang, Liu, Baron, and Boukas, 2011) (e.g., process industry system, mechanical system and biological system). For many systems, uncertainties and time delays are sources of instability, and hence should not be neglected.

As for the strict feedback system with state time delay, constant progress has been made over the past few decades, for instance, see Guan (2012), Hua, Liu, and Guan (2009), Jiao and Shen (2005), Wu (2009) and Yoo, Park, and Choi (2007). Particularly, in Wu (2009), a robust adaptive method was successfully proposed for lower-order time-delay nonlinear systems to achieve global stability. In Hua et al. (2009), by rigorously assuming nonlinear terms to be bounded by a special function, the authors introduced the backstepping technique for a class of nonlinear time-delay systems. To simplify the adaptive control design, an adaptive dynamic surface control (DSC) method was further introduced in Yoo et al. (2007). The above methods were mainly based on Lyapunov–Krasovskii (L–K) method. As another control strategy, LaSalle–Razumikhin (L–R) approach was developed in Jiao and Shen (2005) for adaptive control design of time-delay systems.

When the considered system involves input time delay, the adaptive control problem tends to be more challenging. Until now, there have been a lot of outstanding work, see Bresch-Pietri and Krstic (2010, 2014), Krstic (2010), Zhou, Wen, and Wang (2009), Zhu, Krstic, and Su (2017), Zhu, Su, and Krstic (2015) and the references therein. Specifically, to stabilize a class of uncertain linear systems with input delay, the authors in Zhou et al. (2009) raised a new adaptive backstepping method. Under different system conditions, Zhu et al. (2015) further studied the adaptive control problem and designed a controller using state feedback strategy. Noting that the state signals sometimes could not be measured, Zhu et al. (2017) incorporated the adaptive backstepping technique with the prediction-based boundary control method and constructed a new adaptive output feedback controller. What is more, in Bresch-Pietri and Krstic (2010), the authors presented a novel delay-adaptive prediction-based control method for linear systems with input delay. Later in Bresch-Pietri and Krstic (2014), this innovative method was extended to study a class of nonlinear systems with input delay. For more details of prediction-based control, see Krstic (2010).

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Ever since the pioneering work (Lin and Qian, 2000), many important results have been proposed for high-order nonlinear systems whose Jacobian linearization is neither controllable nor feedback linearizable, see Lei and Lin (2006), Lin, Chen, and Qian (2017), Polendo and Qian (2007) and Su, Qian, and Shen (2017). In recent several years, some of the studies have been further conducted under the framework of time-delay control theory, for example, see Gao, Wu, and Yuan (2016), Liu and Xie (2013) and Zhang et al. (2011). However, when such systems contain uncertainties in systems' parameters, state time delays and input time delays, to the authors' best knowledge, few methods can be employed for control design. The adaptive control problem of delayed high-order nonlinear systems remain unsolved. On the other hand, there exist very few methods discussing the universal adaptive control methods for nonlinear systems with lower-triangular form or upper-triangular form. Then, some natural and nontrivial questions are: *Under some conditions, can we propose a universal adaptive control strategy for nonlinear time-delay systems with lower-triangular or upper-triangular form? When the systems involve disturbances, is it possible to solve the robust adaptive control problem?*

In this paper, we will try to find solutions to these problems. The main contributions are: (i) Universal strategies are proposed to solve the adaptive control problem of nonlinear time-delay systems. Different from the existing results such as Wu (2009), Bresch-Pietri and Krstic (2010), Zhou et al. (2009), Zhu et al. (2017) and Krstic (2010), the first method of this paper allows us to divide the control design and the adaptive law design into two parts and construct them separately. The second method does not need to introduce parameter estimation and further simplifies the control design. Moreover, with the third strategy in Theorem 3, we can successfully construct a robust adaptive controller. (ii) A series of obstacles are encountered and being solved. In order to design the explicit unified controllers, new difficulties such as what kind of the transformations should be introduced, how to choose the dynamic gain, how to construct the L–K functionals, and how to analyze the stability of the closed-loop systems are involved in the process of control design. (iii) Compared with the existing work (Guan, 2012; Wu, 2009; Yoo et al., 2007; Zhu et al., 2015), in this paper, the restrictions on systems' powers $p_i$ are relaxed to $p_i \geq 1 (i \geq 2)$. Moreover, the nonlinear terms can be in a lower-triangular form or upper-triangular form, and the time-delay $d(t)$ needs not be known in advance. Also, the imperfection of the "explosion of complexity" problem is avoided in this paper. It can be seen that the adaptive controllers here are much simpler than those in Guan (2012), Jiao and Shen (2005) and Lv, Sun, and Xie (2015). (iv) As an application, a delayed chemical reactor system is considered to demonstrate the effectiveness of the method.

The remainder of this paper is organized as follows. Section 2 provides some preliminary results. Section 3 presents the adaptive control strategy. Section 4 gives the extension results. Section 5 shows the robust control method. Section 6 gives two simulation examples. Section 7 addresses some concluding remarks. This paper contains two appendices.

**Notations:** In this paper, $\mathcal{R}^n$ denotes the set of real $n$-component vectors. $\bar{x}_i(t) = [x_1(t), \ldots, x_i(t)]^\top \in \mathcal{R}^i$, $i = 1, \ldots, n$. A continuous function $h : \mathcal{R}^+ \to \mathcal{R}^+$ satisfying $h(0) = 0$ is called a class $\mathcal{K}_\infty$ function if it is strictly increasing and $\lim_{s\to+\infty} h(s) = +\infty$. We say $f(t) \in L_2$ if $(\int_0^\infty f^2(s)ds)^{\frac{1}{2}}$ exists. Besides, the arguments of functions (or functionals) are sometimes omitted or simplified, whenever no confusion arises from the context, for example, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or $f$. $\mathcal{R}^{\geq 1}_{\text{odd}} := \{\frac{p}{q} \mid p$ and $q$ are positive odd integers, and $p \geq q\}$. The following parameters will be used later. In detail, $\rho, \lambda, \delta, \bar{\delta}, \beta_0$, $\bar{c}_i, c_j, \gamma_j (i = 1, 2, j = 1, 2, 3)$, $\lambda_0 = \rho \cdot \min\{1, \lambda\}$, $\varrho_1 = \frac{\delta}{\lambda_0} + \frac{2\delta}{(1-\beta_0)\lambda_0}$, $\lambda_1 = \min\{\bar{c}_1, \bar{c}_2\}$, and $\varrho_2 = (\bar{\delta} + \delta_2 + 2c_3\delta_2/(1-\beta_0))/\lambda_1$ are positive constants, $\kappa_i$, $a$, $b$ are some powers of the function $H(M)$.

## 2. Preliminary results

The following definition and lemmas are to be used in this paper.

**Definition 1** (*Yin, Khoo, and Man, 2017*). A function $V \in C(\mathcal{R}^n, \mathcal{R})$ is said to be homogeneous of degree $\tau \in \mathcal{R}$ with the dilation $\Delta_\varepsilon(x_1, x_2, \ldots, x_n) = (\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \ldots, \varepsilon^{r_n}x_n)$ ($r_i, i = 1, 2, \ldots, n$ are fixed constants) if $V(\Delta_\varepsilon(x_1, x_2, \ldots, x_n)) = \varepsilon^\tau V(x_1, x_2, \ldots, x_n)$.

**Lemma 1** (*Polendo and Qian, 2007*). *For $x, y \in \mathcal{R}$, real numbers $m > 0$, $n > 0$, and a positive function $a(x, y)$, there exists a function $\mu(x, y) > 0$ such that $|a(x, y)x^m y^n| \leq \mu(x, y)|x|^{m+n} + \frac{n}{m+n} \cdot \left(\frac{m}{(m+n)\mu(x,y)}\right)^{\frac{m}{n}} |a(x, y)|^{\frac{m+n}{n}} |y|^{m+n}$.*

**Lemma 2** (*Polendo and Qian, 2007*). *For $x, y \in \mathcal{R}$, and $p \in \mathcal{R}^{\geq 1}_{\text{odd}}$, there hold $|x + y|^{1/p} \leq |x|^{1/p} + |y|^{1/p}$, $|x - y| \leq 2^{(p-1)/p}|x^p - y^p|^{1/p}$.*

**Lemma 3.** *For $q_1 \in \mathcal{R}^{\geq 1}_{\text{odd}}$, $q_2 \in \mathcal{R}^{\geq 1}_{\text{odd}}$, and $x \in \mathcal{R}$, $y \in \mathcal{R}$, there exist constants $0 < c_1 < 1$, $c_2 > 0$ such that $(x - y)^{\frac{1}{q_1}} y^{q_2} \leq -c_1 y^{\frac{1}{q_1}+q_2} + c_2 x^{\frac{1}{q_1}+q_2}$.*

**Proof.** By Lemma 2, it follows that $|(x-y)^{\frac{1}{q_1}} + y^{\frac{1}{q_1}}| = |(x-y)^{\frac{1}{q_1}} - (-y)^{\frac{1}{q_1}}| \leq 2|x|^{\frac{1}{q_1}}$. Then, by considering two cases $y \geq 0$ and $y < 0$ with Lemma 2, one can show the conclusion holds.

**Lemma 4.** *For $z_i \in \mathcal{R}$, $i = 1, 2, \ldots, n$, and a constant $0 \leq q \leq 2$ which is a ratio of an even integer to an odd integer, there exist positive constants $\varepsilon_1$ and $\varepsilon_2$ such that $\varepsilon_1\left(\sum_{i=1}^n z_i^{2-q}\right)\left(\sum_{j=1}^n z_j^q\right) \leq \sum_{i=1}^n z_i^2 \leq \varepsilon_2\left(\sum_{i=1}^n z_i^{2-q}\right)\left(\sum_{j=1}^n z_j^q\right)$.*

**Proof.** It is not difficult to get $\left(\sum_{i=1}^n z_i^{2-q}\right)\left(\sum_{j=1}^n z_j^q\right) = \sum_{i=1}^n z_i^2 + \sum_{i,j=1,i\neq j}^n \left(z_i^{2-q} z_j^q\right)$. By the definition of $q$, we see that $z_i^{2-q} \geq 0$, $z_j^q \geq 0$, $i, j = 1, 2, \ldots, n$. This leads to $\sum_{i=1}^n z_i^2 \leq \left(\sum_{i=1}^n z_i^{2-q}\right)\left(\sum_{j=1}^n z_j^q\right)$. On the other hand, by Lemma 1, one has $z_i^{2-q} z_j^q \leq \frac{2-q}{2} z_i^2 + \frac{q}{2} z_j^2$, which shows that there exists a constant $\varepsilon_0 > 0$ such that $\sum_{i,j=1,i\neq j}^n \left(z_i^{2-q} z_j^q\right) \leq \varepsilon_0 \sum_{i=1}^n z_i^2$. This implies that $(\sum_{i=1}^n z_i^{2-q})(\sum_{j=1}^n z_j^q) \leq (1 + \varepsilon_0)\sum_{i=1}^n z_i^2$. Now, let us divide both sides by the constant $1 + \varepsilon_0$, it follows that $\frac{1}{1+\varepsilon_0}\left(\sum_{i=1}^n z_i^{2-q}\right)\left(\sum_{j=1}^n z_j^q\right) \leq \sum_{i=1}^n z_i^2$. Thus, the conclusion holds with $\varepsilon_1 = \frac{1}{\varepsilon_0+1}$, $\varepsilon_2 \geq 1$.

## 3. Adaptive control of time-delay nonlinear systems

Consider the high-order nonlinear time-delay system

$$\begin{cases} \dot{x}_1 = x_2^{p_1} + f_1(\theta, x, u, x(t-d(t)), u(t-d_{n+1}(t))), \\ \dot{x}_2 = x_3^{p_2} + f_2(\theta, x, u, x(t-d(t)), u(t-d_{n+1}(t))), \\ \quad\vdots \\ \dot{x}_n = u^{p_n} + f_n(\theta, x, u, x(t-d(t)), u(t-d_{n+1}(t))), \end{cases} \tag{1}$$

where $x = (x_1, \ldots, x_n)^\top \in \mathcal{R}^n$ and $x(t - d(t)) = (x_1(t - d_1(t)), \ldots, x_n(t - d_n(t)))^\top \in \mathcal{R}^n$ are the state vector and the delayed state vector, $u \in \mathcal{R}$ and $u(t - d_{n+1}(t)) \in \mathcal{R}$ are the control input and delayed input, respectively. $\theta$ is the unknown parameter vector, $d_j(t)$, $j = 1, 2, \ldots, n+1$ are time-varying delays satisfying $0 < d_j(t) \leq d_0$, $\dot{d}_j(t) \leq \beta_0 < 1$ for unknown constants $d_0$, $\beta_0$. The system initial condition is $x(s) = \xi_0(s)$, $s \in [-d_0, 0]$ with $\xi_0(\cdot)$ being a specified continuous function. $p_i \in \mathcal{R}^{\geq 1}_{\text{odd}}$ are the system powers, $f_i : \mathcal{R}^m \times \mathcal{R}^i \times \mathcal{R}^i \to \mathcal{R}$ are unknown continuous functions.