Automatica 89 (2018) 194–200

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

# Filtering for stochastic uncertain systems with non-logarithmic sensor resolution[☆]

Junfeng Zhang[a], Xiao He[a], Donghua Zhou[b,a,*]

[a] Department of Automation, TNList, Tsinghua University, Beijing 100084, China
[b] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China

ARTICLE INFO

*Article history:*
Received 2 April 2016
Received in revised form 22 June 2017
Accepted 17 November 2017

*Keywords:*
Model uncertainty
Non-logarithmic sensor resolution
Soft measurement model
Robust filter

ABSTRACT

The sensor resolution is a most basic parameter for nearly all kinds of sensors which is so important that cannot be ignored for any signal processing problems. In this paper, the robust filtering problem is investigated for a class of stochastic systems with model uncertainty and non-logarithmic sensor resolution. A novel soft measurement model (SMM) is proposed. It has advantages of zero mean sensor resolution-induced uncertainty (SRU) and maximum signal resolution ratio (SRR). Based on the proposed model, a new robust filter (RF) is put forward which takes both model uncertainty and sensor resolution into full consideration. By designing the filter gain appropriately, the upper bound of estimation error covariance is obtained and minimized at each time step. The corresponding filtering algorithm is recursive, thus suitable for real-time online applications. Finally, a simulation study is carried out to demonstrate the effectiveness and applicability of our proposed SMM and RF.



## 1. Introduction

In the past decades, the filtering problem has attracted persistent attention from control community, communication community, signal processing community, etc. There is no doubt that the Kalman filter (KF) is a milestone in the filtering theory and the related literature has grown from a trickle to a torrent (Gustafsson & Hendeby, 2012; Karasalo & Hu, 2011; Mandic, Kanna, & Constantinides, 2015). However, the KF demands the availability of accurate system model, which is almost impossible in practice, thus affecting its real application effects. As a result, a huge amount of results have been reported on robust filtering in the literature. Among them, there are mainly four kinds of robust filters that shed insightful lights: $H_\infty$ filter (Abbaszadeh & Marquez, 2012; Abraham & Kerrigan, 2015; Borges, Oliveira, Abdallah, & Peres, 2010; Lee, Joo, & Tak, 2014), robust Kalman filter (Ahn & Truong, 2009; Li, Ding, & Liu, 2014; Mohamed & Nahavandi, 2012), generalized $H_2$ filter (Palhares & Peres, 2000; Shen, Wu, & Park, 2014; Zhang, Zhu, & Zheng, 2015), and peak-to-peak filter (Ahn, 2014; He & Liu, 2010; Li, Shi, & Karimi, 2015). Their main differences lie in disturbance assumptions and filtering objectives.

Meanwhile, with the rapid development of network technology, a large number of efforts have been devoted to the study of networked control systems (Argha, Li, Su, & Nguyen, 2016; Gupta & Chow, 2010; Trivellato & Benvenuto, 2010; Zhang, He, & Zhou, 2015). They have advantages of reduced weight, simple installation, high flexibility, and low cost. However, in the network environment, the sensors' measurements are usually required to be quantized before transmission. Therefore, the robust filtering problem for quantized systems has become a popular topic. In Gao and Chen (2007), a robust filter was designed for time-invariant systems with polytopic uncertainty and imperfect output measurements (that is measurement quantization, time delay, and packet dropout) in both quadratic and parameter-dependent frameworks. By solving a set of recursive linear matrix inequalities, Shen, Wang, Shu, and Wei (2010) developed a finite-horizon robust filter for time-varying systems with polytopic uncertainty, random nonlinearities and quantization effects. In Liu, Ho, and Niu (2010), the quantization error of the measurement output was transformed into a bounded nonlinearity. Based on this new model, a mode-dependent robust filter was presented for stochastic switching systems. Wang, Li, Yin, Guo, and Xu (2011) introduced a observer design method for multi-input-single-output time-invariant systems under irregular sampling and binary-valued

☆ This work was supported by the NSFC under Grants 61490701, 61473163, 61522309, 61733009, China Scholarship Council, Tsinghua University Initiative Scientific Research Program, and Research Fund for the Taishan Scholar Project of Shandong Province of China. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Andrea Garulli under the direction of Editor Torsten Söderström.

* Corresponding author at: College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China.
*E-mail addresses:* jf-zhang13@mails.tsinghua.edu.cn (J. Zhang), hexiao@tsinghua.edu.cn (X. He), zdh@mail.tsinghua.edu.cn (D. Zhou).

https://doi.org/10.1016/j.automatica.2017.12.005
0005-1098/

sensors. In Yang, Liu, and Shi (2012), a full-order robust filter was proposed for nonlinear systems with state-dependent disturbance. It took account of the effects of sensor saturation, measurement quantization, and packet dropout simultaneously.

In recent years, some research efforts have also been made on the robust filtering problem with quantization effects. They require to assume that the parameter uncertainty has a specific structure, like norm-bounded structure or polytopic structure. The structure and corresponding structural parameters of parameter uncertainty are also required to be known beforehand. Besides, they usually assume that the quantization is caused by quantizer and its form is logarithmic. Furthermore, the sensor resolution is one of the most basic parameters for nearly all kinds of sensors, which cannot be ignored for any signal processing problems. Although the logarithmic quantizer-induced quantization has been studied extensively, the non-logarithmic sensor resolution-induced uncertainty (SRU) has rarely been considered in the filtering community, not to mention its quantitative influence on the filtering performance.

So far, to the best of our knowledge, the robust filtering problem for stochastic linear time-varying (LTV) systems with both non-structural model uncertainty and non-logarithmic SRU has yet not been fully investigated. It is due probably to difficulties in simultaneously processing both non-structural model uncertainty and non-logarithmic SRU. Besides, the stochastic characteristic of model uncertainty and SRU adds substantial challenge to filter analysis and design, especially when the upper bound of estimation error covariance is required to be obtained and minimized at each time step. Our work presents a new robust filter framework and makes it possible to achieve favorable filtering performance using only resolution limited sensors, thus can greatly reduce the costs. The main contributions can be highlighted as follows: (1) *a novel soft measurement model (SMM) is proposed which is optimal in the sense of maximum signal resolution ratio (SRR);* (2) *a new robust filter (RF) is proposed for stochastic uncertain systems which fully utilizes the information of both model uncertainty and sensor resolution;* (3) *the upper bound of estimation error covariance is obtained and minimized iteratively with proper filter gain design;* (4) *our developed filtering algorithm is recursive, thus suitable for real-time online applications.*

**Notations**. Except where otherwise stated, the notations used throughout this paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the $n$ dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $\mathbf{I}_{n\times n}$ stands for the identity matrix with $n$ rows and $n$ columns (1 at the $(i, i)$th entry and 0 elsewhere), and $\mathbf{0}_{n\times m} \in \mathbb{R}^{n\times m}$ stands for the null matrix (0 at all entries). The scalar $x^{(i)}$ stands for the $i$th entry of the vector $\mathbf{x} \in \mathbb{R}^n$. $\mathbb{E}_S\{\mathbf{A}\}$ is the mathematical expectation of a stochastic variable $\mathbf{A}$ over the set $S$. The notation $\mathcal{S}(m, n)$ denotes the set $\{m, m+1, \ldots, n\}, (m \in \mathbb{Z}, n \in \mathbb{Z}, m \le n)$. Given a set $\mathcal{V} = \{n_1, n_2, \ldots, n_m\}, (n_1 \le n_2 \le \cdots \le n_m)$, $\text{row}_{i\in\mathcal{V}}\{\mathbf{A}_i\}$, $\text{col}_{i\in\mathcal{V}}\{\mathbf{A}_i\}$, respectively, represent the block-row matrix $\left[\mathbf{A}_{n_1}\ \mathbf{A}_{n_2}\ \cdots\ \mathbf{A}_{n_m}\right]$ and block-column matrix $\left[{\mathbf{A}_{n_1}}^{\mathrm{T}}\ {\mathbf{A}_{n_2}}^{\mathrm{T}}\ \ldots\ {\mathbf{A}_{n_m}}^{\mathrm{T}}\right]^{\mathrm{T}}$. Given a matrix $\mathbf{A} \in \mathbb{R}^{n\times m}$, $\mathbf{A}\{i_1 : i_2, j_1 : j_2\} \in \mathbb{R}^{(i_2-i_1+1)\times(j_2-j_1+1)}$ denotes the matrix which is composed of matrix $\mathbf{A}$' entries (rows $i_1$ to $i_2$, columns $j_1$ to $j_2$). row $\{\mathbf{A}, i\}$ denotes the vector $\left[a_{i1}\ a_{i2}\ \cdots\ a_{im}\right]^{\mathrm{T}}$. The notations $\mathrm{p}_{\text{row}}(\mathbf{A})$, $\boldsymbol{\mu}_{\mathbf{A}}$, and $\boldsymbol{\Sigma}_{\mathbf{A}}$ denote $\text{col}_{i\in\mathcal{S}(1,n)}\{\text{row}\{\mathbf{A}, i\}\}$, $\mathbb{E}\{\mathbf{A}\}$, and $\mathbb{E}\left\{\mathrm{p}_{\text{row}}(\mathbf{A})\,\mathrm{p}_{\text{row}}(\mathbf{A})^{\mathrm{T}}\right\}$, respectively. Scalars are in italic, and matrices are in bold. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Problem formulation

Consider a class of stochastic discrete-time LTV systems described by the following state-space model:

$$\begin{aligned}\mathbf{x}(k+1) &= \left(\mathbf{A}_c(k) + \mathbf{A}_\delta(k)\right)\mathbf{x}(k)\\ &\quad + \left(\mathbf{B}_c(k) + \mathbf{B}_\delta(k)\right)\mathbf{u}(k) + \mathbf{w}(k),\ k \in \mathbb{N},\end{aligned} \tag{1}$$

where $\mathbf{x}(k) \in \mathbb{R}^{n_\mathbf{x}}$ is the system state, $\mathbf{u}(k) \in \mathbb{R}^{n_\mathbf{u}}$ is the control input, and $\mathbf{w}(k) \in \mathbb{R}^{n_\mathbf{x}}$ is the process noise. $\mathbf{A}_c(k) \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{x}}$, $\mathbf{B}_c(k) \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{u}}$ are known deterministic process parameter matrices of appropriate dimensions. $\mathbf{A}_\delta(k) \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{x}}$, $\mathbf{B}_\delta(k) \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{u}}$ represent corresponding unknown stochastic process parameter uncertainty.

Before proceeding further, let us first give some basic definitions.

**Definition 1.** Suppose that $\mathcal{Y}$ is the measurement of real sensor belonging to the set $\mathcal{V}$, then the sensor resolution $r$ is defined as the smallest change the sensor can detect in the quantity that it is measuring, i.e., $r = \max\left\{s \,\middle|\, \frac{\mathcal{Y}}{s} \in \mathbb{Z}, \mathcal{Y} \in \mathcal{V}\right\}$. And the perfect sensor is referred to the sensor with zero resolution.

**Definition 2.** Suppose that $\mathcal{Y}$ is the measurement of real sensor, $y$ is the measurement of perfect sensor belonging to the set $S(\mathcal{Y})$, and $\Delta(\mathcal{Y})$ is the SRU, satisfying $\mathcal{Y} = y + \Delta(\mathcal{Y})$. Then the SRR is defined as

$$\text{SRR}(\mathcal{Y}) = \frac{\|y\|_2^2}{\mathbb{E}_{S(\mathcal{Y})}\left\{\|\Delta(\mathcal{Y})\|_2^2\right\}}. \tag{2}$$

**Definition 3.** Given two matrices $\mathbf{X} \in \mathbb{R}^{n\times m}$ and $\mathbf{Y} \in \mathbb{R}^{p\times q}$ whose entries are stochastic variables, we call $\mathbf{X}$ and $\mathbf{Y}$ are independent of each other if and only if their entries are independent of each other.

Assume that system (1) is monitored by $N$ different classes of sensors. The perfect measurement model (PMM) of perfect sensors is as follows:

$$\mathbf{y}_i(k) = \left(\mathbf{C}_{c,i}(k) + \mathbf{C}_{\delta,i}(k)\right)\mathbf{x}(k) + \mathbf{v}_i(k),\ i \in \mathcal{S}(1, N), \tag{3}$$

where $\mathbf{y}_i(k) \in \mathbb{R}^{n_{\mathbf{y}_i}}$ is the perfect measurement (PM) of the sensors of class $i$, and $\mathbf{v}_i(k) \in \mathbb{R}^{n_{\mathbf{y}_i}}$ is the corresponding measurement noise. $\mathbf{C}_{c,i}(k) \in \mathbb{R}^{n_{\mathbf{y}_i}\times n_\mathbf{x}}$ is the known deterministic measurement parameter matrix of the sensors of class $i$, and $\mathbf{C}_{\delta,i}(k) \in \mathbb{R}^{n_{\mathbf{y}_i}\times n_\mathbf{x}}$ represents corresponding unknown stochastic measurement parameter uncertainty.

The real measurement model (RMM) of real sensors with resolution $\mathbf{r}_i > \mathbf{0}_{n_{\mathbf{y}_i}\times 1}$ is as follows:

$$\begin{aligned}&\boldsymbol{\mathcal{Y}}_i(k) = \left(\mathbf{C}_{c,i}(k) + \mathbf{C}_{\delta,i}(k)\right)\mathbf{x}(k) + \mathbf{v}_i(k) + \boldsymbol{\Delta}\left(\boldsymbol{\mathcal{Y}}_i(k)\right),\\ &\frac{\mathcal{Y}_i(k)^{(j)}}{r_i^{(j)}} \in \mathbb{Z}, j \in \mathcal{S}\left(1, n_{\mathbf{y}_i}\right),\end{aligned} \tag{4}$$

where $\boldsymbol{\mathcal{Y}}_i(k)$ is the real measurement (RM) of the sensors of class $i$, and $\boldsymbol{\Delta}\left(\boldsymbol{\mathcal{Y}}_i(k)\right)$ is the corresponding SRU. The measurement noise and SRU are two different types of uncertainty with completely different characteristics. The superscript $(j)$ means the $j$th sensor in the $i$th class, and the total number of sensors is $\sum_{i=1}^{N} n_{\mathbf{y}_i}$.

The following assumptions are made throughout this paper.

**Assumption 1.** The initial state $\mathbf{x}(0)$ has the mean $\bar{\mathbf{x}}_0$, covariance $\mathbf{P}_0$, and second moment $\boldsymbol{\Sigma}_0$. The noise $\mathbf{w}(k)$ and $\mathbf{v}_i(k)$ are