Automatica 87 (2018) 345–357

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Funnel control for nonlinear systems with known strict relative degree☆

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ARTICLE INFO

Article history:
Received 12 December 2016
Received in revised form 4 September 2017
Accepted 12 September 2017
Available online 7 November 2017

Keywords:
Nonlinear systems
Relative degree
Adaptive control
Model-free control
Funnel control

ABSTRACT

We consider tracking control for uncertain nonlinear multi-input, multi-output systems which have arbitrary strict relative degree and input-to-state stable internal dynamics. For a given sufficiently smooth reference signal, our aim is to design a low-complexity model-free controller which achieves that the tracking error evolves within a prespecified performance funnel. To this end, we introduce a new controller which involves the first $r-1$ derivatives of the tracking error, where r is the strict relative degree of the system. We derive an explicit bound for the resulting input and discuss the influence of the controller parameters. We further present some simulations where our funnel controller is applied to a mechanical system with higher relative degree and a two-input, two-output robot manipulator. The controller is also compared with other approaches.

1. Introduction

In the present article we consider output trajectory tracking for uncertain nonlinear systems by funnel control. We assume knowledge of the strict relative degree of the system and that the internal dynamics are, in a certain sense, input-to-state stable, resembling the concept introduced by Sontag (1989). The concept of funnel control has been developed in Ilchmann, Ryan, and Sangwin (2002) for systems with relative degree one, see also the survey (Ilchmann & Ryan, 2008) and the references therein. The funnel controller is a low-complexity model-free output-error feedback of high-gain type; it is an adaptive controller since the gain is adapted to the actual needed value by a time-varying (non-dynamic) adaptation scheme.[1] Note that no asymptotic tracking is pursued, but a prescribed tracking performance is guaranteed over the whole time interval. Controllers of high-gain type have various advantages when it comes to "real world" applications; we like to quote from Hackl (2011):

> "Since only structural assumptions on the system are required, high-gain adaptive control is inherently robust and makes it attractive for industrial application".

In particular, the funnel controller proved to be the appropriate tool for tracking problems in various applications, such as temperature control of chemical reactor models (Ilchmann & Trenn, 2004), control of industrial servo-systems (Hackl, 2017; Hackl, Hopfe, Ilchmann, Mueller, & Trenn, 2013; Ilchmann & Schuster, 2009) and rigid, revolute joint robotic manipulators (Hackl, 2017; Hackl & Kennel, 2012), speed control of wind turbine systems (Hackl, 2014, 2015b, 2017), current control for synchronous machines (Hackl, 2015a, 2017), DC-link power flow control (Senfelds & Paugurs, 2014), voltage and current control of electrical circuits (Berger & Reis, 2014), oxygenation control during artificial ventilation therapy (Pomprapa, Alfocea, Göbel, Misgeld, & Leonhardt, 2014) and control of peak inspiratory pressure (Pomprapa, Weyer, Leonhardt, Walter, & Misgeld, 2015).

A longstanding open problem in high-gain adaptive control is the treatment of systems with relative degree larger than one, see (Ilchmann, 1991; Ilchmann & Ryan, 2008; Morse, 1996). In Bechlioulis and Rovithakis (2014), a "Prescribed Performance Controller" for systems with higher strict relative degree has been introduced (and in Theodorakopoulos and Rovithakis (2016) the influence of disturbances is discussed), however trivial internal dynamics are assumed. In Bullinger and Allgöwer (2005), an adaptive λ-tracker is introduced which achieves tracking with prescribed asymptotic accuracy $\lambda > 0$ for a class of systems which are affine in the control, of known relative degree, and with affine linearly bounded drift term. However, the drawback of this controller is

☆ We acknowledge support by the German Research Foundation DFG via the grant BE 6263/1-1 "Adaptive Control of Nonlinear Differential-Algebraic Systems in Multibody Dynamics". The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Nathan van de Wouw under the direction of Editor Daniel Liberzon.

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[1] Note that often only controllers with dynamic gain adaptation are viewed as adaptive controllers of high-gain type.

https://doi.org/10.1016/j.automatica.2017.10.017
0005-1098/

that the transient behavior of the tracking error cannot be influenced. Ilchmann, Ryan, and Townsend (2006) and Ilchmann, Ryan, and Townsend (2007) developed a funnel controller for systems with higher strict relative degree by introducing a "backstepping" procedure in conjunction with a precompensator. This controller achieves tracking with prescribed transient behavior for a large class of systems governed by nonlinear (functional) differential equations. Unfortunately, this backstepping procedure is quite impractical, especially since it involves high powers of a gain function which typically takes very large values, cf. Hackl (2012), Sec. 4.4.3. Backstepping is also used for an adaptive λ-tracker in an earlier work by Ye (1999).

For systems with relative degree two, a proportional–derivative (PD) funnel controller has been introduced in Hackl et al. (2013) (see also the modification in Hackl, 2011), where the backstepping procedure is avoided. The only available generalization of this approach to systems with higher relative degree is the bang–bang funnel controller introduced by Liberzon and Trenn (2013). However, this controller is restricted to single-input, single-output (SISO) systems and the involved compatibility conditions on the funnel boundaries, the safety distances and the settling times are quite complicated. Nevertheless, we stress that the controllers in Hackl (2011), Hackl et al. (2013) and Liberzon and Trenn (2013) are able to achieve prescribed transient behavior of the tracking error derivatives.

In the recent conference paper (Chowdhury & Khalil, 2017) the funnel controller from Ilchmann et al. (2002) is used for SISO systems with higher relative degree by defining a virtual (weighted) output such that the system has relative degree one with respect to this virtual output. Then funnel control is feasible and it is shown that (ignoring the additional use of a high-gain observer) for sufficiently small weighting parameter in the virtual output, the original tracking error evolves in a prescribed performance funnel. However, tuning of the weighting parameter has to be done a posteriori and hence depends on the system parameters and the chosen reference trajectory. Therefore, this approach is not model-free like standard funnel control approaches and the controller is not robust, since small perturbations of the reference signal may cause the tracking error to leave the performance funnel.

In the present paper we introduce a simple funnel controller for systems with arbitrary known relative degree r and (in a suitable sense) input-to-state stable internal dynamics. The controller is based on a simple recursion law and involves the first $r-1$ derivatives of the tracking error.

1.1. Nomenclature

$\mathbb{R}_{\geq 0}$	$:= [0, \infty)$
$\|x\|$	the Euclidean norm of $x \in \mathbb{R}^n$
$\mathcal{L}^\infty_{\text{loc}}(I \to \mathbb{R}^n)$	the set of locally essentially bounded functions $f : I \to \mathbb{R}^n$, $I \subseteq \mathbb{R}$ an interval
$\mathcal{L}^\infty(I \to \mathbb{R}^n)$	the set of essentially bounded functions $f : I \to \mathbb{R}^n$ with norm
$\|f\|_\infty$	$:= \operatorname{ess\,sup}_{t\in I} \|f(t)\|$
$\mathcal{W}^{k,\infty}(I \to \mathbb{R}^n)$	the set of k-times weakly differentiable functions $f : I \to \mathbb{R}^n$ such that $f, \dots, f^{(k)} \in \mathcal{L}^\infty(I \to \mathbb{R}^n)$
$\mathcal{C}^k(V \to \mathbb{R}^n)$	the set of k-times continuously differentiable functions $f : V \to \mathbb{R}^n$, $V \subseteq \mathbb{R}^m$; $\mathcal{C}(V \to \mathbb{R}^n) = \mathcal{C}^0(V \to \mathbb{R}^n)$
$f\|_W$	restriction of the function $f : V \to \mathbb{R}^n$ to $W \subseteq V$

1.2. System class

In the present paper we consider a class of non-linear systems described by functional differential equations of the form

$$
\begin{aligned}
y^{(r)}(t) &= f\big(d(t), T(y, \dot{y}, \dots, y^{(r-1)})(t)\big) \\
&\quad + \Gamma\big(d(t), T(y, \dot{y}, \dots, y^{(r-1)})(t)\big)\, u(t) \\
y|_{[-h,0]} &= y^0 \in \mathcal{W}^{r-1,\infty}([-h, 0] \to \mathbb{R}^m),
\end{aligned} \tag{1}
$$

where $h > 0$ is the "memory" of the system, $r \in \mathbb{N}$ is the strict relative degree, and

(P1): the "disturbance" satisfies $d \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0} \to \mathbb{R}^p)$, $p \in \mathbb{N}$;
(P2): $f \in \mathcal{C}(\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^m)$, $q \in \mathbb{N}$,
(P3): the "high-frequency gain matrix function" $\Gamma \in \mathcal{C}(\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^{m\times m})$ takes values in the set of positive (negative) definite matrices[2];
(P4): $T : \mathcal{C}([-h, \infty) \to \mathbb{R}^{rm}) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_{\geq 0} \to \mathbb{R}^q)$ is an operator with the following properties:

(a) T maps bounded trajectories to bounded trajectories, i.e., for all $c_1 > 0$, there exists $c_2 > 0$ such that for all $\zeta \in \mathcal{C}([-h, \infty) \to \mathbb{R}^{rm})$,

$$\sup_{t\in[-h,\infty)} \|\zeta(t)\| \leq c_1 \Rightarrow \sup_{t\in[0,\infty)} \|T(\zeta)(t)\| \leq c_2,$$

(b) T is causal, i.e., for all $t \geq 0$ and all $\zeta, \xi \in \mathcal{C}([-h, \infty) \to \mathbb{R}^{rm})$,

$$\zeta|_{[-h,t)} = \xi|_{[-h,t)} \Rightarrow T(\zeta)|_{[0,t)} \overset{\text{a.a.}}{=} T(\xi)|_{[0,t)},$$

where "a.a." stands for "almost all".

(c) T is locally Lipschitz continuous in the following sense: for all $t \geq 0$ there exist $\tau, \delta, c > 0$ such that for all $\zeta, \Delta\zeta \in \mathcal{C}([-h, \infty) \to \mathbb{R}^{rm})$ with $\Delta\zeta|_{[-h,t)} = 0$ and $\|\Delta\zeta|_{[t,t+\tau]}\|_\infty < \delta$ we have

$$\left\| \big(T(\zeta + \Delta\zeta) - T(\zeta)\big)|_{[t,t+\tau]} \right\|_\infty \leq c \|\Delta\zeta|_{[t,t+\tau]}\|_\infty.$$

The functions $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ and $y : [-h, \infty) \to \mathbb{R}^m$ are called *input* and *output* of the system (1), resp. Systems similar to (1) have been studied e.g. in Hackl et al. (2013), Ilchmann et al. (2002), Ilchmann et al. (2007) and Ilchmann and Ryan (2009). In the aforementioned references it is shown that the class of systems (1) encompasses linear and nonlinear systems with strict relative degree and input-to-state stable internal dynamics (zero dynamics in the linear case) and the operator T allows for infinite-dimensional linear systems, systems with hysteretic effects or nonlinear delay elements, and combinations thereof. Note that the operator T is usually the solution operator of the differential equation describing the internal dynamics of the system and its property (P4a) thus amounts to the input-to-state stability of the internal dynamics. One important subclass of systems (1) are minimum-phase linear time-invariant systems

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t), \quad x(0) = x^0 \in \mathbb{R}^n \\
y(t) &= Cx(t),
\end{aligned} \tag{2}
$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, which have *strict relative degree* $r \in \mathbb{N}$ *and positive (negative) definite high-frequency gain matrix*, i.e., $CB = CAB = \cdots = CA^{r-2}B = 0$ and $\Gamma := CA^{r-1}B \in \mathbb{R}^{m\times m}$ is positive (negative) definite. The minimum-phase assumption (equivalently, asymptotic stability of the zero dynamics, see Ilchmann and Wirth (2013) is characterized by the condition

$$\forall\, \lambda \in \mathbb{C} \text{ with } \operatorname{Re} \lambda \geq 0 : \ \det \begin{bmatrix} \lambda I_n - A & B \\ C & 0 \end{bmatrix} \neq 0.$$

It is known that systems of this type can be transformed into *Byrnes-Isidori normal form*, see (Ilchmann et al., 2007),

$$
\begin{aligned}
y^{(r)}(t) &= \sum_{i=1}^{r} R_i y^{(i-1)}(t) + S\eta(t) + \Gamma u(t), \ \ y(0) = Cx^0 \\
\dot{\eta}(t) &= Py(t) + Q\eta(t), \qquad\qquad \eta(0) = \eta^0 \in \mathbb{R}^{n-rm}
\end{aligned}
$$

where $R_i \in \mathbb{R}^{m\times m}$ for $i = 1, 2, \dots, r$, $S^\top, P \in \mathbb{R}^{(n-rm)\times m}$, and $Q \in \mathbb{R}^{(n-rm)\times(n-rm)}$ is a Hurwitz matrix, i.e., all eigenvalues of Q have

[2] One may wonder why Γ is not assumed to be uniformly bounded away from zero. The reason is that in the closed-loop system this is established anyway due to the boundedness of the involved signals.