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Intrinsic tetrahedron formation of reduced attitude[☆]

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ABSTRACT

In this paper, formation control for reduced attitude is studied, in which a regular tetrahedron formation can be achieved and shown to be asymptotically stable under a large family of gain functions in the control. Moreover, by further restriction on the control gain, almost global stability of the desired formation is obtained. In addition, the control proposed is an intrinsic protocol that only uses relative information and does not need to contain any information of the desired formation beforehand. The constructed formation pattern is totally attributed to the geometric properties of the space and the designed inter-agent connection topology. Besides, a novel coordinates transformation is proposed to represent the relative reduced attitudes in $\mathcal{S}^2$, which is shown to be an efficient approach to reduced attitude formation problems.

1. Introduction

The formation control problem has attracted considerable attention in the last decades. This trend is not only inspired by a significant amount of similar phenomenon presented in bio-communities, but also motivated by the theoretical challenges posed and its practical potential in various applications, such as formation flying (Ren & Beard, 2004; Scharf, Hadaegh, & Ploen, 2004), target encircling (Chen, Ren, & Cao, 2010), terrestrial or oceanographic exploration (Egerstedt & Hu, 2001), and collaborative surveillance (Tron & Vidal, 2009; Wang, 2013). Among such problems studied, attitude formation is an important one.

Rigid-body attitude control has been widely studied for a long history (Morin, Samson, Pomet, & Jiang, 1995; Wen & Kreutz-Delgado, 1991), and utilized in many engineering applications, such as the control of atmospheric aircrafts, earth-synchronous satellites, and robotic manipulators. In general, the attitude of an unconstrained rigid-body has three degrees of freedom, which can be represented globally and uniquely by a rotation matrix evolving in Lie group $\mathcal{SO}(3)$. However, in some scenarios, not all three degrees of freedom are relevant to the problem. For example, in the control of field-of-view for cameras, orientation of the solar panel or antenna for satellites, and thrust vector for quad-rotor aircrafts, we only consider the pointing direction of a body-fixed axis, and any rotation about this axis is then irrelevant. Motivated by these applications, reduced attitude control problems arise (see Bullo, Murray, & Sarti, 1995; Chaturvedi, Sanyal, & McClamroch, 2011 and references therein), in which reduced attitudes evolve in the 2-sphere $\mathcal{S}^2$.

As the configuration space of (reduced) attitudes is a compact manifold, some topological obstructions emerge in attitude control. In order to refrain from working directly on such a manifold, various attitude parameterizations are applied in attitude control studies, such as Euler angles (Ren, 2007; Shuster, 1993), quaternions (Lawton & Beard, 2002; Ren, 2006), Rodrigues parameters (Zou, Kumar, Krishna, & Hou, 2012). However, unfortunately, no parameterizations (Shuster, 1993) can represent the attitude space both globally and uniquely. In addition to this representation barrier, as both product manifolds $\mathcal{SO}(3)^n$ and $(\mathcal{S}^2)^n$ are compact smooth manifolds without boundary (Bhat & Bernstein, 2000), under any time-invariant continuous feedback controllers, there exists no equilibrium of closed-loop system that is globally asymptotically stable. Thus, almost global stabilization becomes the best possible result for attitude control problems.

Following many significant results on reduced attitude control for a single rigid body (Bullo et al., 1995; Chaturvedi et al., 2011), recently many results have been obtained on cooperative attitude synchronization of multiple rigid bodies. As studies for single rigid-body, most of coordination control for multiple attitudes are based

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on some parametrization (Lawton & Beard, 2002; Paley, 2009; Ren, 2010; Zou, Ruiter, & Kumar, 2016). In order to avoid topological singularities in some certain points, Sarlette, Sepulchre, and Leonard (2009) study consensus control directly in attitude space, however the control proposed only provides local stability of the consensus manifold. Then Tron, Afsari, and Vidal (2012) solve this problem, and achieve consensus with almost global convergence. Compared to consensus, formation control is more general and also more involved. Song, Markdahl, Hu, and Hong (2015) investigate formation control for reduced attitudes with ring inter-agent graph, and show that for different parity of the number of agents, antipodal and cyclic formation are almost globally stable respectively.

As a three-dimensional configuration, tetrahedron formations have attracted a considerable interest in numerous applications and many ongoing projects (Curtis, 1999; Escoubet, Schmidt, & Goldstein, 1997; Hughes, 2008). One of the reasons is that a tetrahedron formation consisting of four sensors is the minimum needed to resolve a physical field gradients in space (Daly, 1998), and especially a regular tetrahedron provides the maximal observational effectiveness (Roscoeand, Vadali, Alfriendand, & Desai, 2013). In the Magnetospheric Multiscale (MMS) mission launched by NASA (Curtis, 1999; Hughes, 2008), in order to build a three-dimensional model of the electric and magnetic fields of the Earth's magnetosphere, four satellites need to construct a regular tetrahedron formation near the apogee of a reference orbit. Another example using the tetrahedron configuration is the Cluster mission developed by European Space Agency (Escoubet et al., 1997).

In this paper, a continuous control law is proposed for reduced attitudes problems, by which a regular tetrahedron formation can achieve asymptotic stability under a quite large family of gain functions in the control. With a further restriction on the control gain, almost global stability of the tetrahedron formation is also obtained. To this end, we introduce a novel coordinates transformation that represents the relative reduced attitudes between the agents. Although involved, it is shown to be an efficient approach to reduced attitude formation problems. Moreover, in contrast to most existing methodologies on formation control (Lawton & Beard, 2002; Mou, He, & Zhang, 2015; Ren, 2010), the proposed method does not need to have the formation errors in the protocol. The desired formation is constructed based on the geometric properties of the manifold $\mathcal{S}^2$ and the designed connection topology. We referred to this type of formation control as intrinsic formation control. Another virtue of the control proposed is that only relative attitude measurement is required. Although only reduced attitudes are considered in the paper, similar design techniques can also be applied to the formation control of point mass systems (Zhang, Song, He, Yao, & Hu, 2016).

The rest of the paper is organized as follows: in Section 2, the necessary preliminaries and notions are introduced. Section 3 proposes the intrinsic formation problem addressed in the paper. In Section 4, the regular tetrahedron formation is rendered almost globally asymptotically stable. Following that, an illustrative example is provided in Section 5, and the conclusions are given in Section 6.

2. Notation and preliminary

In this paper, we model the inter-agent connectivity as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, n\}$ is the set of nodes, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. A graph $\mathcal{G}$ is referred to as an undirected graph if $(j, i) \in \mathcal{E}$, for every $(i, j) \in \mathcal{E}$, otherwise $\mathcal{G}$ is directed. We also define the neighbor set of node i as $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$, and we say j is a neighbor of i, if $j \in \mathcal{N}_i$.

2.1. Attitude and reduced attitude

In the three-dimensional Euclidean space, the (full) attitude of a rigid body i can be determined by a length-preserving linear transformation between coordinates frame $\mathcal{F}_i$ and $\mathcal{F}_w$, where $\mathcal{F}_i$ is the instantaneous body frame of agent i and $\mathcal{F}_w$ presents an inertial reference frame. This transformation is specified by a *rotation matrix* $R_i \in \mathbb{R}^{3\times 3}$, and its columns are, respectively, coordinates of three orthogonal unit basis of frame $\mathcal{F}_i$ resolved to frame $\mathcal{F}_w$. Furthermore, all rotation matrices together form the special orthogonal group $\mathcal{SO}(3) = \{R \in \mathbb{R}^{3\times 3} : R^T R = I, det(R) = 1\}$, under the operation of matrix multiplication.

In reduced attitude applications, we only consider the pointing direction of a body-fixed axis, and ignore the rotations about this axis. Let $\boldsymbol{b}_i \in \mathcal{S}^2$ denote the coordinates of i's pointing axis relative to the body frame $\mathcal{F}_i$, where $\mathcal{S}^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ is the 2-sphere. Then i's pointing axis coordinates resolved in frame $\mathcal{F}_w$ are $\Gamma_i = R_i \boldsymbol{b}_i$. Since $R_i \in \mathcal{SO}(3)$, we still have $\Gamma_i \in \mathcal{S}^2$. This vector Γ_i is referred to as the *reduced attitude* of rigid body i, on account of the neglect of the rotations about one axis in pointing applications.

The kinematics of the reduced attitude Γ_i is governed by Song et al. (2015)

$$\dot{\Gamma}_i = \widehat{\omega}_i \Gamma_i, \tag{1}$$

where $\omega_i \in \mathbb{R}^3$ is i's angular velocity relative to the inertial frame $\mathcal{F}_w$, and hat operator $\widehat{(\cdot)}$ satisfies, for any $x = [x_1, x_2, x_3]^T \in \mathbb{R}^3$,

$$\widehat{x} := \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}. \tag{2}$$

For any two points $\Gamma_i, \Gamma_j \in \mathcal{S}^2$, we define $\theta_{ij} \in [0, \pi]$ and $k_{ij} \in \mathcal{S}^2$ as

$$\theta_{ij} = \arccos(\Gamma_i^T \Gamma_j), \quad k_{ij} = \frac{\widehat{\Gamma_i}\Gamma_j}{\sin(\theta_{ij})}.$$

In the definition of k_{ij}, we stipulate k_{ij} to be any unit vector orthogonal to Γ_i, when $\theta_{ij} = 0$ or π.

Note that θ_{ij} is also the geodesic distance between Γ_i and Γ_j in $\mathcal{S}^2$, and $\Gamma_j = exp(\theta_{ij}\widehat{k_{ij}})\Gamma_i$. The relationship among three reduced attitudes follows from the spherical cosine formula (Todhunter & Leathem, 1914):

Lemma 2.1. *For any three reduced attitudes $\Gamma_i, \Gamma_j, \Gamma_k \in \mathcal{S}^2$, the following relationship will always hold:*

$$\cos(\theta_{ij}) = \cos(\theta_{ik})\cos(\theta_{jk}) + \sin(\theta_{ik})\sin(\theta_{jk})k_{ik}^T k_{jk}.$$

In this paper, we will also use a frequently mentioned parametrization of Γ_i based on RPY angles system (Sciavicco & Siciliano, 2012),

$$\Gamma_i = \begin{pmatrix} \cos(\psi_i)\cos(\phi_i) \\ \sin(\psi_i)\cos(\phi_i) \\ \sin(\phi_i) \end{pmatrix} \tag{3}$$

where $\psi_i \in [-\pi, \pi)$, $\phi_i \in [-\pi/2, \pi/2]$.

2.2. Several results on matrices

In this part, we list several results used in the paper on matrices.

Definition 2.2. $A, B \in \mathbb{C}^{n\times n}$, we say that A and B can be *simultaneously diagonalized* if there exists a nonsingular matrix $P \in \mathbb{C}^{n\times n}$ such that

$$P^{-1}AP = D_1, \; P^{-1}BP = D_2,$$

where D_1 and D_2 are both diagonal matrices.